GENERALIZED SINGULAR VALUE DECOPOSITION IN MULTIDEIMENSIONAL CONDITION MONITORING OF SYSTEMS

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Summary

With the modern metrology, we can measure almost all variables in the phenomenon field of a working machine, and much of measuring quantities can be symptoms of machine condition. On this basis, we can form the symptom observation matrix (**SOM**) for condition monitoring. From the other side we know that contemporary complex machines may have many modes of failure, so called **faults**, which form the fault space. This multidimensional problem is not a simple one, even if we apply some modern tool like **SVD** for the fault extraction purpose. So the question remains if one can learn considering similar problem when having SOM of similar machine observed just before. In this way, we can consider the application of generalized GSVD to the machine condition monitoring problems, and uncover some new possibilities.

Keywords: machine condition, multidimensional, generalized SVD, observation space, fault space, condition similarity.

UOGÓLNIONY ROZKŁAD WARTOŚCI SZCZEGÓLNYCH W WIELOWYMIAROWEJ DIAGNOSTYCE STANU SYSTEMÓW

Streszczenie

Obecnie potrafimy mierzyć większość procesów pola zjawiskowego pracującej maszyny, a wiele z tych procesów może dostarczyć symptomów jej stanu technicznego. Wychodząc stąd możemy tworzyć symptomową macierz obserwacji (SOM) do celów diagnostyki maszyn, czyli oceny ewolucji jej stanu technicznego w czasie życia θ. Ale współczesne maszyny mają wiele uszkodzeń rozwijających się współbieżnie, stąd tez propozycja diagnostyki wielowymiarowej i zastosowania rozkładu (SVD), co pokazano już w wielu pracach. Powstaje pytanie czy potrafimy uzyskana wiedzę wykorzystać i nauczyć się diagnozować lepiej maszyny, które już są rozpoznane diagnostycznie za pomocą SVD. Taki właśni problem postawiono stosując uogólniony rozkład SVD, umożliwiający porównanie dwu macierzy obserwacji, znanej uprzednio i właśnie rozwijającej się. Tak możliwość istnieje, a stawia przed nami nowe wymogi nauczenia się nowej semantyki wspólnego języka GSVD.

Słowa kluczowe: stan techniczny, wielowymiarowość, uogólnione SVD, przestrzeń uszkodzeń, przestrzeń obserwacji, podobieństwo stanu.

1. INTRODUCTION

The multidemsionality of fault space in machine condition monitoring is nowadays well formulated and explored, for example by the application of neural nets [4], singular value decomposition [6], or principal component analysis [2]. Much worse it looks when considering the decision making process in multidimensional case, where we have some method of data fusion and the concept of symptom reliability applied to generalized fault symptom obtained from the application of SVD [10]. So, there is a room for looking to other promising methods of condition symptoms processing and decision making in a multidimensional case. This paper looks for the possible application of generalization of SVD method, which takes into account the other SOM of the similar object, with the same number of symptoms (*columns*), but the number of rows (*observation*) may differ. This may be the situation of learning from the previous usage of the same object or even similar one. The accessible list of references to **GSVD** application is not big one. One can see a few in connection with engineering, but there are some papers of **GSVD** application in physics and biology, as we will see later on. In such situation the paper introduce the GSVD concept on the basis of previous SVD application, and from these introductory results one can notice the possible application of GSVD in machine condition monitoring, particularly when looking for the similarity of machine wear symptoms and indices.

[1] GeneralSVD06 - Intended to Diagnostic Congress 08.

2. SINGULAR VALUE DECOMPOSITION AND EXTRACTION OF FAULT SYMPTOMS

Having in mind the above, let us take into consideration a critical machine in operation, where we have the possibility to observe several '*would be*' symptoms[1] of condition. During its working life $\boldsymbol{0 < \theta < \theta_b}$, ($\theta_b$ *–anticipated breakdown time*), several independent faults (*usually a few*); $\boldsymbol{F_t(\theta)}$, $t = 1,2,..u$, are evolving and growing. Hence, we would like to **identify** and **assess** the advancement of these faults by forming and measuring the symptom observation vector; $\boldsymbol{[S_m] = [S_1,...,S_r]}$, which may have components different physically, like vibration amplitudes (*displacement, velocity, acceleration*), the temperature, machine load, life time $\boldsymbol{\theta}$, etc.

