

DAMAGE IDENTIFICATION IN PRESTRESSED STRUCTURES USING PHASE TRAJECTORIES

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Summary

The work deals with the utilization of the diagnostics of prestressed structures. Considered in work damage were: change (the decrease) the tension force as well as the crack in the prestressed element. Phase trajectories were determined by simulation on analytical Bernoulli-Euler model of beam. In order to compare the sensitivity proposed method of diagnosis (changes in phase trajectories) the other vibro-based damage detection symptoms are shown (the first natural frequency).

Keywords: prestressed structures, diagnostics, crack, phase trajectories.

IDENTYFIKACJA USZKODZEŃ KONSTRUKCJI SPRĘŻONYCH Z WYKORZYSTANIEM OBSERWACJI TRAJEKTORII FAZOWYCH

Streszczenie

Praca dotyczy wykorzystania trajektorii fazowych do diagnostyki struktur sprężonych. Analizowanymi w pracy uszkodzeniami były: zmiana (obniżenie) siły sprężającej oraz pęknięcie elementu sprężonego. Trajektorie fazowe wyznaczone zostały za pomocą symulacji na modelu Bernoulli-Eulera belki. Pęknięcie zamodelowano w oparciu o mechanikę pęknięcia i twierdzenie Castigliano. W celu porównania wrażliwości proponowanej metody diagnostyki (obserwacja zmian trajektorii fazowych) porównano ją ze zmianami częstości drgań własnych uszkodzonego elementu sprężonego.

Słowa kluczowe: struktury sprężone, diagnostyka, pęknięcie, trajektorie fazowe.

1. INTRODUCTION

The detection of damages in concrete prestressed elements of building constructions determines the serious challenge for present technique. The used at present non-destructive procedures of diagnosis the damages, it means the method: visual observations, ultrasonic, radiographical, or magnetical analysis, possess the many essential limitations. The most often to effective their utilization, it is necessary to carry out many of additional actions connected with correctness of diagnosis process, as well as the a priori knowledge about the place of potential damage. From here using these methods in places with difficult access, and on early stage of damage evolution is limited and burdened with large uncertainty.

Mentioned above conditions required research on finding the new methods of damage detection, the global one, which based on changes of dynamic response of constructions, would permit to estimate degree of damage. This approach was checked out with the changes of different useful measures of wave propagation in prestressed constructions (eigenfrequency, eigenvectors, damping factor, amplitudes of forced vibration), going out with

assumption, that they are the function of physical properties of diagnosed construction (mainly the stiffness) connected with the current state of the system. They are the object numerous publications [3, 4]. But in processes of diagnosis connected with these methods the essential difficulty is the matters connected with nonstationarity of measuring conditions, or the changes of construction conditions (temperature, humidity, unknown loads of diagnosed construction) could cause the change of dynamic properties. It imposes the necessity of use a complicated algorithms and the techniques of the measured signals processing, dedicated the processes of symptoms separation.

Helpful information in building such diagnostic of damage detection system could be found by computer simulations. Computer models are based on accepted models of defects and could lead to new observational premises.

The content of present article with this direction of investigations is connected. Going out with methodology of investigation of technical stability [2] and conception of it connection with tasks of diagnostics [1], we seek on the numeric experiments of prestressed beam, the relationships of type "defect - symptom", which could be helpful

for process of damage detection of studied constructional elements.

2. PRINCIPLE OF PRESTRESS

Principle of prestress in shown in fig. 1

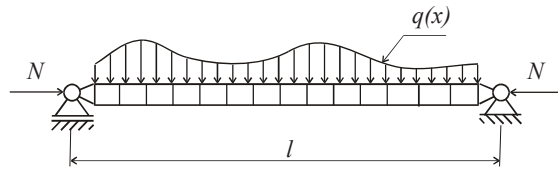


Fig. 1. Principle of prestress

If a set of separate elements was subjected by a compressive force, it can to carry its own weight as well as possible external transverse load - $q(x)$. These loads lead to appearing of a tension zone in transverse section. Showed in fig. 1 set of elements does not possess any tensile strength, but it gains this strength thanks to compressive force N .

