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MODELS OF PERIODICALLY SWITCHED ON AND OFF ELECTRICAL EQUIPMENT OPERATION

Key words

Mathematical models, periodically switching on and off, electrical equipment, time to failure.

Summary

Mathematical models of periodically switched on and off electrical equipment operation are presented in the paper. Models of electrical devices in the initial instant of the working state, when idle (not working) at the initial instant or not having idle moments between switching are characterised. The basic defined value for these cases has been the time to failure.

Introduction

The presented analysis deals with models of periodically switched on and off electrical devices. Primarily they are contactors steering the work of mechanic, electric, pneumatic and other kinds of devices (Fig. 1 and Fig. 2).

η_n^j denotes independent random variables sequence describing consecutive working periods of a *j*th electrical device; whereas, ς_n^j refers to an independent random variables sequence describing the devices consecutive idle periods (staying off the working period).

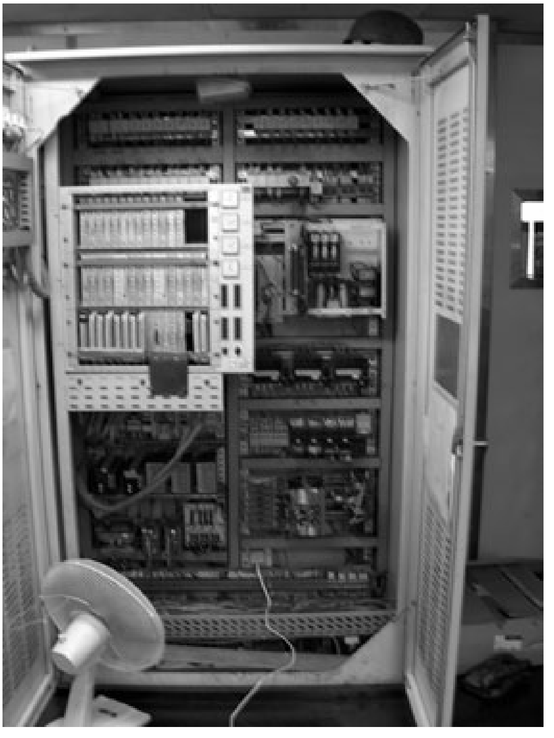

Fig. 1. A view of an electrical cubicle with set of contactors in engine control room

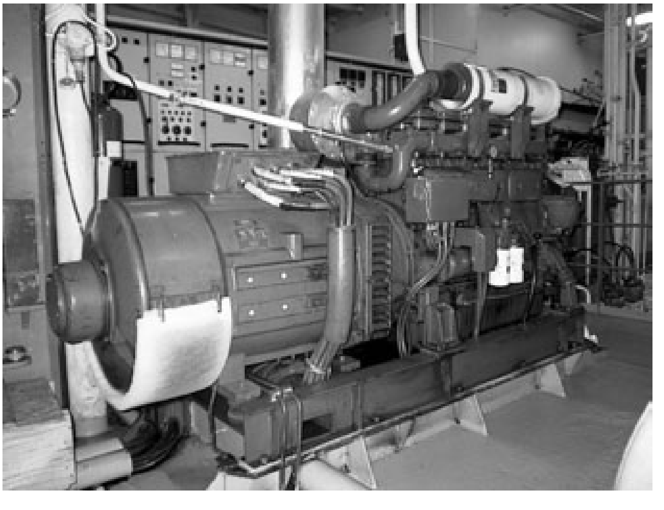

Fig. 2. A general view of one of the generating sets in a marine electrical power plant

Assuming that η_n^j random variables have identical cumulative distribution function:

$$\partial_j(t) = P\{\eta_n^j < t\}, \quad t \geq 0; \;\; n = 1, 2, ..., \;\; j = 1, 2..., N \tag{1}$$

and probability density $g_j(t) = \partial_j^{'}(t)$ for $t \geq 0$, analogically, ς_n^j random variables have an identical cumulative distribution function:

$$H_j(t) = P\{\varsigma_n^j < t\}, \quad t \geq 0; \;\; n = 1, 2, ..., \;\; j = 1, 2..., N \tag{2}$$

and probability density $h_j(t) = H_j^{'}(t)$ for $t \geq 0$. η_n^j and ς_n^j random variables have been assumed to be independent.