In order to track machine condition (*faults evolution*) by these observations, we are making equidistant reading of the above symptom vector in the lifetime moments; $\boldsymbol{\theta_n,\ n = 1, \ldots\ p,\ \theta_p \le \theta_b}$, forming in this way the rows of a rectangular symptom observation matrix (**SOM**). From the previous research and papers [6], we know, that the best way of **SOM** preprocessing is to center it (*subtract*), and normalize (*divide it*) to the symptom initial value; $\mathbf{S_m(0) = S_{0m}}$, of each given symptom (*column of* ***SOM***).

It is also known from this research, that amount of diagnostic information in **SOM** increases if we append the lifetime θ column, as the first approximation of system logistic vector **L** and the load [7]. Finally, in order to minimize stochastic disturbances in readings we will apply also the three points moving average procedure to the successful symptom readings, as it was shown and validated in the last paper [14].

So, after such preprocessing we will obtain the dimensionless symptom observation matrix (**SOM**) in the form:

$$SOM \equiv O_{pr} = [S_{nm}],\ S_{nm} = \frac{\mathbf{S}_{nm}}{\mathbf{S}_{0m}} - 1, \qquad (1)$$

where bold non italic letters indicate primary measured and averaged dimensional symptoms.

As it was already said in the introduction, we apply now to the dimensionless **SOM** (1), the Singular Value Decomposition (**SVD**) [9, 15], to obtain singular components and singular values in the form of matrix formulae;

$$O_{pr} = U_{pp} * \Sigma_{pr} * V_{rr}^{T}, \qquad (2)$$

(***T****- matrix transposition*),
where $\boldsymbol{U_{pp}}$ is $\boldsymbol{p}$ dimensional orthonormal matrix of left hand side singular vectors, $\boldsymbol{V_{rr}}$ is $\boldsymbol{r}$ dimensional orthonormal matrix of right hand side singular vectors, and the diagonal matrix of singular values $\boldsymbol{\Sigma_{pr}}$ is as below

$$\Sigma_{pr} = diag\ (\ \sigma_1,\ \ldots,\ \sigma_l\ ),\ and\ \ \sigma_1 > \sigma_2 > \ldots > \sigma_u > 0, \qquad (3)$$

$\boldsymbol{\sigma_{u+1} = \ldots\ \sigma_l = 0,\ \ l = max\ (p,\ r),\ \ u \le min\ (p,\ r),\ u < r < p.}$

Mathematically it can be shown also, that every perpendicular matrix has such decomposition (2), and it may be interpreted also as the product of the three matrices [15], namely

$$O_{pr} = (Hanger)\ X\ (\ Stretcher)\ X\ (Aligner). \qquad (2a)$$

This is very metaphorical description of **SVD** transformation, but it seems to be useful analogy for statistical reasoning and diagnostic decision making in our case.

In terms of machine condition monitoring the above decomposition means, that from the $\boldsymbol{r}$ primarily measured symptoms (*dimension of* ***observation space***) we can extract only $\boldsymbol{z \le r}$ independent sources of diagnostic information describing evolving generalized faults $\boldsymbol{F_t}$, creating in this way **fault space** (*see Fig. 1)*. As it is seen from Fig. 1 upper right picture, only a few developing faults are making essential contribution to total fault information, the rest of generalized faults are below the standard 10% level of noise. What is important here, that such **SVD** decomposition can be made currently, after each new observation (*reading)* of the symptom vector $[S_m]$*;* $\boldsymbol{n = 1,\ \ldots\ p}$, and in this way we can trace the fault life evolution in any operating mechanical system.

Diagnostic interpretation of SVD results

From the current research and implementation of this idea [11], we can say, that the most important fault oriented indices obtained from **SVD** is the first pair: ($\boldsymbol{SD_t,\ \sigma_t}$), $t=1,2$. This pair presents the lifetime evolution of all independent sources of information contained in our SOM. We interpret them as the fault development life curves $\boldsymbol{F_t(\theta)}$. From the other side we need also some measure of total damage advancement in diagnosed object in a form $\boldsymbol{\sigma_t(\theta)}$.

The first fault indices $\boldsymbol{SD_t}$ can be named as discriminant or the generalized symptom of the fault $\boldsymbol{t}$, and one can get it as the **SOM** product and the first singular vector of the matrix $\boldsymbol{V}$, as below

$$SD = O_{pr} * V_{rr} = U_{pp} * \Sigma_{pr}, \qquad (4)$$

and for the one column component of this matrix we will have simply

$$SD_t = O_{pr} * v_t = \sigma_t \cdot u_t\ .\ t=1,...z. \qquad (5)$$

We know from **SVD** theory [9, 15], that all singular vectors $\boldsymbol{v_t}$, $\boldsymbol{u_t}$ are normalized to one, so the energy norm of this new discriminant (*vector*) is simply

$$Norm\ (SD_t) \equiv //SD_t// = \sigma_t\ ,\ t = 1,\ ...,z. \qquad (6)$$

If the number of observation is growing in the life time, so the above discriminant $\boldsymbol{SD_t(\theta)}$ can be also named as lifetime **fault profile**, and in turn singular value $\boldsymbol{\sigma_t(\theta)}$ as a function of the lifetime seems to be its damage advancement (*energy norm*).

