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A SEARCH FOR THE WEIGHTS AND PARAMETERS OF THE PROCESSES CONSTITUTING CAVITATION EROSION

Key words

Cavitation erosion, cavitation loading, erosion processes.

Summary

A statistical description of the cavitation erosion process has been presented. The energy delivered to a material due to cavitation loadings was assumed equal to the sum of the energies used in the processes constituting the erosion process. A statistical dependence of the cavitation erosion effectiveness on the loading and material parameters was regarded. The mathematical formulation of the process presented in the paper makes a prediction of cavitation erosion possible, provided that energy input factor, partition coefficients and statistical parameters of constituent processes are known. Some suggestions on the search for partition coefficient values were revealed.

Introduction

The cavitation erosion process is analysed from the point of view of energy consumption in constituent processes of a stochastic nature. A kinetic approach is applied. Mass loss rate is assumed proportional to the difference of the rates of the energy supply to the surface layer and the energy used for the crack closure process as well as other geometrical/environment processes participating in retarding the crack development.

The goal and rationale for using this approach, the prediction of cavitation erosion efficiency, is possible if the values of energy partition coefficients and energy input factor and the statistical parameters – dependent on the material parameters – are known. The effort is justified due to the lack of satisfactory solutions of such problems as a prediction of the erosion progress, setting up a relationship between material properties and its cavitation resistance in uniform functional formulation or distinguishing the features decisive for the constitution of the cavitation resistance at a given stage and conditions of the process.

Method and scope of the work, having a priori known the forms of the probability functions of the constituent processes an adjustment of the theoretical erosion curves to the experimental ones, derived from the International Cavitation Erosion Test. This has led to the assessment of the employed parameters for Armco iron and establishing their appropriate physical meaning. The forms of the probability functions for the cracking were also confirmed experimentally for Armco iron. Energy partition for various component processes strictly results from material constitutive properties; however, in the search for appropriate coefficients, the variational calculus was postulated to work, provided that the real erosion intensity is known.

As a result of the research carried out, the procedures of finding some parameters (indispensable for erosion efficiency prediction) were tested and their feasibility was confirmed for Armco iron.

The general idea of the work consists in delivering a simplified description of the cavitation erosion, having caught the statistical dependence of the material destruction on its mechanical parameters and the loading conditions.

1. Outline of the physical problems

Surface erosion of materials placed within the cavitation cloud zone is caused by forces generated due to single or collective bubble implosion [1]. Cyclic or eternal vanishing of bubbles is a result of an increase in the ambient pressure [2].

Due to the randomness of the pulses, impact angles and local properties of the target microstructure, cavitation erosion is a stochastic process, depending both on the loading regime and properties of the eroded material. By its characteristics, cavitation erosion is a fatigue process, as it was proved by Richman [3, 4].

It seems admissible to consider cracking a process of external source energy transfer into the surface energy of crevices [5]. Cracking initiated on the body surface is an efficient process, both for brittle and elasto-plastic materials. The process frequently takes a form of spallation [6]. Simultaneous to the process of the surface layer erosion, the process of nucleation and the association of micro-cracks within deeper layer occurs. An increase in the micro-crack's density is driven by cavitation loading until their critical level is achieved and an extraction of a piece of the material occurs. The paramount role of the micro-cracks in the cavitation erosion process is linked to the very high rate of the loading force cycles and the fatigue nature of the process [7]. Therefore, the fracture of heterogeneous solids is preceded by the nucleation of cracks on their surface and the generation of a set of micro-cracks, indispensable for the creation of the critical cracks. The effectiveness of the processes mentioned above depends – in minor extent – on the state of the surface, i.e. the surface energy and geometry of the solid edge.

At the same time, fatigue crack propagation in metallic materials is slowed down or even retarded due to the cracks closure phenomenon [8, 9].

The range of the force action in elasto-plastic materials is non-local with the resultant stress field built up within confined volume of the body [10]. Thus, one can refer the amount of energy delivered to the body to the critical level of the material strength.