When ν_j denotes a random variable describing the number of a *j*th device switching off until the device failure, it has a discreet distribution with the probability function:

$$P_j(k) = P\{\nu_j = k\} \qquad \text{for} \quad k = 1, 2... \tag{3}$$

When the device is at the initial working state, idle state (not working) at the initial instant, and without “idle moments” between switchings on have been analysed [1, 2].

1. A model of an electrical device operation at the initial instant of its working state

When analysing a *j*th device at the initial instant of the working state, $t_1, t_2, ..., t_\nu$ refers to the device consecutive switching off moments until a t_ν random moment of its failure (Fig. 3), and $S_n^j = \eta_1^j + \eta_2^j + ... + \eta_n^j$ refers to the *j*th device total working time until its *n*th switching off, and $V_{n-1}^j = \varsigma_1^j + \varsigma_n^j + ... + \varsigma_{n-1}^j$ refers to the *j*th device total idle time until its *n*th switching off. The random variables distributions defined by means of η_n^j and ς_n^j have the following shape [3-5]:

$$P\left\{S_n^j > t\right\} = \int_0^t \partial_{n-1}^j (t-u) d\partial_j(u) = \partial_n^j(t) \qquad \text{for} \quad t \geq 0 \tag{4}$$

$$P\left\{V_{n-1}^{j} > t\right\} = \int_{0}^{t} H_{n-2}^{j}(t-u)\,dH_{j}(u) = H_{n-1}^{j}(t) \qquad \text{for} \quad t \geq 0 \tag{5}$$

where:

$\partial_{n}^{j}(t)$ – n-multiple functions entanglement $\partial_{j}(t), \partial_{1}^{j}(t) = \partial_{j}(t)$;

$H_{n-1}^{j}(t)$ – (n-1)-multiple functions entanglement $H_{j}(t), H_{1}^{j}(t) = H_{j}(t)$.

When $\tau_{j}\left(j = 1, 2, \ldots, N\right)$ denotes a jth device up time and τ_{n}^{j} refers to the period of time until the device failure – under the condition that the failure occurs at the nth switching off of the device – the conditional random variable τ_{n}^{j} equals:

$$\tau_{n}^{j} = \begin{cases} \eta_{1}^{j} & \text{for} \quad v_{j} = 1 \\ S_{n} + V_{n-1} & \text{for} \quad v_{j} \geq 2 \end{cases} \tag{6}$$

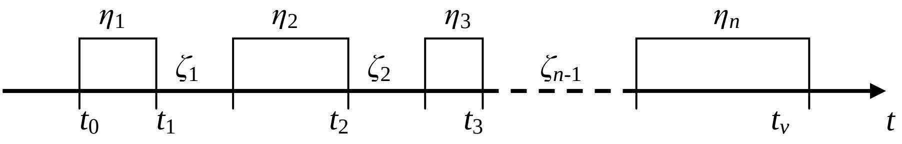

Fig. 3. A model of an electrical device operation at the initial instant of its working state

The distribution of τ_{j} – the unconditional random variable defining the time to failure has the following shape [3-5]:

$$\begin{aligned} F_{j}(t) &= P\left\{\tau_{j} < t\right\} = P\left\{\eta_{1}^{j} < t\right\} \cdot P\left\{v_{j} = t\right\} + \sum_{k=2}^{\infty} P\left\{\tau_{k}^{j} < t\right\} \cdot P\left\{v_{j} = k\right\} = \\ &= P_{j}(t) \cdot \partial_{j}(t) + \sum_{k=2}^{\infty} P_{j}(k) \cdot P\left\{\left(S_{k}^{j} + V_{k-1}^{j}\right) < t\right\} = P_{j}(t) \cdot \partial_{j}(t) + \\ &+ \sum_{k=2}^{\infty} P_{j}(k) \cdot \int_{0}^{t} \partial_{k}^{j}(t-u)\,dH_{k-1}^{j}(u) \end{aligned} \tag{7}$$

2. A model of an electrical device operation at the initial instant of its idle state

Analysing the case when a *j*th electrical device at the initial instant is idle (Fig. 4) and applying the assumptions from the previous sections, we achieve τ_n^j - a conditional random variable in the following form:

$$\tau_n^j = S_n + V_n \tag{8}$$

The distribution of a random variable τ_j describing the time until a *j*th device failure for $t \geq 0$ has the following shape [3-5]:

$$\begin{aligned} F_j(t) = P\{\tau_j < t\} &= \sum_{k=1}^{\infty} P\{\tau_j^k < t\} \cdot P\{v_j = k\} = \sum_{k=1}^{\infty} P\{(S_k^j + V_k^j) < t\} = \\ &= \sum_{k=1}^{\infty} P_j(k) \cdot \int_0^t \partial_k^j (t-u) dH_k^j(u) \qquad \text{for } t \geq 0 \end{aligned} \tag{9}$$