In fig. 2 the distribution of stresses was showed in the most loaded section of prestressed element. The distribution of reduced stress depends on point where the compressive force is applied.

In the first case the force is applied in beam axis therefore the stress (from compression) has solid value. In the second case the compressive force is applied in $1/3$ height of beam, what causes to create the complex state of stress - the bending with the tension - therefore linear distribution of stress.

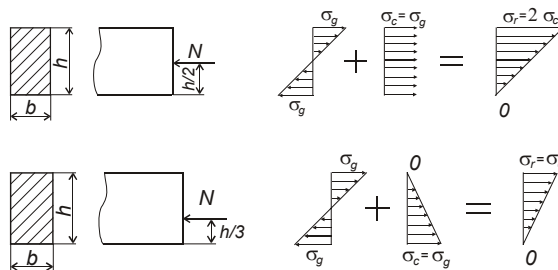


Fig. 2. Stress distribution

In both cases of compression, in no phase of element work, in no fibre of transverse section the tensile stress will not appear. Only larger or smaller compressive stress will appear.

In dependence however on the point of the compressive force applying to obtain the same effect it is necessary, in first case (the force in centre of the beam height), to apply the twice larger force N , and also the maximum compressive stress is twice larger than in the second case. In case of eccentric compression it is possible to apply material about twice-smaller ultimate compressive stress k_c so, it is more economical solution.

Assuming, that the force is applied in $1/3$ height of beam we can define the maximal value of the compressive force:

$$N_{\max} = b \cdot h \cdot k_c$$

where:

b, h – the dimensions of the beam cross section,
 k_c – the ultimate compressive strength.

3. DESCRIPTION OF THE PROBLEM

Analysed beam model in fig. 3 is shown

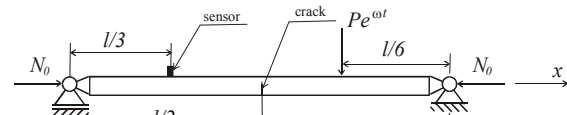


Fig. 3. Model of analysed beam

The equation of motion of the beam can be written as

$$X^{(4)}(x) + \beta X''(x) - \lambda^4 X(x) = 0 \quad (1)$$

where: $\beta = \frac{N}{EI}$; $\lambda^4 = \frac{\rho A}{EI}$; ρ - mass density of the beam, $A = b \times h$ - the cross-sectional area, EI - bending stiffness, N - compressive force.

The solution of equation (1) has form:

$$X(x) = P \cosh \kappa_1 x + Q \sinh \kappa_1 x + R \cos \kappa_2 x + S \sin \kappa_2 x \quad (2)$$

where:

$$\kappa_1 = \sqrt{\frac{-\beta + \sqrt{\beta^2 + 4\lambda^4}}{2}}; \quad \kappa_2 = \sqrt{\frac{\beta + \sqrt{\beta^2 + 4\lambda^4}}{2}}$$

Integral constants P, Q, R, S depends on boundary conditions.

In work, two kinds of prestressed element damage were considered: the loss of prestress, that is the decrease the force N_0 and a crack in the element (without the loss in prestress).

As a damage symptoms the change of phase trajectories were used. The trajectories are determined in section with co-ordinate $x = l/3$ (sensor in fig. 3).

As excitation a harmonic force with amplitude P and frequency ω was accepted. About frequency was assumed that it has been smaller than the first natural frequency of the beam.

Proposed method sensibility has been compared with sensibility of method based on measurement the first natural frequency.

4. LOSS IN PRESTRESS

The compressive force is most often introduced by tensioning of a prestressed concentrate wire either before hardening of concentrate - prestressing, or after hardening - posttensioning.

Every prestressed element may lose some of its prestress force due to creep and relaxation from a long period of service under design or overloaded vehicles.

Fig. 4a shows the changes of phase trajectories, and fig. 4b shows the first natural frequency change for the prestressed beam with force $N_0 = 0.7 N_{\max}$.

An analysed damage was the change (percentage decrease) of the prestressed force.