2. Energy partition approach

The cavitation erosion process consists of component processes as energy trapping due to plastic deformations, the transformation of energy absorbed into the surface energy of microcracks and cracks developed, including cracks induced on the material surface. Energy used for plastic micro-deformations in the vicinity of the cracks tips contribute to the attenuation of the erosion. Having assumed that the statistical distributions of the component processes are known, the material volume loss can be expressed as proportional to the quantity (1):

Volume loss rate ~ integral over the loaded area of the formula

$$\begin{aligned}&\lambda_0\int_0^t g(\tau)d\tau+\lambda_1\int_0^t\int_0^w f_1(w-\tau)g(\tau)d\tau dw+\lambda_3\int_0^t\int_0^u h_1(\tau)d\tau du+\\&\lambda_2\int_0^t\int_0^z h(z-s)\int_0^s f_1(s-\tau)g(\tau)d\tau ds dz-\lambda_4\int_0^t\int_0^y g_1(y-\tau)g(\tau)d\tau dy\end{aligned}\tag{1}$$

multiplied by the real effective intensity loading; e.g. the stream of delivered energy, which contribute to the material destruction.

Where, the subsequent integrals refer to the rates of energy absorption appropriately in plastic deformations, in micro-cracks production, in cracks generation on the material surface, in micro-cracks joining and the cracks closure.

The probability distribution $g(\tau)$ and the probability density distributions $f_1(\tau)$, $g_1(\tau)$, $h(\tau)$, $h_1(\tau)$ are time realisations of the component processes. Due to the assumption that the highest probability of crack generation, joining and closure refers to the action of the single impulse and that collective interactions are not of very high importance, the Rice statistical distribution function was applied for $f_1(\tau)$, $g_1(\tau)$, and $h(\tau)$. Keeping in mind that the more loading pulses, the more probable damage of the surface layer, the Weibull distribution was chosen to describe the process of crack production at the material surface ($h_1(\tau)$). The distributions refer to the probabilities of the process events which occur within the unit area of the surface loaded and under the unit energy stream delivered to the material (intensity I).

Coefficients λ_i are the weights of the constituent processes – they express the energy partition among the processes at any moment. Under the severe simplification, they are independent of time. Therefore, the following relation is valid: $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$.

In order to predict the cavitation erosion process, based on the presented approach, one should know the values of the distribution parameters, weight coefficients λ_i and intensity I.

Experimental investigations revealed the relationships between the distribution parameters and the strength parameters of the material [11]. An example is presented in Fig. 1.

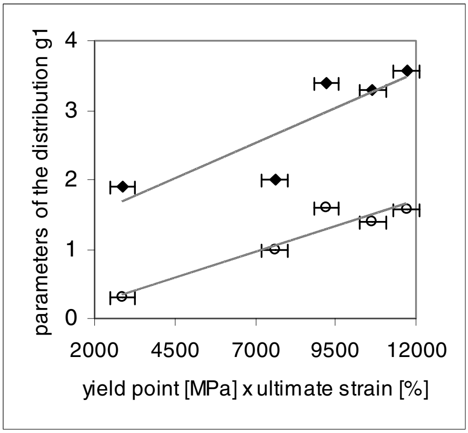

Fig. 1. Dependence of the parameters of the Rice distribution function g_1 on the product of the yield point [MPa] and the ultimate strain [%]. The relationship was found by approximations of the experimental curves of various materials

The distribution of coefficients λ_i is specific for given experimental conditions. It is influenced by the loading conditions – the characterisation of the loading impingement and depends on the material properties. The way of finding the coefficients λ_i is to solve the reverse problem when boundary conditions are known. The proposed method of proceeding is a variational method. Facing the same difficulties in the case of finding the intensity I value, a similar problem is to be solved.

3. Variational method

Under the assumption that mass loss rate (σ_1) is proportional to the difference of the rates of the energy supply to the surface layer (σ) and the energy used for crack closure process (as well as other geometrical/environment processes participating in retarding the cracks development), the following equation is written:

$$\sigma_1 = \lambda\sigma - (1-\lambda)(g_1 * \sigma) \tag{2}$$

where, g_1 is a probability density distribution for the crack closure process and $g_1 * \sigma$ is a convolution.