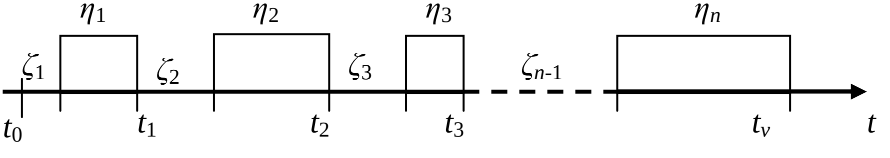

Fig. 4. A model of an electrical device operation at the initial moment of its idle state

3. A model of an electrical device operation excluding idle times

In the analysis below electrical devices in which idle times occur to be negligibly short when compared to their working state are presented.

ξ_n^j denotes a sequence of independent random variables describing times between a *j*th device consecutive switching on (Fig. 5). The conditional random variable τ_n^j describing the time to a *j*th device failure (under the condition that the failure occurs at the moment of the device *n*th switching off) is as follows:

$$\tau_n^j = \xi_1^j + \xi_2^j + \ldots + \xi_n^j \tag{10}$$

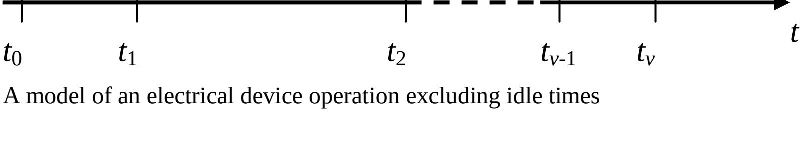

Fig. 5. A model of an electrical device operation excluding idle times

When applying the formerly accepted assumptions for modelling the operation of an electrical device including idle states to modelling an electrical device operation excluding idle times, the following is obtained:

$$\xi_k^j = \eta_k^j + \varsigma_k^j \tag{11}$$

When we assume that random variables ξ_j^n have an identical cumulative distribution function of $K_j(t) = P\{\xi_n^j < t\}$ for $t \geq 0$ and probability density $k_j(t) = K'(t)$ for $t \geq 0$ and like formerly denoting ν_j as a discreet random variable defining the number of a *j*th device switchings off until its failure, we may determine the distribution of an unconditional random variable τ_j describing the time to a *j*th device failure [3-5].

$$F_j(t) = P\{\tau_j < t\} = \sum_{n=1}^{\infty} P\{\tau_n^j < t\} \cdot P\{\nu_j = n\} = \sum_{n=1}^{\infty} P_j(n) \cdot K_n^j(t) \quad \text{for} \quad t \geq 0 \tag{12}$$

where:

$K_n^j(t)$ – *n*-multiple cumulative distribution function convolution $K_j(t)$,

$K_1^j(t) = K_j(t)$.

When, according to the former analysis, the total working time of a *j*th device until its *n*th switching off is denoted as:

$$S_n^j = \eta_1^j + \eta_2^j + \ldots + \eta_n^j \tag{13}$$

the distribution of the electrical device's total working time until its failure has the following shape [3-5]:

$$\Phi_j(t) = P\{S_\nu^j < t\} = \sum_{n=1}^{\infty} P\{S_n^j < t\} \cdot P\{\nu_j = n\} = \sum_{n=1}^{\infty} P_j(n) \cdot \partial_n^j(t) \quad \text{for} \quad t \geq 0 \tag{14}$$

where:

$\partial_n^j(t)$ – *n*-multiple cumulative distribution function convolution

$\partial_j(t), \partial_1^j(t) = \partial_j(j)$.

Conclusions

In the above analysis it has been assumed that random variables ξ_n^j, describing periods between consecutive switchings off of a *j*th electrical device have an exponential distribution of a λ_j parameter just like random variables η_n^j describing working periods of a *j*th device which have exponential distribution of a μ_j parameter. The random variable ν_j defines the device switching off number until its failure which allows us to assume that the random variable distribution is a geometric distribution of a p_j parameter.

