STABILITY IN TECHNICAL DIAGNOSTICS

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Summary

The aim of the paper is the presentation of new methodological approach in the machine state monitoring system building process. It is related to the attempt of state change monitoring task affiliation to the behavior (dynamics) testing task after some time from certain starting point. The works draws attention to the solutions of technical stability theory, which might be useful tool of machine state recognition algorithm building. It describes its relation to the tasks defining monitoring system building that is: possible exploitational deviations with the dynamic behavior of the monitored object, the problem of diagnostic symptoms choice and the quantification levels choice allowing making the diagnostic decisions. It also covers possible methods of their implementation. It points to the purposefulness of phase image change of the diagnostic signals control, assigning to them the value of useful tool of process identification of origin and development of faults in the monitored object.

Keywords: stability, monitoring, dynamic behaviour, damage identification.

STATECZNOŚĆ W DIAGNOSTYCE TECHNICZNEJ

Streszczenie

Celem pracy jest prezentacja nowego podejścia metodologicznego w procesie budowy systemów monitorujących stan maszyn. Wiąże się on z próba powiązania zadania monitorowania zmian stanu kontrolowanego obiektu, z zadaniem badania jego zachowań (dynamiki) po upływie pewnego czasu od wybranego punktu startowego. Praca kieruje uwagę na rozwiązania teorii stateczności technicznej, które mogą być użytecznym narzędziem budowy algorytmów rozpoznawania zmian stanu monitorowanej maszyny. Opisuje jej powiązanie z określającymi budowę systemu monitorującego zadaniami tj.: badaniami możliwych zaburzeń eksploatacyjnych z zmianami zachowań dynamicznych monitorowanego obiektu, problemem wyboru symptomów diagnostycznych oraz doborem poziomów ich kwantyfikacji, umożliwiającej podejmowanie decyzji diagnostycznych. Omawia możliwe metody dla ich realizacji. Wskazuje na celowość kontroli zmian obrazów fazowych kontrolowanych sygnałów diagnostycznych, przypisując im walor użytecznego narzędzia identyfikacji procesu powstawania i rozwoju uszkodzeń monitorowanego obiektu.

Słowa kluczowe: stateczność techniczna, monitorowanie, zachowania dynamiczne, identyfikacja uszkodzeń.

1. INTRODUCTION

The function analysis of the monitored object heavily depends on its construction parameters and its exploitation conditions determining its dynamic behavior. Given such conditions monitored object's behavior might be foreseen by theoretical means, as for mechanical object and the deviation from the planned movement estimated thus assessing their acceptability from the exploitational point of view. The defining procedures might bring a new dimension to the monitoring process, bringing it down to simple analysis.

Lets assume that the monitored object is defined by the differential equations:

$$x_1 = f_1\,(x_1,.. \quad .., x_n\,, t)$$

$$\tag{1}$$

$$x_n = f_n\,(x_1\,,.. \quad , x_n\,, t)$$

where:

$$x^i = \frac{d\ x_i}{d\ t}\ ;\ i = 1, \ldots, n$$

$\boldsymbol{f}$ **:** $\mathbf{W} \to \mathbf{R}$ is some open subset of $\mathbf{R^n} \times \mathbf{R}$.

Its behavior is given by the equation set (1) solution and its differentiable function $\mathbf{\Phi_t = (\Phi_{t1},..}$ $\mathbf{..,\Phi_{tn})}$ defined in the range $\boldsymbol{I} \subset \mathbf{R}$, for which $(\mathbf{\Phi_t}\, , \boldsymbol{t}) \in \boldsymbol{U},$ for each $\boldsymbol{t} \in \boldsymbol{I}$ fulfilling the condition:

$$\Phi(t)/dt = f(\Phi(t), t) \text{ for each } t \in I \qquad (2)$$

The projections (1) and (2) describe each point of phase space, their dynamics and location and also the behavior of the tested system for each coordinate. The describing solution of the monitored dynamic system (1) is given by function $\Phi_t(\boldsymbol{x(0)})$, where $\boldsymbol{x(0)} = (x_1,(0),x_2(0),\ldots,x_n(0))$ i 100 s the initial condition.

Its form $\Phi_t(\boldsymbol{x(0)})$ assigns in any moment $\boldsymbol{t}$, the position of the initial point $\boldsymbol{x(0)}$ after time $\boldsymbol{t}$ in phase space and its trajectory reflects the evolution of the tested system.