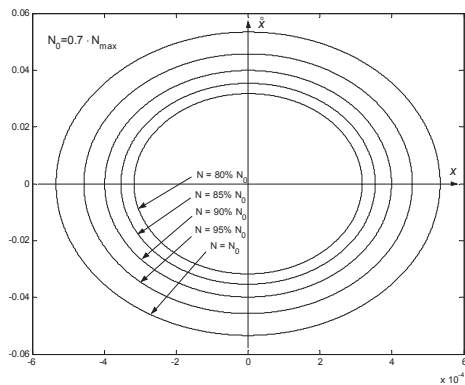


Fig. 4a. Damaged beam phase trajectories

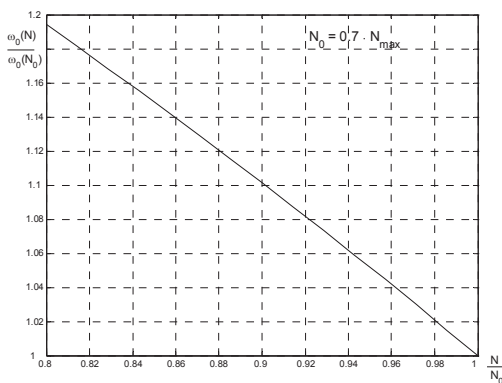


Fig. 4b. Natural frequency changing

Fig. 5a shows the changes of phase trajectories, and fig. 5b the first natural frequency change for the beam with prestress force $N_0 = 0.5 N_{max}$

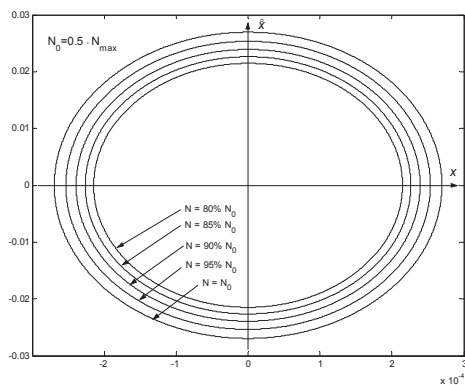


Fig. 5a. Damaged beam phase trajectories

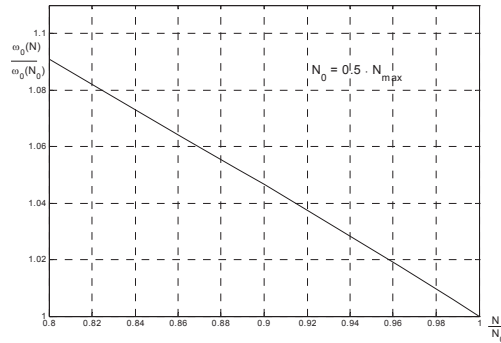


Fig. 5b. Natural frequency changing

Fig. 6a shows the changes of phase trajectories, and fig. 6b the first natural frequency change, for the beam with force $N_0 = 0.3 N_{max}$

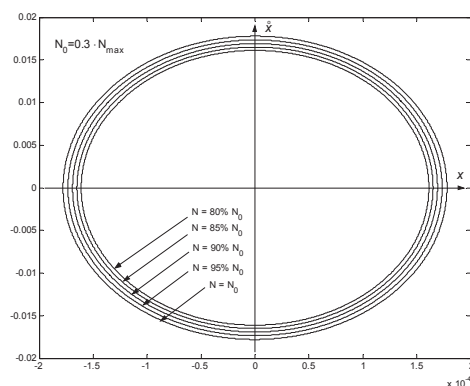


Fig. 6a. Damaged beam phase trajectories

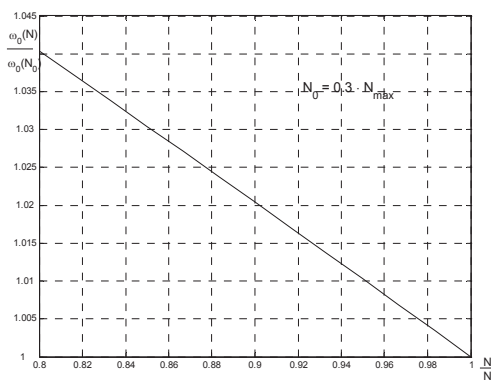


Fig. 6b. Natural frequency changing

Analysis of curves showed in figs 4, 5 and 6 leads to conclusion that the observation of phase trajectories permits to detect the smaller change in compressing force than the first natural frequency change. Therefore the proposed method of diagnostics is more sensitive than the most often used method based on the eigenfrequency change method.

