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AN ALGORITHM OF NAVIGATIONAL DATA INTEGRATION

Key words

Navigational data fusion, Kalman filter, integrated navigational system.

Summary

An algorithm devised to merge data obtained from several measuring instruments into one signal is described. Such a signal has applications in integrated navigational systems in objects equipped with a number of sensors measuring the same signals. The algorithm was verified by its implementation in the Matlab environment. A sea-going vessel is used as the object.

Introduction

The second part of the 20th century witnessed the beginning of integration in the field of navigation and its globalisation. The first integrated navigational systems were developed. These systems make use of various sources of navigational information in order to determine an accurate position of the vessel. Basic objectives of the integration include the following:
- The enhanced reliability of system operation,
- The reduction of measuring errors,
- The assurance of continuous operation, and
- An increased frequency of data acquisition.

In this connection, it becomes necessary to systematise the collection of various navigational data, so that they can be subsequently processed while controlling the object. The problem of the joint analysis of data obtained from a variety of navigational devices is a typical problem of multi-sensor data fusion [3]. One possible solution to the above problem can be reached by the application of an algorithm for data fusion using a multi-sensor Kalman filter.

1. Description of the algorithm

Let us consider a discrete stochastic system with a few sensors as follows:

$$\begin{gathered}\mathbf{x}(t+1)=\mathbf{\Phi}\cdot\mathbf{x}(t)+\mathbf{w}(t)\\ \mathbf{y}_i(t)=\mathbf{H}_i\cdot\mathbf{x}(t)+\mathbf{v}_i(t)\qquad i=1,2,\ldots l\end{gathered}\tag{1}$$

where:

- $\mathbf{x}(t)\in R^n$ – state vector,
- $\mathbf{y}_i(t)\in R^{m_i}$ – measurement vector of i–th sensor ($1\le m_i\le n$),
- $\mathbf{\Phi},\mathbf{H}_i$ – constant matrices of proper dimensions,
- $\mathbf{w}(t),\mathbf{v}_i(t)$ – disturbance vectors with a characteristic of white Gaussian noise with expected zero values and covariance matrices $\mathbf{Q}$ and $\mathbf{R}_i$, respectively.

The fusion of data set l of sensors is expressed by the weighted mean as follows:

$$\tilde{\mathbf{x}}(t)=\mathbf{A}_1(t)\cdot\hat{\mathbf{x}}_1(t)+\mathbf{A}_2(t)\cdot\hat{\mathbf{x}}_2(t)+\ldots+\mathbf{A}_l(t)\cdot\hat{\mathbf{x}}_l(t)\tag{2}$$

where:

- $\mathbf{A}_i(t)$ – weight matrices,
- $\tilde{\mathbf{x}}(t)$ – state estimates fusion vector,
- $\hat{\mathbf{x}}_i(t)$ – estimates of the state vector.

The estimates of state vector $\hat{\mathbf{x}}_i(t)$ for i-th subsystem (defined by a given sensor) is obtained by using the Kalman filter [1, 2]:

$$\begin{gathered}\hat{\mathbf{x}}_i(t\backslash t-1)=\mathbf{\Phi}\cdot\hat{\mathbf{x}}_i(t-1\backslash t-1)\\ \mathbf{E}_i(t)=\mathbf{y}_i(t)-\mathbf{H}_i\cdot\hat{\mathbf{x}}_i(t\backslash t-1)\\ \mathbf{P}_i(t\backslash t-1)=\mathbf{\Phi}\cdot\mathbf{P}_i(t-1\backslash t-1)\cdot\mathbf{\Phi}^T+\mathbf{Q}\\ \mathbf{K}_i(t)=\mathbf{P}_i(t\backslash t-1)\cdot\mathbf{H}_i^T\cdot\left[\mathbf{H}_i\cdot\mathbf{P}_i(t\backslash t-1)\cdot\mathbf{H}_i^T+\mathbf{R}_i\right]^{-1}\\ \mathbf{P}_i(t\backslash t)=\mathbf{P}_i(t\backslash t-1)-\mathbf{K}_i(t)\cdot\mathbf{H}_i\cdot\mathbf{P}_i(t\backslash t-1)\\ \hat{\mathbf{x}}_i(t\backslash t)=\hat{\mathbf{x}}_i(t\backslash t-1)+\mathbf{K}_i(t)\cdot\mathbf{E}_i(t)\end{gathered}\tag{3}$$

where:

$\hat{\mathbf{x}}_i(t \backslash t-1)$ – state vector estimate determined without the knowledge of measuring data at instant *t*,

$\mathbf{E}_i(t)$ – innovation vector at instant *t*,

$\mathbf{P}_i(t \backslash t-1)$ – covariance matrix for filtration errors determined without the knowledge of filter gain at instant *t*,

$\mathbf{K}_i(t)$ – filter gain matrix at instant *t*,

$\mathbf{P}_i(t \backslash t)$ – updated covariance matrix for filtration errors,

$\hat{\mathbf{x}}_i(t \backslash t)$ – updated estimate of the state vector.

Weight matrices are determined from this formula [4, 5] as follows:

$$\mathbf{A}_i(t) = \left[\sum_{j=1}^{l} \mathbf{P}_{jj}^{-1}(t)\right]^{-1} \cdot \mathbf{P}_{ii}^{-1}(t) \tag{4}$$

where:

$\mathbf{P}_{ij}(t)$ – matrix of the cross-covariance of filtration errors between *i*-th and *j*-th subsystem of the system described by the equation (1).

Matrices of the cross-covariance of filtration errors are determined from the following formula [4, 5]:

$$\mathbf{P}_{ij}(t) = \left[\mathbf{I}_n - \mathbf{K}_i(t)\cdot\mathbf{H}_i\right]\cdot\left[\mathbf{\Phi}\cdot\mathbf{P}_{ij}(t-1)\cdot\mathbf{\Phi}^T + \mathbf{Q}\right]\cdot\left[\mathbf{I}_n - \mathbf{K}_j(t)\cdot\mathbf{H}_j\right]^T \tag{5}$$

where

$\mathbf{I}_n$ – $n \times n$ unit matrix.

Defined by the equations (2), (3), (4), (5) the algorithm is optimal, since it minimises the trace of fusion estimate error variance matrix [5].

2. Application of navigational data integration algorithm

The algorithm was verified by its implementation in the Matlab environment [6]. The state vector estimates fusion was examined in three cases, in which the measurements were obtained from the following:

- Two sensors, and the signal was linear,
- Two sensors, and the signal was non-linear, and
- Four sensors, and the signal was linear.

A sea-going vessel was chosen as an object. In all the computational experiments the following was assumed:

$$\mathbf{x}(t)=\begin{bmatrix} x(t) \\ y(t) \\ v_x(t) \\ v_y(t) \end{bmatrix}; \qquad \mathbf{\Phi}=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

where:

(x, y) – Cartesian coordinates of vessel's position $[m]$,

(v_x, v_y) – coordinates of vessel's speed $\left[m/s^2\right]$.

In addition, the following was assumed for the first computational experiment:

$$\mathbf{Q}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \ \mathbf{H}_i=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T; \ \mathbf{R}_i=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}; \quad 1 \le i \le 2 \tag{7}$$

For the second computational experiment, in turn, the following was adopted:

$$\mathbf{Q}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}; \ \mathbf{H}_i=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T; \ \mathbf{R}_i=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}; \quad 1 \le i \le 2 \tag{8}$$

An additional assumption for the third experiment was as follows:

$$\mathbf{Q}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \ \mathbf{H}_i=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T; \ \mathbf{R}_i=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}; \quad 1 \le i \le 4 \tag{9}$$