[1] **Symptom** is a measurable quantity taken from the phenomenal field of the machine, which sees to be correlated to machine condition, we are looking for.

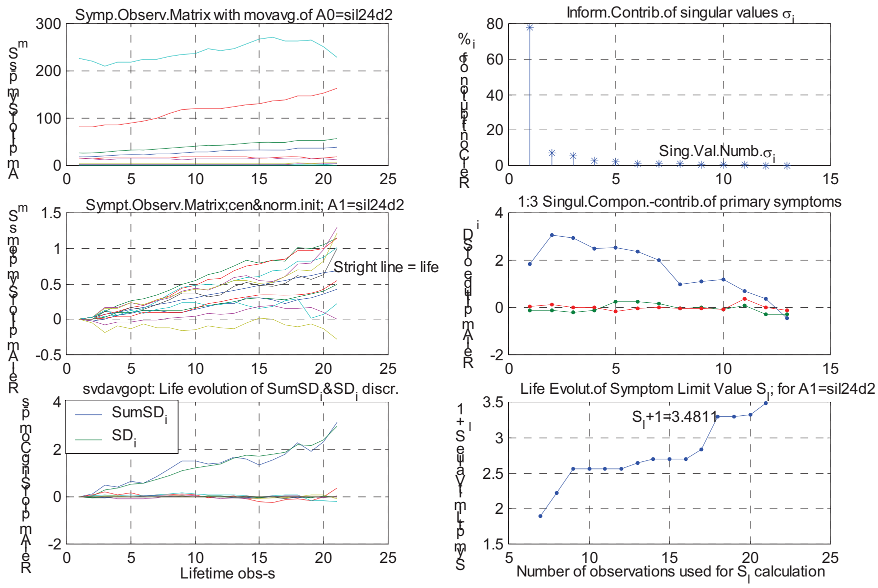

Fig. 1. Illustration of inference possibilities in multidimensional observation by the application of SVD

The similar fault inference can be postulated to the meaning, and the evolution, of summation quantities, what can mean the total damage profile ***SumSD$_i$(θ)***, and total damage advancement ***Sumσ$_i$(θ)***, as follows;

$$SumSD_i(\theta) = \sum_{i=1}^{z} SD_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta)\cdot u_i(\theta) = P(\theta),$$

$$Norm\ (SumSD_i(\theta)) \equiv //\Sigma\ SD_i\ (\theta)// \le //\Sigma\ \sigma_i(\theta)\cdot u_i(\theta)// = \Sigma\, \sigma_i(\theta)\ ,$$

hence;

$$Sum\sigma_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta) \sim \sum_{i=1}^{z} F(\theta)_i = F(\theta)\cdot \qquad (7)$$

But it is worthwhile to add, that the meaning of the last relation with $\sigma_t(\theta)$eems to be not fully validated experimentally, as yet. It seems to be also, that the condition inference based on the first summation measure; *Sum(SD$_i$)* may stand for the first approach to multidimensional condition inference, as it was clearly shown in the previous papers (*see for example* [17]), and shortly illustrated on Fig. 1 below.

3. GENERALIZED SINGULAR VALUE DECOMPOSITION GSVD

The diagnostic application of generalized **SVD** [9], as far as for today, is not known at all. However, it seems to be different from the ordinary SVD. This is because we have not one but **two** symptom observation matrices **A** and **B**. Let us assume we have primary $SOM_p = \boldsymbol{A}$ and auxiliary $SOM_a = \boldsymbol{B}$, and we will try to align SVD decomposition to both matrices, treating the first as primary **SOM** and the second as auxiliary. Even so, the matrices may differ, having different number of rows (*observations*), but they must have the same number of columns (*symptoms*).

Going to the definition of **GSVD** we have following relationships [16].

If we define; gsvd(A,B) ≡ [U,V,X,C,S], than it gives the following decompositions of:

$$\boldsymbol{A = U*C*X^T}, \qquad (8)$$

$$\textbf{and ; } \boldsymbol{B = V*S*X^T}, \qquad (9)$$

with singular values diagonal matrix:

$$\boldsymbol{\Sigma = C * S^{-1}}, \ \textbf{\textit{ascending ordered,}} \qquad (10)$$

and additional identity relation:

$$\boldsymbol{C^T * C + S^T * S = I.} \qquad (11)$$

It maybe important to show, when the **GSVD** becomes **SVD,** because we know already diagnostic interpretation of the second decomposition.

