WEIGHT AND SIGNIFICANCE OF OBJECT DYNAMIC MODELS IN CONSTRUCTING MONITORING SYSTEMS

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Summary
The author's approach to tasks of constructing monitoring systems of the state changes of machines, devices and structures is presented in the paper. The weight and significance of the identification process of the monitored object dynamic model – as a factor which in an universal and coherent way preserves the realisation process of several tasks assigned to this system operations – is emphasised. Purposefulness of applying methods of the technical stability testing for estimation of dynamic behaviours of the monitored object, was indicated. Its usefulness for the selection of the proper monitoring symptoms, for determining their criterion values and for the estimation of their exceeding the permissible values - are shown in the paper.

Keywords: monitoring of machine technical state, the algorithms of monitoring procedures, modelling of dynamic state, stability, signal processing and analysis.

ROLA I ZNACZENIE MODELI DYNAMICZNYCH OBIEKTU
W BUDOWIE SYSTEMÓW MONITORUJĄCYCH

Streszczenie
W artykule zaproponowano autorskie podejście do zadań konstruowania systemów monitorujących zmiany stanów maszyn, urządzeń, konstrukcji. Zwrócono uwagę na rolę i znaczenie procesu identyfikacji modelu dynamicznego monitorowanego obiektu, jako czynnika, który w sposób uniwersalny i spójny zabezpiecza proces wykonawczy szeregu zadań przypisanych funkcjonowaniu tych systemów. W szczególności, wskazano na celowość zaangażowanie metod badania stateczności technicznej do oceny zachowań dynamicznych kontrolowanego obiektu. Pokazano jej użyteczność dla procesu realizacji wyboru właściwych w procesie monitorowania symptomów, określenia dla nich wartości kryterialnych, jak i tworzenia procedur ocen wyjścia ich poza dopuszczalne granice.

Słowa kluczowe: monitoring stanu technicznego, algorytmy procesu monitorującego, modelowanie stanu dynamicznego, stateczność, analiza i przetwarzanie sygnałów

1. INTRODUCTION

Monitoring of state changes of machines and devices – corresponding to the recognition of early stages of defects and identification of disturbances in their operations – is a complex research task. In practice, it is realised the most often via measuring and tracing dynamic changes of processes occurring in the object under inspection. This is done by the properly selected sensors in the monitoring system.

The construction process is related to:

- Selection of measuring rules and recording of the determined process variables,
- Selection of the conversion methods for recognition of causes of the observed states,
- Assuming relevant analytical algorithms of their trends,
- Establishing criterion references enabling the proper classification of the recognised states.

Procedures of filtration of measurement disturbances, algorithms of change predictions of controlled signals or expert systems allowing for a certain automation of the diagnostic changes occurring in the monitored machine - are more and more often implemented in solutions. Their realisations are based on various model formalisms determining their functioning as well as on the knowledge collected during their operations. The proper selection of algorithms in the construction process is decisive for the universality and functionality of the monitoring system.

Currently, in the collection of construction rules of the monitoring systems available in the market, there is none coherent theory enabling the proper connection of conditions of loosing the ability to safe operations with the rules of selecting diagnostic symptoms. Constructional and exploitational features of the inspected object corresponding to the requirements of undisturbed estimation of the monitored processes and prediction processes with a determined time advance - are not sufficiently taken into account in the construction of monitoring systems. Trials of looking for solutions, in which the essence of operating of the monitoring system constitute relationships between the state of the inspected system and the form of the monitored signal [3, 4, 7] conjugated with the criteria selection for the alarm signals [5, 6] or for switching off - are not numerous.

Thus, it seems that an important realisation approach to the construction of monitoring systems is its coupling with the identification of the dynamic model of the inspected object and with the analysis of its technical stability [11]. Such approach ensures the possibility of undertaking and solving several tasks in the building process of the monitoring system. The frame of constructional operations related to the building of the monitoring system, it means related to the selection of proper diagnostic symptoms, of the levels of their quantification for the purpose of diagnostics decisions or the requirements of the filtration of measuring disturbances – can be based on them.

The aim of this paper is to indicate certain possibilities within this scope. It does not pretend to show all results of the author researching these problems, but aims to indicate the existence of the universal platform giving the possibility of undertaking and solving several tasks in constructing monitoring systems – referring to the common modelling formalisation.

2. OBJECT MODELLING – THE BASE FOR CONSTRUCTING MONITORING SYSTEM

Several tasks determining functioning of monitoring systems, can be reduced to the problem of analysis the possible trajectories of the inspected dynamic system under the influence of disturbances in the exploitation process, caused by undesirable exploitation inputs (loads and external disturbances), or by changes of structural parameters (it means: mass, elastic and damping parameters) due to failures.