By simple transformation the Equation 2 takes the form:

$$\sigma_1 = -\int_0^t \{\lambda'\sigma - \lambda\sigma' + \lambda g_1(t-\tau)\sigma(\tau) + \lambda' g_1 * \sigma\} d\tau \tag{3}$$

Applying the variational method to Equation 3, one can obtain the following formula for λ, which may be useful when σ function is known from the other condition and the boundary conditions for λ are imposed. In that case, the expression is derived as follows:

$$\ln\lambda = -\int_0^t \frac{g_1}{2+\phi(\tau)} d\tau \tag{4}$$

where, $\phi = \dfrac{\partial(g_1 * \sigma)}{\partial\sigma}$.

If partition coefficient λ is a function of energy supply rate σ, the variational formulation leads to the equation as follows:

$$\sigma'\left(\frac{d\lambda}{d\sigma} + \sigma\frac{d}{d\sigma}\left(\frac{d\lambda}{d\sigma}\right)\right) + \sigma'\frac{d\lambda}{d\sigma} - g_1\left(1-\lambda-\frac{d\lambda}{d\sigma}\right) - \frac{d}{dt}\left(\sigma\frac{d\lambda}{d\sigma} + \lambda\right) = 0 \tag{5}$$

being in general difficult to solve.

4. Experimental investigations

Correctness of the formula (1) was verified by comparison of the data calculated and the experimental data obtained during the realisation of the International Cavitation Erosion Test [12]. Experimental and theoretical cumulative erosion curves for Armco iron are presented in Fig. 2. The experimental curve was taken at the vibratory rig [13].

Experimental verification of the assumption that distributions f_1 for Armco iron was of the Rice type was completed according to the method described in [14]. The picture of the sample used for investigations and an example of the identified crack are shown in Fig. 3. Normalised cumulative curves of the micro-cracks total length increase within the surface layer of the sample (cumulative distribution function for f_1) as presented in Fig. 4.

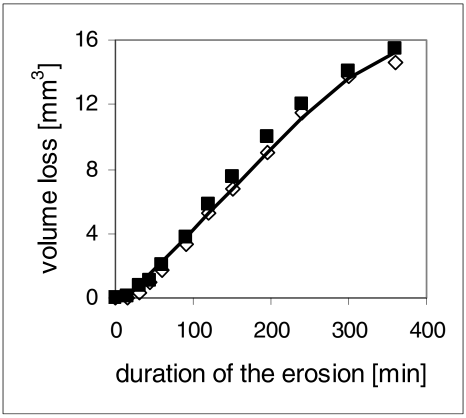

Fig. 2. Cumulative erosion curve of Armco iron tested at the vibratory rig at the Institute of Fluid-Flow Machinery. Experimental data was obtained for ICET. The theoretical curve was derived for the following model parameters:

$\nu_1 = 5$, $\nu_2 = -0{,}04$, $\nu_3 = 1{,}2$, $\nu_4 = 1{,}7$, $\nu_5 = 2$, $\nu_6 = 2$, $\nu_7 = 4$, $\nu_8 = 4$, $\nu_9 = 1{,}4$, $\nu_{10} = 3{,}3$,
$\lambda_0 = 0{,}229$, $\lambda_1 = 0{,}095$, $\lambda_2 = 0{,}008$, $\lambda_3 = 0{,}002$, $\lambda_4 = 0{,}667$; $I = 123{,}99$

$\nu_{1\text{-}10}$ are values of the statistical parameters of distributions: g, f_1, h_1, h, g_1 respectively. I means the factor of the loading intensity

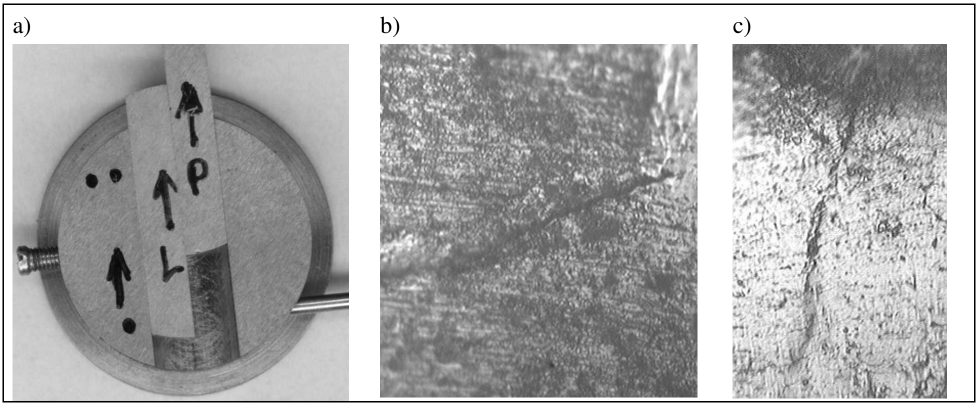

Fig. 3 (a) the sample used for investigation; (b) crack under the loaded area, almost parallel to the sample surface; (c) crack under the loaded area perpendicular to the sample surface