So the dynamic behavior analysis of the tested object might be reduced to its trajectory analysis and related to them change assessment task as a response to initial condition change or acting driving forces

2. THE RELATIONSHIPS OF TECHNICAL STABILITY PROBLEM TO THE CONTROLLED OBJECT'S STATE CHANGE MONITORING PROCESS

From the controlled object's state change monitoring point of view there are important questions of assessment of possible behavior of the trajectories in the span of observation time. They relate to the question if the trajectories reflecting the technical state of the tested object, starting from any point of the surroundings of $\boldsymbol{x}(0)$ after time $\boldsymbol{t}$, will appear again in proximity of this point. It might be formed as a question of trajectory attraction area, that is the neighborhood $\boldsymbol{U}$ of set $\boldsymbol{A} \subset \boldsymbol{W}$ where for each $\boldsymbol{x}(0) \in \boldsymbol{U}$ the trajectory $\Phi_t(\boldsymbol{x(0)})$ remains in $\boldsymbol{U(A)}$ and tends to $\boldsymbol{A}$, when $\boldsymbol{t \to \infty}$.

The answers to those questions might be related to the stability testing of the state space points and their trajectories. A good testing criteria to solve a number of tasks appearing in the monitoring process of the machine state change might a criteria of technical stability [6], deciding about the resistance of the controlled object to the deviations appearing during the exploitation time. It decides about the limitations of the dynamic system movement solutions with existing deviations influencing the system during the exploitation time caused by initial conditions change or the acting forces. The result of small deviations from the stable state is widely assumed to be a stability criteria.

For such assessment it is necessary to make some assumptions:

- acceptable deviation of movement trajectory from its stationary state (from the point of view of safe exploitation of the analyzed object);
- acceptable range of changes for initial conditions;
- predicted level of external and internal disturbances constantly influencing the controlled object during its exploitation.

The question of stability of the analyzed technical object with forces $f(\dot{x}, x, t)$ and disturbances $R(\dot{x}, x, t)$ acting on it, and its movement described by the equation:

$$\ddot{x} = f(\dot{x}, x, t) + R(\dot{x}, x, t) \qquad (3)$$

demands analysis of its solution and answering the questions:

- Does the system have zero-solution and what is the course of the solution in the neighborhood of the zero-solution?
- What are the areas of initial conditions, where solutions coming out of them have the same qualitative course?
- What is the influence of perturbations $R(\dot{x}, x, t)$ of right side of the equation (1) on qualitative course of the solution?

Taking into consideration that requirements, the definition of technical stability for mechanical system described with the differential equations:

$$\dot{x} = f(x, t) + R(x, t) \qquad (4)$$

where $\boldsymbol{x, f, R}$ are vectors in the $\mathfrak{R}^n$ space:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} \boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \\ \ldots \\ f_n \end{bmatrix} \boldsymbol{R} = \begin{bmatrix} R_1 \\ R_2 \\ \ldots \\ R_n \end{bmatrix} \qquad (5)$$

where functions $\boldsymbol{f(t, x)}$ $\boldsymbol{R(t, x)}$ are defined in a range included in n+1-dimensional space:

$$t \geq 0,\ (x_1, x_2, \ldots, x_n) \in \mathbf{G} \subset \mathbf{E}_n \qquad (6)$$

where: $\mathbf{E}_n$ denotes linear normed n-dimensional space and functions $R_i(t, x_1, x_2, \ldots, x_n)$ are disturbances acting constantly, with assumption:

$$\| R(t, x_1, \ldots x_n) \| \leq \delta \qquad (7)$$

might be phrased as follows.

Let there be two ranges Ω and ω included in G such that Ω is closed, limited and includes origin of coordinates and ω is open and included in Ω.

Let us assume that the solution of the analyzed system (4) is x(t) with the initial condition $x(t_0)=x_0$.

If for each x_0 belonging to ω, x(t) stays in the range Ω for $t \geq t_0$ with disturbance function satisfying inequality (7), then the system (4) is technically stable in terms of range ω, Ω and limited constantly acting disturbances (7). According to this definition of technical stability, each movement trajectory derived from the range ω is supposed to stay in the range Ω for $t \geq t_0$.

For monitoring systems, allowing monitored signals to momentarily exceed the accepted levels, the term of technical stability might be weakened to the condition where each trajectory exceeding the range ω is supposed to stay in the range Ω for $t_0 \leq t < T_0$, where T - t_0 is the duration of the movement. With such a condition we deal with a technical stability in limited time.

3. TECHNICAL STABILITY TESTING AND ITS DIAGNOSTIC RELATIONS

From the point of view of technical diagnostics, including the need of application of the technical stability theory solutions for the monitoring system development [1, 2], there are interesting questions of algorithm building for condition recognition for technical stability loss of the overviewed object. It could be realized by solving the system of differential equations (4) describing dynamic behavior of the monitored object or analyzing the course of their solution with qualitative methods.

The latter method might be tightly connected to the machine state changes monitoring process. It is related to the examination of the phase portraits of the solutions, that is curves $x(t),\ \overset{o}{x}(t) = y$ on the plane $x, \overset{o}{x}$ called phase plane, which might be the subject of the monitoring. Such methods are usually included to the set of topological methods of solutions testing of differential equations (4). They allow dynamic behavior analysis of the monitored object, with the disturbances constantly acting and non-linear, which are significant for the fault appearing process [7, 9, 12], including their early phases.