They determine the possible solutions of the set of differential equations modelling state changes in the monitored object and related to them the estimation of changes of the monitored dynamic processes.

Their forms result from the selected model formalisation of the inspected object, it means, from the description of the set of elements being in a mutual cooperation with the surroundings, which is characterised by a set of measurable features (containing the total information on the state of the inspected object), which are changing with time.

Generally the model describing the cooperation can be represented by a pair (**X** , **f**), in which **X** denotes the vector of state space, while ***f*** is a vector of this space representation:

$$\dot{x}_i = f_i\,(t, x_1, x_2, \ldots, x_n)\ , \quad i=1,2,\ldots,n \qquad (1)$$

in which functions f_i are determined in a space:

$$t \geq 0, \qquad (x_1, \ldots, x_n) \in G \subset E_n$$

where E_n denotes the linearly normalised n-dimensional space.

When using the identified model of the monitored object it is possible to recognise theoretically its behaviour under the influence of disturbances. The deviations of the inspected processes versus the programmed ones and caused either by parameter changes or by variability of inputs acting on the object - are classified.

Taking into account their model connections with the measuring observations the Kalman [15] model formalism can be assigned to them, ensuring the realisation process of the optimal filtration and prediction tasks of the monitored diagnostic signals.

The identified model of the inspected object given by Equation (1), under conditions of acting random disturbances, allows - in addition - the estimation of the time when the analysed operation enters into zones assigned to alarm states [19, 20]. The assigned for them analyses can be related to the existing analytical methods of the dynamic systems stochastic trajectories, including solutions based on the method of *„functionals supporting the obtainable zone„*. The analysis of the monitoring process of the state changes of the hydrodynamic bearing [1, 2] being the part of the dynamic system: '*Rotor – Bearings – Supports – Foundation'* can constitute a good example.

The model describing dynamic behaviours of the object, when its steady state is disturbed, can become the basic tool at constructing the monitoring system. It generates indications concerning the symptoms selection for the observation of the object state changes as well as the selection criterion values protecting the classification of undesirable state changes. Realisation of this operations results from analysis of dynamic behaviours of the object done when testing its stability. Various understanding of the stability notion and methods of its testing properly related to the required functional properties of the monitored object – can be applied [11, 13, 14].

The criterion of the technical stability [11] can be a good testing criterion for realisation of several tasks, which appear in processes of constructing monitoring systems. It widens the notation of stability in the Lagrange's and Lapunow's meaning into conditions, which can appear in the investigated technical reality. They seem to be especially essential for selecting the method of the monitored state changes realisation in the inspected object. It takes into account the quantification of motion disturbances characteristic for the tasks assigned by the monitoring system. They are – from the point of view of the correctly functioning inspected object - conditioned by the assumed permissible perturbations of input forces, or object characteristics as well as by the permissible changes of initial conditions – related to the acceptable changes of the structure parameters of the monitored object.

In the case when the solutions of the technical stability theory are applied for the realisation the monitoring of the object state it is necessary to assume:

- Permissible deviations of the motion trajectory from the steady state (related to the safe exploitation of the analysed object);
- Permissible range of the initial conditions changes;
- Levels of expected external and internal disturbances permanently acting on the inspected object, which dynamic behaviours are described by the equation:

$$\dot{x} = f(x,t) + R(x,t) \qquad (2)$$

in which **x**, **f**, **U** are vectors determined in space $\Re^n$:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} \quad \boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{bmatrix} \quad \boldsymbol{R} = \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_n \end{bmatrix} \qquad (3)$$

and their functions *f(t,x) R (t,x)* are determined in the zone contained in the $(n+1)$ dimensional space:

$$t > 0,\ x \subset G \subset E_n \qquad (4)$$

It is assumed that input functions of permanently acting disturbances *R (t,x)* in the zone (4) is limited:

$$\| \boldsymbol{R}(t,x) \| < \delta \qquad (5)$$

where δ is a positive number, while $\| \bullet \|$ denotes the Euklides norm of vector ***R(t,x)***.

At this type of model dynamic behaviour the notion of technical stability for the system modelled is understood as follows:

Let there be two zones Ω and ω - contained in *G*. Zone Ω is closed, bounded and contains the origin of the system, while ω is open and contained within Ω.

Let us assume, that the solution of the analysed system (2) is $x(t)$, of an initial condition: $x(t_0) = x_0$.

If for every x_0 belonging to ω, ***x(t)*** remains in zone Ω for $t \geq t_0$, at the disturbance function satisfying inequality (5), the system (2) is *technically stable* due to zones ω, and the limited, constantly acting disturbances (5).

According to this definition of the technical stability, each motion trajectory, which exits zone ω, has to remain in zone Ω for $t \geq t_0$.