We have from (9);

$$S^{-1} * V^T * B = X^T\,. \qquad (12)$$

Using it with the decomposition of $\boldsymbol{A}$ as in the relations (8) one can get;

$$\boldsymbol{A = U*C*X^T = U*C*S^{-1}\ V^T *B = U*\Sigma *V^T*B}\ . \qquad (13)$$

So, if the auxiliary matrix is the identity matrix, i. e. $\boldsymbol{B = I}$,

we have the already known **SVD**, with the properties shown in the paragraph of 2.1 above.

But using $\boldsymbol{A}$ as a self reference matrix, when $\boldsymbol{B=A}$, we obtain from (8) and (9) immediately,

$$\boldsymbol{A = U*C*X^T = V*S*X^T}. \qquad (14)$$

And it can be possible only if : **C=S** and **U=V.** Finally, with this assumption we have:

$$\boldsymbol{\Sigma = C * S^{-1} = I}\,. \qquad (14a)$$

Hence, if both matrices primary and auxiliary are identical they singular vales obtained from **GSVD** are all equal one! This means that using this property we can investigate the similarity between two **SOM**s, so between respective diagnosed objects. From the other side we can investigate if there is some possibility to learn, using already known knowledge from one object to diagnose the other, not known already.

But before deep penetration of this possibility, let us create the similar condition related (*life*) quantities for both matrices SOM_A and SOM_B .We may try to obtain fault related discriminants from **GSVD** in the same manner as in case of one **SOM** used with usual **SVD** (*see (4) and (5)*.
Several approaches to accomplish this task was made, and one of the best which gives similar results to the case of single **SOM**, (i.e $B=I$) is proposed here as below:

$$SD_A = A * X * S^{-1} = U * C * X^T * X * S^{-1} ,$$

$$SD_B = B X * S^{-1} = V * S * X^T * X * S^{-1} , \qquad (15)$$

and of course matrices have ascending column norms, in contrary to ordinary SVD.

The above-proposed relations give sometimes a little greater numeric results than for one **SOM** case, but the qualitative life course of singular vectors and symptom limit value S_l is much similar. Also this is in some agreement with information contribution (σ_t) of the singular vectors v_t , u_t .

From the relation (13) it is seen that in a case $B \neq I$ all matrices of **GSVD**, that means U, Σ, V , must be aligned to the properties of A and B symptom observation matrices. Hence, we can infer that application of **GSVD** in diagnostic may be interpreted also as some kind of learning process. That mean for example that, based on previous observation (auxiliary $SOM = B$) we are trying to compare the current wear process observed by primary $SOM = A$. That is we should look now for some measures of similarity between matrices A and B.

Having now the possibility of real application of GSVD in condition monitoring let us look for the help at the other branches of science, namely bioinformatics, where one can find already the application of **GSVD** [18]. Following this paper and relation (10) with the additional condition of (14) we can find, that for identical observation matrices **A=B** all singular values of **GSVD** are equal unity; $\Sigma = C * S^{-1} = I$. If we interpret this as the tangent of the angle α between two **SOM**s (***A,B***). So in the case of their identity we have $\alpha_o = \pi/4 = 45^o$, and in all other cases we will have some angular measure of similarity differing from the angle 45^o . Centering it to zero, for the general case of similarity, we can write in a matrix notation,

$$\Sigma_\alpha = C * S^{-1} - I = tg\ \alpha ; \quad \text{and } \alpha = arc\ tg\ (\Sigma_\alpha) . \qquad (16)$$

Once more, we may suppose from the above, that if the defined measure is equal zero for a some singular value σ_i of primary **SOM** it may mean that, that the wear process associated with the given singular value is similar as it was previously, for the known already case of auxiliary **SOM = B** .

We can invent the other similarity measures. Moreover, for the **SOM** identity case as in (14) we have **U = V**, and the correlation coefficient calculated between columns or rows is equal one. So, for the general case of **A, B** matrices we can define correlation coefficient between columns (**U, V**) for the matrices defined by GSVD. Using the commonly shared numbers of rows of both matrices (*observations*) $l=min(n,p)$ for a current life time moment θ one can write it in a **Matlab®** notation, with some weighting matrix Σ;

$$C_{uv} = corcoef(U(1:l,:),V(1:l,:)) * \Sigma \qquad (17)$$

In general, this means, that as for now we have two independent measures of similarity between primary and auxiliary **SOM**. The first measure (16), shows the angles α_i in the observation spaces (**A**, **B**), between axes defined by matrices **C** and **S** of **GSVD.**

The second measure (17) shows the same similarity but seen here as the column by column correlation coefficients of (**U**, **V**) matrices defined by **GSVD,** and weighted by multiplication of singular value matrix Σ.. Such weighting gives much better differentiation of values of similarity measures for different objects.

