CRACK DETECTION IN BEAM LIKE STRUCTURES PART II. QUANTIFICATION OF CRACK DEPTH

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Summary

In the first part of this work the some methods of the location of a transverse crack in beam like structure based on wavelet analysis of mode shape is described. In this part of work this quantity was used for identification of crack depth. An input quantity for identification the frequency response function FRF was chosen. The crack is substituted by rotational spring, which flexibility c_g is calculated by using Castigliano theorem and laws of the fracture mechanics. Based on inverse model of forced vibration cracked beam the depth of crack was identified.

Keywords: crack identification, forced vibration.

DETEKCJA PĘKNIĘCIA W ELEMENTACH BELKOWYCH
CZĘŚĆ II. IDENTYFIKACJA GŁĘBOKOŚCI PĘKNIĘCIA

Streszczenie

W pierwszej części pracy opisano metodę wyznaczenia lokalizacji pęknięcia w oparciu o ciągłą i dyskretną transformatę falkową zastosowaną do analizy wektorów własnych. W tej części pracy wykorzystano tę wielkość do identyfikacji głębokości pęknięcia. Jako wielkość wejściową procesu identyfikacji wykorzystano charakterystykę amplitudowo – częstotliwościową. Jako model pęknięcia przyjęto przegub sprężysty, którego podatność wyznaczono na podstawie praw mechaniki pękania i twierdzenia Castigliano. Wykorzystując taki opis pęknięcia wyznaczono model odwroty drgań wymuszonych belki, z którego wyznaczono głębokość pęknięcia

Słowa kluczowe: identyfikacja pęknięcia, drgania wymuszone.

1. INTRODUCTION

The detection of damages in structural elements determines the serious challenge for present technique. The used at present non-destructive procedures of diagnosis the damages, it means the method: visual observations, ultrasonic, radiographical, or magnetical analysis, possess the many essential limitations. The most often to effective their utilization, it is necessary to carry out many of additional actions connected with correctness of diagnosis process, as well as the a priori knowledge about the place of potential damage. From here using these methods in places with difficult access, and on early stage of damage evolution is limited and burdened with large uncertainty.

Mentioned above conditions required research on finding the new methods of damage detection, the global one, which based on changes of dynamic response of constructions, would permit to estimate degree of damage.

A crack diagnostic system can have four levels [1]:

- level 1: the presence of crack,
- level 2: the geometric location of crack,
- level 3: the quantification of the crack depth,
- level 4: the prediction of the remaining service live of the structure.

In part 1 of this work some methods for detection of the transverse crack location in beam like structure are described. These methods are based on discrete and continuous wavelet transform of a modal vector.

In this part of work this quantity was used for identification of crack depth (3-rd level of crack diagnostics). An input quantity for identification the frequency response function FRF was chosen. The crack is substituted by rotational spring, which flexibility c_g is calculated by using Castigliano theorem and laws of the fracture mechanics. Based on inverse model of forced vibration of cracked beam the depth of crack was identified.

2. FORCED VIBRATION OF THE CRACKED BEAM

The equation of the harmonic forced vibration of the beam with crack in co-ordinate $x=x_p$ has form:

$$X^{(4)} - \lambda^4 X = c_g \cdot X''(x_p) \cdot \delta''(x, x_p) + \\ - F \cdot \delta(x, x_f) \tag{1}$$

The solution of the equation (1) can be found in class of generalized function. The function given by (2) is a solution of equation (1):

$$\begin{aligned} X(x) &= X_0(x) + \frac{c_g}{2\lambda} X''(x_p)\cdot \\ &\cdot[\sinh\lambda(x-x_p)+\sin\lambda(x-x_p)]H(x,x_p) + \\ &-\frac{F}{2\cdot EI\cdot\lambda^3}\cdot \\ &\cdot[\sinh\lambda(x-x_f)+\sin\lambda(x-x_f)]H(x,x_f) \end{aligned} \tag{2}$$

where:
$X_0 = P\cosh\lambda x + Q\sinh\lambda x + R\cos\lambda x + S\sin\lambda x$,
$\delta(x,x_p)$ - Dirac delta function at $x = x_p$, $H(x,x_p)$ - Heaviside step function at $x = x_p$, $\lambda = \omega^2\rho A/EI$, ρ - beam material density, A – cross-sectional area.