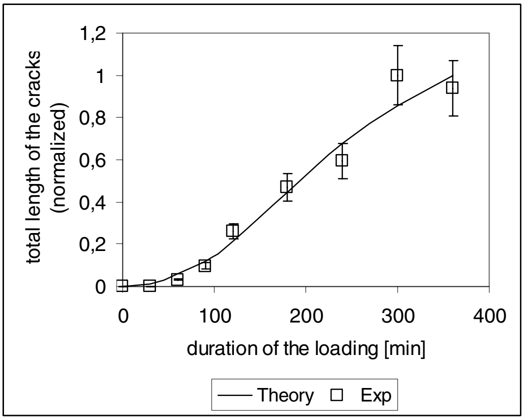

Fig. 4. Increase in total length of the cracks within the cross section of the surface layer of the sample of Armco subjected to cavitation loading

Experimental values were normalised to the maximal value detected up to 360 min of cavitation. The theoretical curve was drawn as the cumulative Rice distribution function for distribution parameters: 1,2 and 1,7 (as in Fig. 2).

5. Discussion

Results of the experiments for Armco iron (Fig. 2) support the thesis that the cavitation erosion process can be described in statistical formulation as the assemblage of the component processes. The problem of the search for the partition coefficients, the effective intensity of the loading and the statistical parameters is complex and is postulated to be partially solved. Thus, based on physical premises, special types of statistical distributions are suggested to be used -- the Rice and the Weibull types. Results presented in Fig. 4 indicate that the choice of the Rice distribution is right. Moreover, the relationships between the material parameters and the statistical distribution parameters can be established, enabling their definition in an easy way. Such relationships are conformed for distribution g_1 (Fig. 1). Unfortunately, there is no easy method of finding the partition coefficient for component processes. In this paper, the variational method is proposed in order to define the coefficient λ_4, which refers to the process of the crack closure.

An approach presented in the paper would be completed if the other mentioned variables were defined. Especially, the lack of the intensity of the effective loading I is essential.

Conclusions

1. Based on the mathematical formulation presented in the paper, the prediction of cavitation erosion efficiency is possible, if the values of energy partition coefficients and input factor and the statistical parameters are known. The difficulties are met in deriving their values theoretically and some experimental data is needed. At this approach, the model is a phenomenological one.
2. All parameters are specific for loading, geometry and material conditions, and may change their values in time. Therefore, the method is still approximate and its uncertainty is difficult to assess.
3. Cavitation erosion should be dealt with as a separate type of material damage due to the short time of pulse duration, the very small area of pulse interaction with solid surface and the randomness of the impingement, all allowing a large number of different ways of destruction.

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Acknowledgements

This work was accomplished under the research project No N52201031/3368 financed by the Polish Ministry of Science and Higher Education.

Recenzent:
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Procesy składowe erozji kawitacyjnej – poszukiwanie współczynników partycji i parametrów statystycznych

Słowa kluczowe

Erozja kawitacyjna, obciążenie kawitacyjne, procesy erozyjne.

Streszczenie

Przedstawiono uproszczony opis procesu erozji kawitacyjnej. Założono, że energia dostarczona do materiału w wyniku obciążeń kawitacyjnych jest równa sumie energii zużywanej w procesach składowych niszczenia i uwzględniono statystyczną zależność efektywności erozji od obciążenia i parametrów materiałowych ciała. Przedstawiono zapis matematyczny procesu, pozwalający na przewidywanie szybkości niszczenia materiału, pod warunkiem znajomości czynnika dostarczania energii, współczynników partycji i parametrów statystycznych rozkładów procesów składowych. Zaproponowano kilka sposobów poszukiwania wartości wymienionych parametrów.