According to the above assumptions, we obtain the following:

$$K_j(t) = P\{\xi_n^j < t\} = 1 - e^{-(\lambda_j \cdot t)}, \qquad t \ge 0 \tag{15}$$

and respectively:

$$k_j(t) = \lambda_j \cdot e^{-(\lambda_j \cdot t)}, \qquad t \ge 0 \tag{16}$$

$$\partial_j(t) = P\{\eta_n^j < t\} = 1 - e^{-(\mu_j \cdot t)}, \qquad t \ge 0 \tag{17}$$

and

$$g_j(t) = \mu_j \cdot e^{-(\mu_j \cdot t)}, \qquad t \ge 0 \tag{18}$$

$$P_j(n) = P\{v_j = n\} = p_j \cdot (1 - p_j)^{(n-1)} \tag{19}$$

The probability density of a random variable τ_j has been determined on the basis of (12), due to which the following has been obtained:

$$\begin{aligned} f_j(t) &= F'(t) = \sum_{n=1}^{\infty} P_j(t) \cdot k_n^j(t) = \sum_{n=1}^{\infty} p_j \cdot (1-p_j)^{(n-1)} \cdot \frac{\lambda_j \cdot t^{(n-1)}}{(n-1)!} \cdot e^{-(\lambda_j \cdot t)} = \\ &= p_j \cdot \lambda_j \cdot e^{-(\lambda_j \cdot t)} \cdot \sum_{n=1}^{\infty} \frac{[\lambda_j \cdot (t - p_j)]^{(n-1)}}{(n-1)!} = \rho_j \cdot \lambda_j \cdot e^{-(\lambda_j \cdot t)} \cdot e^{\lambda_j \cdot (1-p_j) \cdot t} = \\ &= \rho_j \cdot \lambda_j \cdot e^{-(\lambda_j \cdot p_j \cdot t)} \end{aligned} \tag{20}$$

Thus, the distribution of a random variable τ_j defining the time until a *j*th device failure is an exponential distribution of $\lambda_j \cdot p_j$.

In the same way, on the basis of (14), a random variable S_v^j defining *a j*th device total working time has the exponential distribution of $\mu_j \cdot p_j$ parameter in the form of:

$$\Phi_j(t) = \Phi'_j(t) = \sum_{n=1}^{\infty} p_j(n) \cdot g_n^j(t) = \sum_{n=1}^{\infty} p_j \cdot (1-p_j)^{(n-1)} \cdot \frac{\mu_j^n \cdot t^{(n-1)}}{(n-1)!} \cdot e^{-(\mu_j \cdot t)} = \\ = p_j \cdot \mu_j \cdot e^{-(\mu_j \cdot p_j \cdot t)} \tag{21}$$

Knowing the distribution of a random variable τ_j, due to (12) and then (20), it is possible to receive the *j*th electrical device reliability function, which has the following shape:

$$R_j(t) = 1 - F_j(t) = e^{-(\lambda_j \cdot p_j \cdot t)} \quad , \qquad t \geq 0 \tag{22}$$

Whereas, any electrically powered device system reliability function being the result of relations has the following shape:

$$R(t) = \prod_{j=1}^{N} e^{-(\lambda_j \cdot p_j \cdot t)} = \exp\left[-t \cdot \sum_{j=1}^{N} \lambda_j \cdot p_j \right] \quad , \qquad t \geq 0 \tag{23}$$

The presented analysis deals only with a selected aspect of the electrical device's operation of the specific and the simplest distributions of working and idle states accepted on the basis of research and literature [1-6]. When other assumptions are made referring to distributions, the methodology may be the same but the calculations would be much more complicated.

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Modele pracy urządzeń elektroenergetycznych o cyklicznych czasach załączeń i wyłączeń

Słowa kluczowe

Modele matematyczne, cykliczne czasy załączeń i wyłączeń, urządzenia elektroenergetyczne, czas do uszkodzenia.

Streszczenie

Przedstawiono modele pracy urządzeń elektroenergetycznych o cyklicznych czasach załączeń i wyłączeń. Scharakteryzowano modele pracy urządzeń, gdy w chwili początkowej jest ono w stanie pracy; w stanie spoczynku (nie pracuje) lub nie posiada w czasie pracy tzw. czasów spoczynku między załączeniami. Podstawową wielkością, jaką określono dla tych przypadków, był czas pracy do uszkodzenia urządzenia.