There is another possibility of similarity measure calculation, using the vector of singular values taken from diagonal matrix Σ as in (10). Extracting its diagonal, and treating it as the vector we will have it in the form $Sig=diag(\Sigma)$. We know that in the case of identity all singular vales are equal **1**. So, it will be good if we subtract this identity value from the previous vector. Calculating now the norm of such new vector and normalizing it to the not subtracted value we can define the index of similarity of SOM matrices in GSVD, as below

$$SI = 1 - (Sig - 1)^T * (Sig - 1)/(Sig^T * Sig) \qquad (18)$$

One can see from the above, that such similarity index ranges from zero to unity, being one if both matrices in GSVD are identical

We will see these possibilities of inferring from the previous observations (*auxiliary SOM*) on the examples below. This will indicate how these measures of similarity of matrices **A** and **B** behaves, and how sensitive they are to the abbreviated data in a SOM, and to the data taken from the another object.

4. COMPARATIVE PROPERTIES OF GSVD IN CONDITION MONITORING, AND POSSIBLE DIAGNOSTIC APPLICATION AND MEANING OF GSVD

As we have mentioned earlier, generalized **SVD** can use auxiliary diagnostic observation. So it is possible to use another SOM obtained previously from the same object, or from the similar diagnosed object. This statement is by analogy to other applications in computational biology [18], as for the author knowledge, no condition monitoring (**CM**) application is known to this date. Starting at beginning let us take the simplest possible case of the same object treated by specially elaborated program written in Matlab® called **gsvdavg.m.** In addition, we have shown earlier, that for the real **CM** industrial data with some instability of

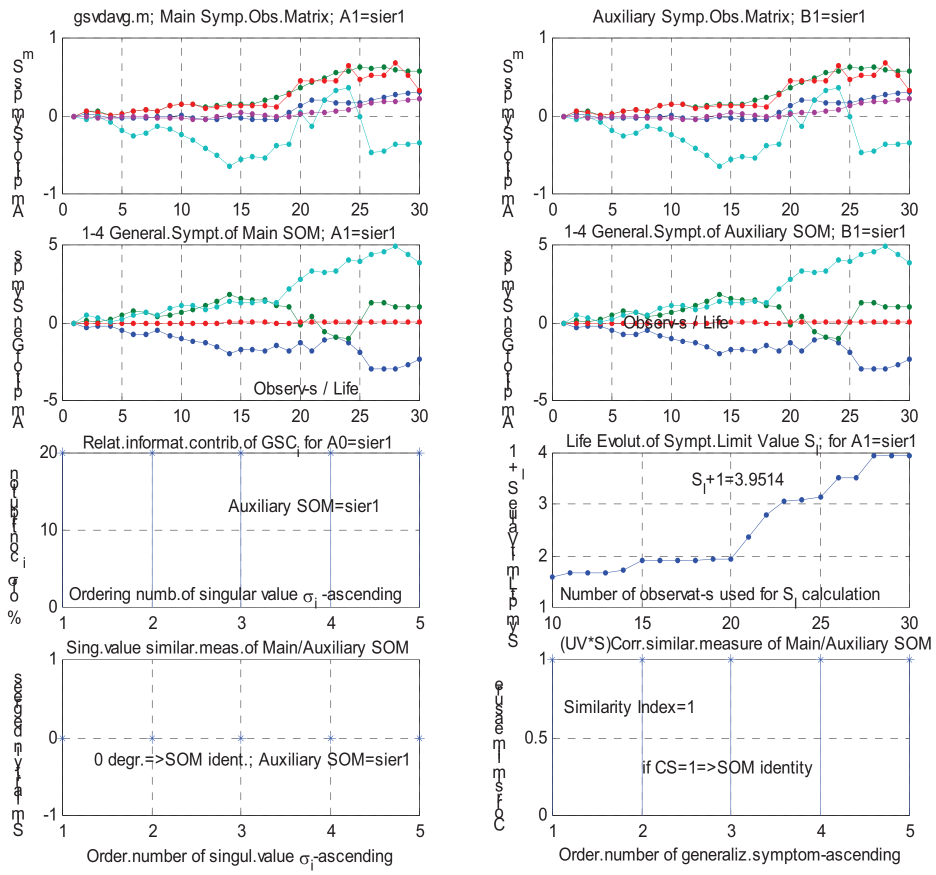

Fig. 2. Industrial fan as an example of application of GSVD applied to the same symptom observation matrix

symptom readings it is good to apply the moving average operation (**avg**) for the primary **SOM** [14]. In the presented example here, we have taken the huge industrial fan, which pumps the air to shaft of the copper mine, and has been working 32 weeks with one reading per week of the vibration symptom vector (*5 components*). Fig. 2 present this example elaborated by special GSVD program and subdivided into the 8 pictures described below.