The initial state vector was as follows:

$$\mathbf{x}(t=0)=\begin{bmatrix}0\\0\\1\\1\end{bmatrix} \tag{10}$$

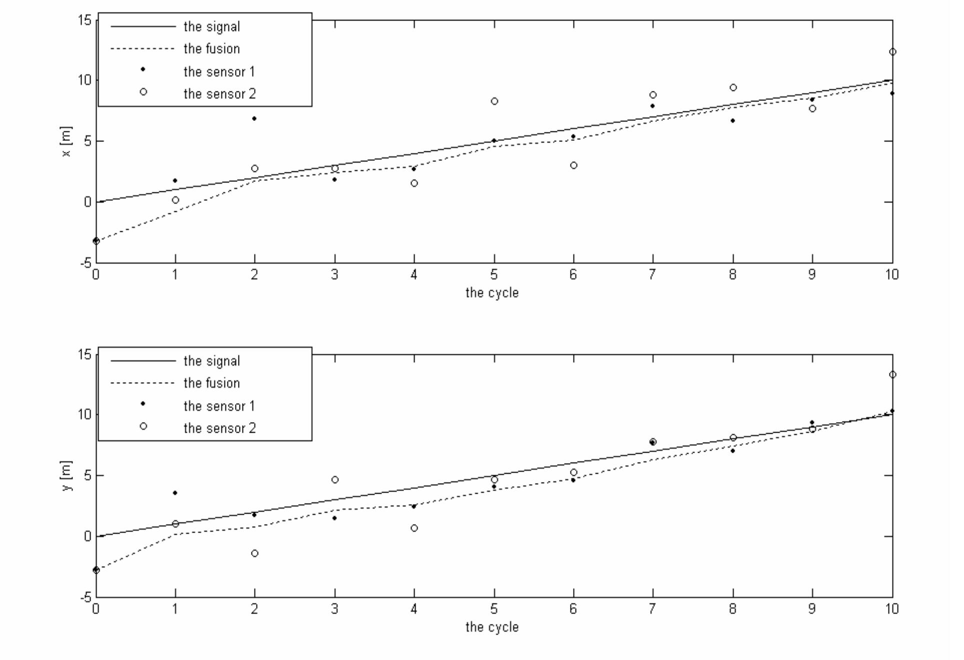

Fig. 1. Cartesian coordinates of the ship's position for measurement data from two sensors and a linear signal

The following was assumed as filtration error covariance matrices and filtration error cross covariance matrices (due to earlier definition of matrices **Q** and $\mathbf{R}_i$):

$$\mathbf{P}_0=\begin{bmatrix}3 & 0 & 0 & 0\\0 & 3 & 0 & 0\\0 & 0 & 0.1 & 0\\0 & 0 & 0 & 0.1\end{bmatrix} \tag{11}$$

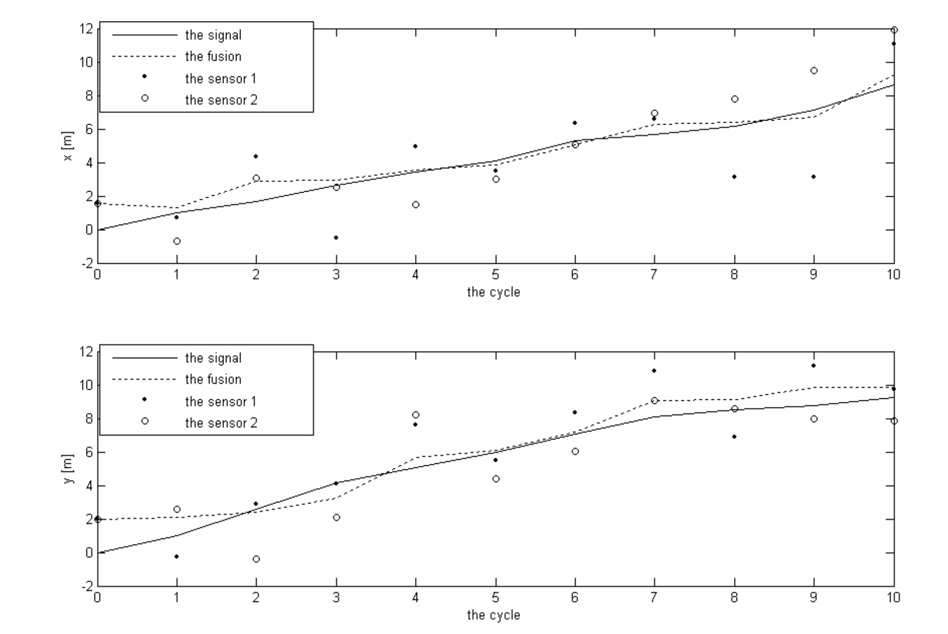

Fig. 2. Cartesian coordinates of the ship's position for measurement data from two sensors and a non-linear signal