Amplitude of forced vibrations In measured point with coordinate $x = c$ has form:

$$\begin{aligned} X(c) &= X_0(c) + \frac{c_g}{2\cdot\lambda}\cdot \\ &\cdot[\sinh\lambda(c-x_p)+\sin\lambda(c-x_p)]H(c,x_p) + \\ &-\frac{F}{2\cdot EI\cdot\lambda^3}\cdot \\ &\cdot[\sinh\lambda(c-x_f)-\sin\lambda(c-x_f)]H(c,x_f) \end{aligned} \tag{3}$$

where:
$X_0(c) = P\cosh\lambda c + Q\sinh\lambda c + R\cos\lambda c + S\sin\lambda c$.

Integration constants *P, Q, R, S* depends on the beam boundary conditions.

The course of action at the identification of crack depth for which an input quantity is the amplitude of forced vibrations is shown on the cantilever beam example.

2.1. Cantilever beam

Analysed beam model in fig. 1 is showed

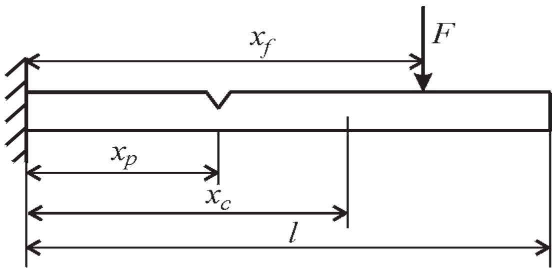

Fig. 1. Cantilever beam with crack

For beam showed on fig. 1 the boundary conditions are described by equations $X(0)=0$, $X'(0)=0$, $X''(l)=0$, $X'''(l)=0$.

The boundary conditions equation for beam can be written in matrix form:

$$\mathbf{M}\cdot\mathbf{CS} = \mathbf{W} \tag{4}$$

where matrix M:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ \cosh\lambda l & \sinh\lambda l & -\cos\lambda l & -\sin\lambda l & a_{35} \\ \sinh\lambda l & \cosh\lambda l & \sin\lambda l & -\cos\lambda l & a_{45} \\ \mathrm{ch}\lambda x_p & \mathrm{sh}\lambda x_p & \cos\lambda x_p & \sin\lambda x_p & -1 \end{bmatrix}$$

where:

$$a_{35} = \frac{c_g}{2\lambda}\cdot\left[\sinh\lambda(l-x_p) - \sin\lambda(l-x_p)\right]$$

$$a_{45} = \frac{c_g}{2\lambda}\cdot\left[\cosh\lambda(l-x_p) - \cos\lambda(l-x_p)\right]$$

vector of constants CS:

$$\mathbf{CS}^T = \left[P \quad Q \quad R \quad S \quad X''(x_p)\right]^T$$

and vectors of W:

$$\begin{bmatrix} 0 \\ 0 \\ \frac{F}{2\cdot EI\cdot\lambda^3}\cdot\left[\sinh\lambda(l-x_f) - \sin\lambda(l-x_f)\right] \\ \frac{F}{2\cdot EI\cdot\lambda^3}\cdot\left[\cosh\lambda(l-x_f) - \cos\lambda(l-x_f)\right] \\ \frac{F}{2EI\lambda^3}\left[\sinh\lambda(x_p-x_f) + \sin\lambda(x_p-x_f)\right]H(x_p,x_f) \end{bmatrix}$$

If excitation frequency is different from any natural beam frequency the Cramer's rule can be used for solving of equation (4).

For solution the identification problem, some procedure will be proposed.

3. INVERSE MODEL OF CRACKED BEAM

For solution the identification problem will be proposed following methodology, which permits to create of computer algorithm:

1. in main matrix M one should replace value c_g with 1 (one) – this matrix is named A;
2. one should construct matrix named B, from matrix A by eliminate last row and last columns;
3. at such denotation main matrix determinant can be written as:
$$|\mathbf{M}| = c_g\cdot(\det(\mathbf{A}) + \det(\mathbf{B})) - \det(\mathbf{B})$$
4. one should construct 5 others matrix C_i obtained from matrix A by replacing *i*-th column by vector W (i = 1,2,3,4,5);
5. in way described in point 2 one should construct from matrices C_i other matrices named D_i;
6. introducing denotation:
$L_1 = \det(\mathbf{A}) + L_2$; $L_2 = \det(\mathbf{B})$;
$L_3 = \det(\mathbf{C_1}) + L_4$; $L_4 = \det(\mathbf{D_1})$;
$L_5 = \det(\mathbf{C_2}) + L_4$; $L_6 = \det(\mathbf{D_2})$;
$L_7 = \det(\mathbf{C_3}) + L_4$; $L_8 = \det(\mathbf{D_3})$;
$L_9 = \det(\mathbf{C_4}) + L_{10}$; $L_{10} = \det(\mathbf{D_4})$;
$L_{11} = \det\mathbf{C_5}$);