The first left top picture presents averaged centered and normalized primary symptom observation matrix. As it can be seen, the variability of observed symptoms is not a great one, ranging from zero up to ± 1, although some of the symptom life curve changes the sign of values, their oscillations is not a big one, as a result of an introductory performed averaging operation (***avg***). The same is shown on the top right picture for the auxiliary **B** matrix, and as both were assumed identical, it is the same picture as at the top left. The next two pictures, the second row from the top, present us the results of **GSVD** calculation, and here we show only the biggest four generalized fault symptoms $F_t(\theta)$, t=(1,4), calculated according to formula (15). They are of course the same, due to our identity assumption. The next row of pictures is quite different, from the left one can see singular values, and they are equal each other due to assumed matrix identity. Further on the right picture presents calculated symptom limit value S_l , and one can notice it seems to be quite good evolution of this diagnostically important quantity.

The last row of pictures, at the left, shows the same singular values as above but treated as the tangent of the angle of similarity between the two spaces of symptom observation matrices (16), primary **A** and auxiliary **B**. And of course for the identical matrices the α angle is equal zero. The bottom right picture gives another measure of similarity (17), the transformed correlation coefficient between generalized fault symptom of primary and auxiliary matrices, multiplied additionally by the respective part of singular value (**S** matrix). This is in order to make the measure more sensitive. Of course, for the case of identical matrix this measure is also equal one for every singular vale. Therefore, it seems to that in this way as above, we can investigate the similarity between two SOM, and to decide if the fault development during the machine operation is the same as previously or only slightly similar to the previous case.

Knowing this let us take not identical **SOM**s but similar one, like for example auxiliary SOM the same as primary but with smallest number of rows. For the good illustrative purposes it can be the same **SOM** sier1, but with the smaller number of observations. Fig. 3 illustrates this case, and the organization and the meaning of the individual pictures are the same as previously.

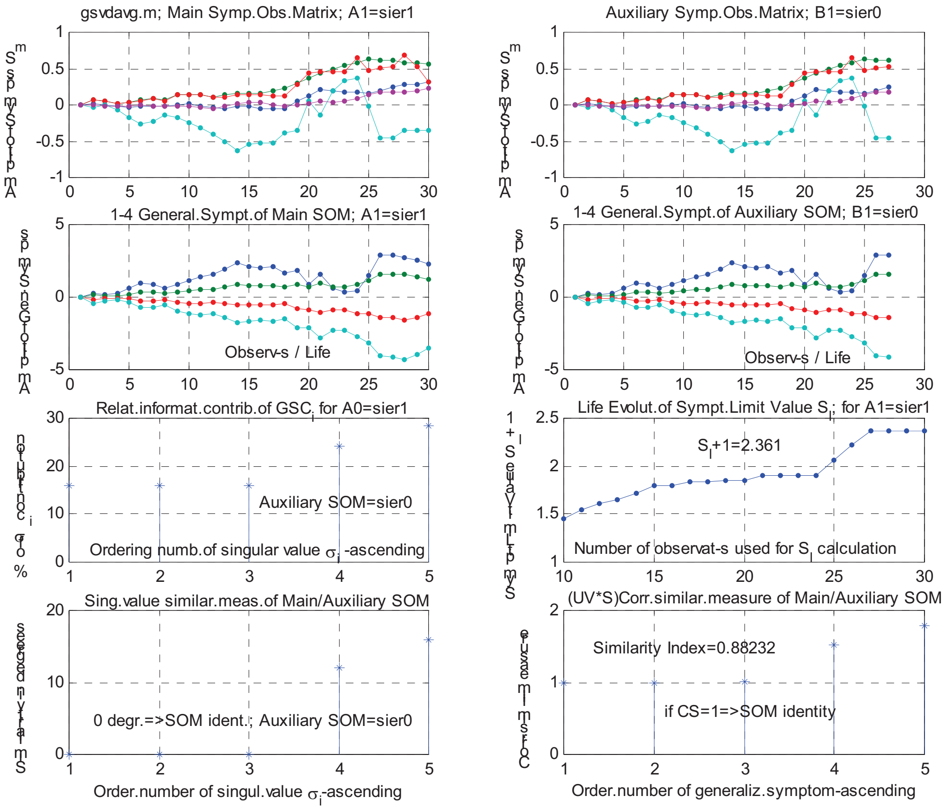

Fig. 3. GSVD comparison of the same SOM's but with the different number of observations (the last sixth cancelled)