For the monitored objects, for which instantaneous exits of the monitored signals outside levels considered permissible - are safe, the notion of the technical stability can be weakened into the condition, in which each trajectory of motion which exits zone ω has to remain in zone Ω for $t_0 \leq t \leq T_0$, where $T - t_0$ is a time of motion. At such condition we are dealing with the ***technical stability in a finite time***.

Realisation of the monitoring process of changes of the inspected object state can be also looked for on the grounds of the ***stochastic technical stability*** definition introduced by W. Bogusz [11].

It is formulated as follows, in reference to the object described by the model:

$$\dot{x} = f(x, t, \xi(t,\alpha)) \qquad (6)$$

This model takes into account random disturbances α, acting on it, in a form of a certain stochastic process $\xi(t,\alpha)$ of the following features:

$$\xi(t,\alpha) = \sup_{x \in G} |\boldsymbol{R}(t, x, \alpha)| \qquad (7)$$

In reference to such conditions of functioning of the inspected object, two zones Ω and ω contained in E_n are defined, such that zone ω is limited and open, Ω is limited, closed and contains origin of coordinates and also takes into account the condition $\omega \subset \Omega$. Let us assume that the positive number ε satisfying inequality $0 < \varepsilon < 1$ exists. Initial conditions for $t = t_0$ are $x(t_0) = x_0$, while the solution assigned to them: $x(t, t_0, x_0)$. If each solution $x(t, t_0, x_0)$ of initial conditions belonging to zone ω belongs also to zone Ω with the probability higher than $1-\varepsilon$, the analysed object is technically, stochastically stable it means: ***stochastically stabilised*** versus zones ω, Ω and process $\xi(t)$ with the probability $1-\varepsilon$:

$$P\{ \boldsymbol{x}(t, t_0, \boldsymbol{x}_0) \in \Omega \} > 1 - \varepsilon \quad \text{for } \boldsymbol{x}_0 \in \omega \qquad (8)$$

The presented above mathematical formalisation of the problem of monitoring state changes of the object described by model (2), supplemented by relevant statements facilitating technical stability testing, can be used as the approach methodology to constructing monitoring system. Thus, it forms the logical frames for solving problems occurring at searching for the proper constructions of monitoring systems.

3. METHODS OF TESTING THE TECHNICAL STABILITY – TOOLS IN CONSTRUCTING MONITORING SYSTEMS

Realisation of the idea of utilising solutions of the technical stability theory in the process of constructing monitoring systems [5, 6] means introducing – into their solutions – algorithms of testing conditions of loosing the technical stability by the inspected object. This requires the identification of the dynamic model of the monitored object, being the grounds for comparative references of the observed states. Algorithms of testing the technical stability enable their classification. The main determinants of the monitoring process are identifications of the dynamic model of the inspected object. The identification process should be conjugated with the selected methods and algorithms of testing the technical stability, which can become the useful tool for the recognition of alarm states in the inspected object.

Solutions included in the so-called 'topologic' methods of solutions of differential equations can be applied in the constructing of monitoring systems. They require examination of trajectories, it means curves [$x(t),\ \dot{x}(t)=y$] on the phase plane $x,\ \dot{x}$, related to the dynamic model of the monitored object. Their analysis can constitute the basis for determination of the permissible dynamic behaviour of the monitored object, at the described level of permanently acting disturbances and allowable perturbations of the steady state. Those perturbations are related to changes of the parameters value, which might be significant for appearing of defects, including their early stages.

The investigating procedures are based on certain topologic facts, related to certain invariants of homomorphic rearrangements, formulated in the form of statements. They enable the qualitative estimation of dynamic behaviours of the analysed object and conditions for the technical stability loss - corresponding to them.

The Lapunow's method [11, 16, 17] is the most often applied procedure. The properties of the scalar function $V(x,t)$ **-** properly selected for the dynamic description of the inspected object - are used. Investigating its derivative along solutions (behaviours) of the system (2) determines its stability.

This law states: if there is a scalar function $V(x,t)$ of class C^1 determined for each $\boldsymbol{x}$ and $t \geq 0$ in the vicinity of the equilibrium point, meeting the conditions:

- $V(\boldsymbol{x}, t) > 0$, for $x \neq 0$
- $\dot{V}(x,t) \leq 0$ along solutions (4) , for
  $\boldsymbol{x} \notin \boldsymbol{G} - \omega$ (9)
- $V(\boldsymbol{x_1}, t_1) < V(\boldsymbol{x_2}, t_2)$ for $\boldsymbol{x_1} \notin \boldsymbol{\omega}$ and $\boldsymbol{x_2} \notin \boldsymbol{G} - \boldsymbol{\Omega}$; $t_1 < t_2$

then the object described by (2) and meeting conditions (3-5) is technically stabilised.