integration constants can be obtain from equation:

$$P = \frac{c_g\cdot L_3 - L_4}{c_g\cdot L_1 - L_2}; \quad Q = \frac{c_g\cdot L_5 - L_6}{c_g\cdot L_1 - L_2};$$

$$R = \frac{c_g \cdot L_7 - L_8}{c_g \cdot L_1 - L_2}; \quad S = \frac{c_g \cdot L_9 - L_{10}}{c_g \cdot L_1 - L_2};$$

$$X''(x_p) = \frac{L_{11}}{c_g \cdot L_1 - L_2}$$

7. for location of crack x_p which was determined in part one of this work, the flexibility c_g can be obtain from equation:

$$c_g = \frac{L_P - L_2 \cdot Z}{L_N - L_1 \cdot Z + L_H} \tag{5}$$

where:

$$L_p = L_4 \cosh \lambda c + L_6 \sinh \lambda c + L_8 \sin \lambda c + L_4 \cos \lambda c$$

$$L_N = L_3 \cosh \lambda c + L_5 \sinh \lambda c + L_7 \sin \lambda c + L_9 \cos \lambda c$$

$$L_H = \frac{L_{11}}{2\lambda} \cdot \left[\sinh \lambda(c - x_p) + \sin \lambda(c - x_p)\right] \cdot H(c, x_p)$$

$$Z = POM(c) + \frac{F}{2 \cdot EI \cdot \lambda^3} \cdot$$
$$\cdot [\sinh \lambda(c - x_f) - \sin \lambda(c - x_f)] H(c, x_f)$$

POM(*c*) – is measured value of vibration amplitude in beam coordinate $x = c$.

Using described above methodology the flexibility c_g can be determined for each frequency and corresponding vibration amplitudes of FRF.

4. CRACK DEPTH IDENTIFICATION

Some methods based on discrete and continuous wavelet transform for the crack location determination in part one of this work have been described. The results of this identification is used to the quantification of the crack depth. The crack depth identification based on the described above inverse model of beam.

In fig. 2 the FRF of cantilever beam obtained as a computer simulation by finite element analysis.

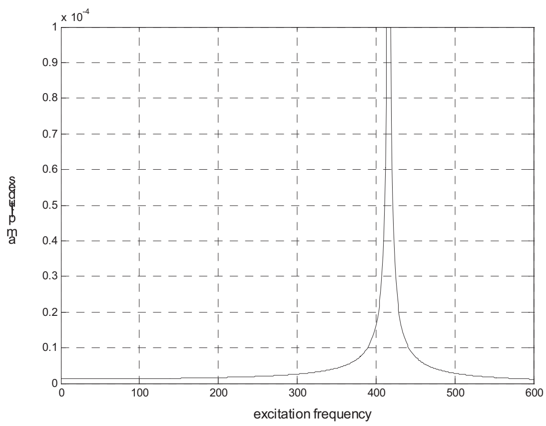

Fig. 2. FRF for the cantilever beam

In fig. 3 the identified local flexibility c_g, determined (from equation 5) for each frequency below the first natural frequency and corresponding amplitudes of vibration of FRF is showed.

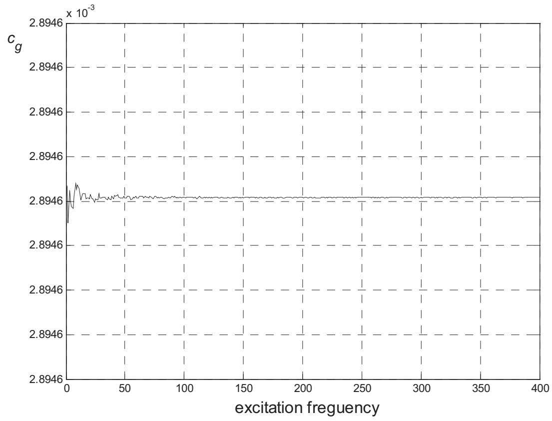

Fig. 3. Identified local flexibility c_g as a function of frequency excitation

Having identified the flexibility c_g (as a mean value identified for each frequency) the depth of crack have been determined.