Their testing procedures, based on some topological facts, related to the existence of some constants of homomorphic transformations formed as theorems, allow qualitative assessment of dynamic behavior of the analyzed object and related to it conditions of technical stability loss.

The most often used method is Lapunov method [10], where properties are used of properly chosen, for the dynamics description of the controlled object, scalar function ***V(x,t)***. Its derivative testing along the solutions (behaviors) of the equation set (4) determines the decision leading to its stability.

The theorem on which it is based states that if there exist a scalar function ***V(x,t)*** of the class $\mathbf{C}^1$, defined for each $\boldsymbol{x}$ and $t \geq 0$ fulfilling requirements:

$$V(x, t) > 0 \text{ for } \boldsymbol{x} \neq \mathbf{0}$$

$$\dot{V}(x,t) \leq 0 \text{ along solutions of (4)}$$

$$\text{for } \boldsymbol{x} \notin \boldsymbol{G} - \omega \qquad (6)$$

$$V(x_1, t_1) < V(x_2, t_2)$$

$$\text{for } \boldsymbol{x_1} \notin \boldsymbol{\omega} \text{ and } \boldsymbol{x_2} \notin \boldsymbol{G} - \boldsymbol{\Omega};\ \boldsymbol{t_1} < \boldsymbol{t_2}$$

then the object described by (4) is technically stable.

Referring results of that theorem to the issue of creating foundation for machinery state monitoring system, the Lapunov function ***V(x,t)*** should be built and should be checked by means of measured trajectories x, y of the tested object conditions (6). In building the Lapunov function ***V(x,t)*** directions from [3, 8] might be useful, or an effort made to define its form as total energy of the tested object.

Another way of testing the properties of the monitored trajectories from the point of view of the stability assessment of the monitored dynamic system is its testing by means of two functions [4]:

$$\Phi(x, y) = x\,y + \dot{x}\,\dot{y}\ ;\ \Psi(x, y) = x\,\dot{y} - \dot{x}\,y \quad (7)$$

Of which the positive or negative definition allow to assess the character of the monitored movement. Their dependent values allow assigning to the points of trajectories a direction characteristic for the point of entry, exit or slip related to the analyzed curve, which helps in determining the ranges ***G*** and ***Ω*** in the range of the monitored phase space.

The construction foundation for quantifier of the monitored trajectories properties from their stability point of view might also be searched based on the topological retract method – Ważewski method [12]. In that method ranges are built, limited with curves of the points of entry and exit of the equation set (4) solutions, from the ranges accepted as allowable.

As it emerges from the synthetic review of the technical stability testing methods [5] their usage for the monitoring system development is related to two tasks [1]:

1. Creation of the measurement tools providing observation of the phase portraits changes for the dynamic behavior of the monitored node of the system, defined by measurement:

$$x(t), \quad \overset{\circ}{x}(t) = y$$

2. Building of the quantifier for the monitored courses by implementation of the technical stability testing algorithms, based on the Lapunov function method or two functions Φ(x, y) and ψ(x, y) method or the retract method.

From the practical realization of the monitoring system point of view, the solution for the task one is not problematic. There are more inconveniences with determining positively or negatively defined Lapunov function for the monitored construction and the differential equation set describing its dynamics. The significant simplification of the task appears when the monitoring system with defined location of the measurement sensors is dedicated to modal parameters change of the controlled object. In case of using the two functions Φ(x, y) and ψ(x, y) method there may some difficulties appear in solving their functional equations, necessary to draw their zero-adjustment curves. A significant advantage of the method is fact that both functions for a non-linear system have identical form, which makes the method more universal for different applications. The usage of the retract method in turn forces the necessity of building some curve-limited range on which there are only points of entry or points of exit which might appear as a significant application problem.

4. SUMMARY

Diagnosed object's state change testing – considered as a task of nonlinear dynamic system behavior analysis which state changes with time – creates a new applicational perspective for monitoring system construction. It might be related to the task of technical stability loss conditions testing in the analyzed object. In technical application it means the necessity of phase image testing of the dynamic behavior of the tested object, related to the chosen conditions of the stability loss assessment. It is considered in relation to the assumed range of constantly acting disturbances and the initial condition range. It well defines the process of transition of the object into the state of functional incapacity. It relates fully to the non-linear physics of the phenomenons describing the process. It ties realized recognition with the dynamic state of the monitored object and with its constructional and exploitational parameter changes, which makes it universal.

The practical application of the phase trajectory change control method seems to be very useful tool of fault emerging and development process identification. It might be its main quality factor and is easily adoptable to practical application. It is not filtering non-linear effects and phenomenons of frequency structure change of the monitored diagnostic signals related to the fault development, which might be its very unique advantage.

It might be easily adapted to the task solutions related to the estimation of the time needed for the monitored variables to leave their acceptable range or issues demanding stability loss probability estimation. It also allows monitored object's behavior analysis by random disturbances and to phrase estimations of their infallibility.

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