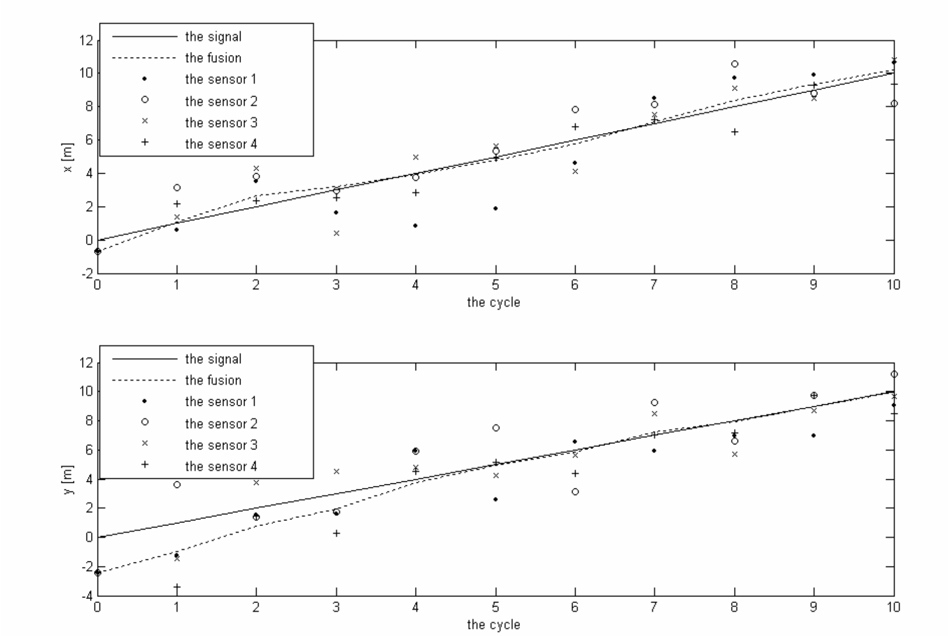

Fig. 3. Cartesian coordinates of the ship's position for the case when measurements data were obtained from four sensors and the signal was linear

The measurement vector for the *i*-th sensor was obtained by disturbing the state vector with white Gaussian noise having the expected zero value and variance matrix $\mathbf{R}_i$. As initial estimates for all the sensors, the same vector was adopted, obtained by disturbing the initial state vector with white Gaussian noise with the expected zero value and covariance value $\mathbf{P}_0$.

The diagrams present examples of the discussed computational experiments (Fig. 1, Fig. 2, Fig. 3). They confirm the correct operation (in laboratory conditions) of the described algorithm. Additionally, it may be inferred that a greater number of measuring instruments will imply a more accurate fusion of measurement estimates.

Conclusion

The above presented algorithm of navigational data integration, enabling a combination of data received from a number of various measuring devices into one signal ensures an increased reliability of system operation and reduces measurement errors, which has been confirmed by the computational experiments.

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Reviewer:
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Algorytm integracji danych nawigacyjnych

Słowa kluczowe

Fuzja danych nawigacyjnych, filtr Kalmana, zintegrowany system nawigacyjny.

Streszczenie

W artykule opisano algorytm umożliwiający połączenie w pojedynczy sygnał danych pozyskanych z kilku różnych urządzeń pomiarowych. Znajduje to zastosowanie w systemach nawigacji zintegrowanej dla obiektów posiadających kilka sensorów, mierzących te same sygnały. Algorytm został zweryfikowany poprzez zaimplementowanie go w środowisku Matlab. Jako obiekt wybrano statek morski.