Comparing now Fig. 2 and 3 one can notice the essential difference at the last two rows of pictures only. That means, that GSVD singular values are not identical, the value and the course of symptom limit values S_l is also not the same. What is more, the similarity measures at the last row of pictures are not as previously, because the last two singular values are not identical. We can infer that shorter SOM produce such differentiation, although there is no other difference, only in the number of the rows (*observations*). Almost the same situation is noticeable when we exchange the calculation sequence of primary and auxiliary matrix. Above, the first three singular values give the measures of identity 0 (*left picture*) and 1 (*right picture*), and in case of the matrix exchange the last three singular values gives the sign of matrix identity.

Let us now pass to the diagnostic objects of the same type but different copies of it. Fig. 4 present the comparison of two different exemplars of railroad diesel engines with the different primary and auxiliary SOM. This gives of course the difference in generalized symptoms (*second row of pictures*), with the last two singular values essentially different from zero. The angular measure of similarity (*picture bottom left*) is spread here from -50^o to $+50^o$ degrees, giving no essential message to us, but the correlation measure of similarity indicates also one singular value close to unity (*picture bottom right*). Finally, the course of symptom limit value (*picture second right*) is evolving gradually, and growing rapidly at the end of the life of both systems.

Let as now compare another class of diagnosed objects namely rolling bearings at durability-test stand. Fig. 5 gives here the results of comparison in the same way of pictures organization as before for the diesel engines.

As one can notice from the first row of pictures the durability (*expected lifetime*) of bearings is different here, but the measured life curves are similar, and the same can be said with respect of generalized symptoms at the second row of pictures. Again one can notice, that there is one singular value essentially different in quantity from the others (*third row-left picture*), and the symptom limit value S_l evolving gradually. The last row of pictures is similar, like for the diesel engines, namely the angular measure of similarity is spread from -50 to +50 degrees, and correlation measure of similarity indicates one singular value greater than 1. It may mean that there is only **one way of degradation,** common to both tested bearings. But if we reverse the matrix order (*primary-auxiliary*) the last indication on similarity changes a little giving one singular values close to 1, and the second close to -1.