Table 1 summarizes the modelled and identified crack depth with relative error of identification.

Table 1 Crack depth identification

modelled crack depth	identified flexibility c_g	identified crack depth	relative error
0.1	$2.8946*10^{-3}$	0.099	1.0 %
0.2	$11.295*10^{-3}$	0.201	0.5 %
0.3	$26.447*10^{-3}$	0.300	0 %
0.4	$51.792*10^{-3}$	0.401	0.04 %
0.5	$95.267*10^{-3}$	0.498	0.4 %

In fig. 4 the FRF with some error due to measure and signal processing is showed.

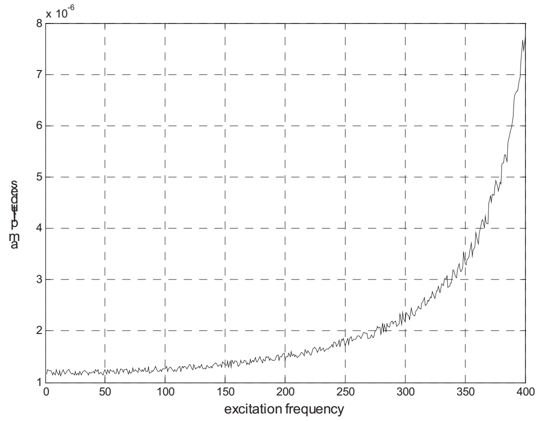

Fig. 4. FRF with some errors

In fig. 5 the identified local flexibility c_g, determined (from equation 5) for each frequency below the first natural frequency and corresponding amplitudes of vibration of FRF is showed.

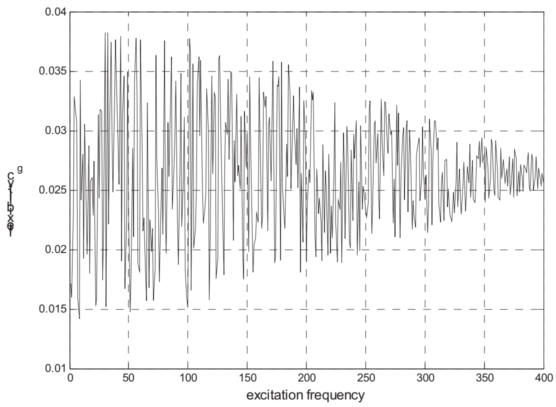

Fig. 5. Identified local flexibility c_g as a function of frequency excitation

Table 2 summarizes the modelled and identified from amplitudes determined with error (addition 3 % of amplitude multiplied by random value from (-1;1) range) crack depth with relative error of identification.

Table 1 Crack depth identification

modelled crack depth	mean value of c_g	identified crack depth	relative error
0.1	$2.832*10^{-3}$	0.098	2 %
0.2	$11.281*10^{-3}$	0.202	1 %
0.3	$26.321*10^{-3}$	0.299	0.33 %
0.4	$52.026*10^{-3}$	0.401	0.25 %
0.5	$95.490*10^{-3}$	0.501	0.2 %

5. SUMMARY

In part 2 of this work some method of crack depth determination (3-rd level of crack diagnostics) is described. Presented method is based on the crack location identification described in part 1 of this work.

The depth identification is based on FRF and inverse model of beam.

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dr inż. **Leszek MAJKUT** (ur. 1970), absolwent Wydziału Elektroniki, Automatyki, Informatyki i Elektrotechniki AGH (1995). Pracę doktorską dotyczącą wpływu lokalnej zmiany sztywności na amplitudę drgań i widmo częstości obronił w 1999 na Wydziale Inżynierii Mechanicznej i Robotyki AGH w Krakowie. Obecnie prace badawcze dotyczące diagnostyki wibroakustycznej, problemów związanych z odwracaniem modeli diagnostycznych oraz szeroko pojętej teorii drgań (drgania, wibroizolacja, hałas), ze szczególnym uwzględnieniem układów ciągłych i dyskretno-ciągłych prowadzi w Katedrze Mechaniki i Wibroakustyki AGH.