Referring the results of this law to the problem of realisation assumptions for the monitoring systems of the machine state, the Lapunow's function $V(x,t)$ should be formulated and by means of the experimentally determined trajectories *x*, *y* of the object under testing, the conditions given by Equation (9) verified.

Essential, helpful element in the process of building a state classifier can be the results of the law given in paper [15], since it provides an estimation of the velocity with which the dynamics of the monitored system proceeds to the zero equilibrium point.

The law states that, in the case when the Lapunow's function:
$V(x) > 0,\ x \in \Re^n,\ x \neq 0,$ is such that $V(x) < 0$ for $x \in \Re^n,\ x \neq 0$ at which the following relation occurs:

$$\gamma = \max_x(-V(x)/\dot{V}(x)),\ x \in \Re^n,\ x \neq 0$$

the inequality:

$$V(x,t) \leq V(x,0)e^{-t/\Upsilon} \qquad (11)$$

which allows to estimate the velocity, with which the analysed dynamic system proceeds to the zero equilibrium point – is the true one.

The results of this law enable to select properly the delay time of the monitoring system for reacting for an instantaneous disturbance.

Another way of investigating properties of the monitored trajectories – estimating the dynamic system stability – is using two functions [12]:

$$\Phi\ (x, y) = xy + \dot{x}\dot{y}\ ;\ \Psi(x, y) = x\dot{y} - \dot{x}y \qquad (12)$$

Φ

The first function is a derivative of the square vector of the distance of the point on the trajectory from the origin of coordinates, while the second is the value of the phase velocity moment versus the origin of coordinates, which positive or negative determinant allows to estimate the character of the monitored motion. Their values, positive or negative, enable assigning to the trajectory points the directions characteristic for input, output or slip points versus the analysed curve, which allows to determine *G* and Ω zones within the monitored phase space.

Assumptions for building quantifiers of the monitored trajectories properties (from the point of view of their stability) can be also looked for on the basis of the topologic retracting method, Ważewski [18]. In this method, developed by W. Bogusz [9], the zones bounded by curves of input and output points of the system *(2)* from areas considered to be permissible - are constructed.

As it results from the synthetic presentation of the examination methods of the technical stability [9], their application in constructing the monitoring system is related to three tasks [5]:

1. Identification of the dynamic model of the monitored construction node of the object under inspection.
2. Creation of the measuring instrumentation protecting observations of trajectory changes of the dynamic behaviour of the monitored object node, determined by the measurement:
   $$x(t),\ \dot{x}(t) = y$$
3. Constructing quantifiers for the monitored courses by the implementation of algorithm of

examining the technical stability, based either on the method of the Lapunow's function, or two functions $\Phi(x, y)$ and $\psi(x, y)$, or the retracting method.

The solution of the first and second task does not generate significant difficulties in the realisation of the monitoring system. More difficult is assigning of positively or negatively determined Lapunow's function to the dynamics describing the monitored object, given by the identified set of differential equations.

An essential element of estimation the usefulness of phase trajectories in the defect recognition process is the requirement of tracing changes in the trajectory shape of the inspected processes. Its realisation, in the case of using the currently available – in the market – monitoring systems for inspecting vibrations of machines, devices or constructions, would require the reorganisation of their functioning. There would be a need of securing the possibility of measuring phase trajectories, which means measuring displacements and vibration velocities on the selected constructional nodes of machines. Analysis of their changes is the basic information carrier, which enables tracing the technical stability loss of the monitored object, caused by inadmissible disturbances in its proper functioning.

Thus, an essential task in the estimation of usefulness of the technical stability theory in the development of diagnostic systems is the verification of behaviours of phase trajectories of the monitored objects under conditions of defect occurrences [9, 10]. Computer simulation experiments provide certain directions and information [8] on changes of phase trajectories caused by defects.

4. CONCLUSIONS

The author's approach to tasks of designing and constructing of the systems monitoring changes of machines, devices and structure states – is presented in the paper. The attention was directed towards the importance of the dynamic model identification process of the monitored object. Its presence in the constructing procedure of the monitoring system preserves - in an universal and coherent way - the realisation process of diagnostic tasks. It enables the possibility of the simulation base formation for the recognition of inefficiency of the monitored objects, protects the optimal filtration and prediction of changes in the processes under inspection and formulates methodological guidelines for the realisation of the monitoring process.

Application of methods of technical stability investigations for assessments of dynamic behaviours of the tested object generates selection indications of symptoms necessary in the monitoring system, determination their criteria values as well as formation of estimation procedures of their exceeding the permissible values.

As it seen from the short review given above, several problems related to constructing the assigned algorithms remains still unknown and constitutes an interesting research field. The author hopes, that the indicated research idea will be developing and the results will produce more effective rules of monitoring the state changes in machines, structures and devices.

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