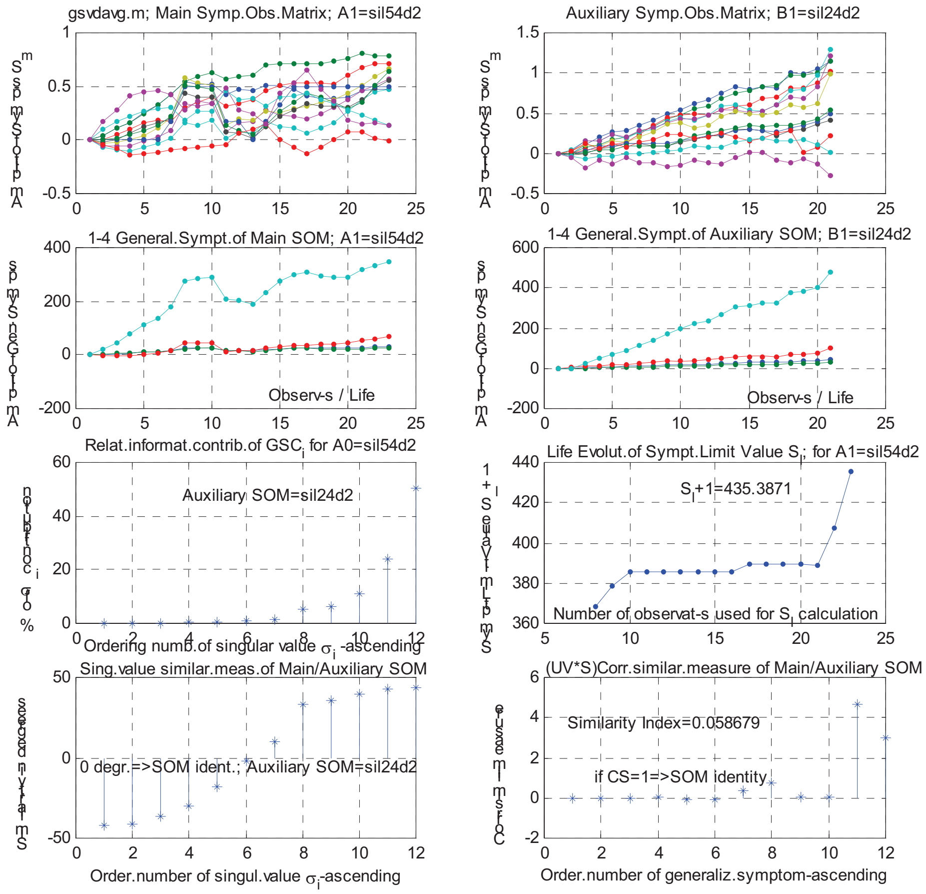

Fig. 4. GSVD comparison of two different exemplars of the same diesel engines

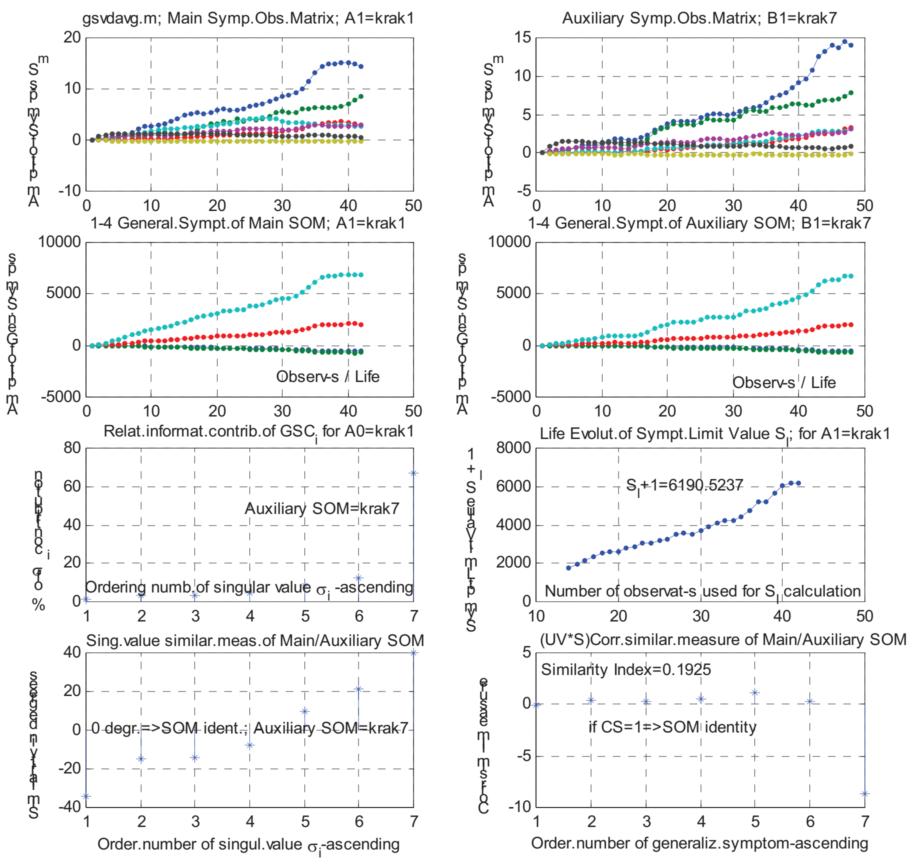

Fig. 5. GSVD similarity of rolling bearings at the durability test stand

Altogether 10 ball bearings were tested at the durability stand, and they damage advancement were described by the same symptom observation vector and SOM with different numbers of rows only. In addition, in every case the measures of similarity between bearings behave like on the last row of the fig. 5. In particular, the angular measure of similarity is spread −50 to + 50 degrees, and correlation measure indicates all singular values close to zero with the exception of the last one being close to unity or much bigger.

Well, these are some introductory diagnostic meaning and possible application of Generalized SVD, and the question now remains, is that all what can be done in condition monitoring? I am sure not, we should investigate the other possible application not only in condition monitoring but also in quality monitoring and comparison, for example.

5. CONCLUSIONS AND FURTHER PROBLEMS

As one can infer from the above consideration and examples of application, there is some possibility of **GSVD** application in machine condition monitoring. This is based mainly on looking at similarities in a machine wear processes and symptoms of its condition. For this purpose, several measure of similarity were defined and calculated for the cases of examples taken from the real monitored objects. There are some promising results. However, it seems to be too early to formulate some solid conclusions concerning GSVD use in machine condition monitoring. Some more approaches and trials seem to be needed to formulate such conclusions.

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