5. CRACK MODEL IN PRESTRESSED BEAM

The model of crack is based on fracture mechanics laws and Castigliano theorem [9, 10].

The fracture mechanics studies allows finding relations between global quantity G - Energy Release Rate determining the increase in elastic strain energy for infinitesimal crack surface increase [11]:

$$G = \frac{\partial U}{\partial A_p}$$

and local quantity K - Stress Intensity Factor (SIF), which is function of crack depth a :

$$G = \frac{1-\nu^2}{E} \cdot K_I^2$$

where:

- G - energy release rate represents the elastic energy per unit crack surface area,
- A_p - area of crack,
- ν - Poisson ratio,
- E - Young modulus,
- K_I - Stress Intensity Factor (SIF) of mode I .

In case of prestressed element one have to take under consideration fact that the normal stress come from both bending moment and the longitudinal (prestressed) force, that is

$$K_I = K_{I_g} + K_{I_w}$$

K_{I_g} - Stress Intensity Factor of mode I for bending moment M_g ,

$$K_{I_g} = \sigma_g \cdot \sqrt{\pi \cdot a} \cdot F_{I_g} \left(\frac{a}{h} \right)$$

where:

- σ_g - normal stress from bending moment,
- a - depth of crack,
- F_{I_g} - correction function [6],

K_{I_w} - SIF of mode I for axial force N_0 ,

$$K_{I_w} = \sigma_w \cdot \sqrt{\pi \cdot a} \cdot F_{I_w} \left(\frac{a}{h} \right),$$

where:

- σ_w - normal stress from axial force,
- F_{I_w} - correction function [6].

Total increase the elastic strain energy due to the crack has form

$$U = \int_{A_p} G dA_p.$$

The above designations demonstrate that elastic strain energy will depend on: square of bending moment $M_g(x_p)$, square of longitudinal (prestressed) force N_0 and product of both $M_g(x_p)$ and N_0 .

Hence, the crack has been modelled as [2x2] flexibility matrix containing c_g and c_w coefficients on the main diagonal, and flexibility coefficients c_{gw} and c_{wg} outside the diagonal. Relation between displacements (longitudinal $u(x)$ and lateral $y(x)$) from the right and left hand side of the cross-section with crack and longitudinal force N_0 and bending moment $M_g(x_p)$ in this cross-section is given by matrix relation [9, 10]:

$$\begin{bmatrix} c_g & c_{gw} \\ c_{wg} & c_w \end{bmatrix} \begin{bmatrix} M_g(x_p) \\ N_0 \end{bmatrix} = \begin{bmatrix} y'(x_p^+) - y'(x_p^-) \\ u(x_p^+) - u(x_p^-) \end{bmatrix}$$

Individual flexibilities included in the flexibility matrix can be calculated using the Castigliano theorem:

$$c_g = \frac{\partial^2 U}{\partial M_g^2(x_p)} \quad c_w = \frac{\partial^2 U}{\partial P_w^2(x_p)}$$

$$c_{gw} = \frac{\partial^2 U}{\partial M_g(x_p) \partial P_w(x_p)}$$

$$c_{wg} = \frac{\partial^2 U}{\partial P_w(x_p) \partial M_g(x_p)}$$

According to the Schwarz's theorem the sequence of differentiation has no effect on the final result, which means that $c_{gw} = c_{wg}$.

6. DIAGNOSTICS OF CRACKS IN PRESTRESSED ELEMENT

Described in section 5 model of crack was used to analysis of changes of phase trajectories as well as first frequency the prestressed beam. As it is showed in fig. 3 crack is in beam centre ($x_p = l/2$).

Fig. 7a shows the changes in phase trajectories, and fig. 7b the change in first natural frequency for the beam with prestress force $N_0 = 0.7 N_{max}$. An analysed damage was the change (increase) the relative crack depth - $\alpha = a/h$.

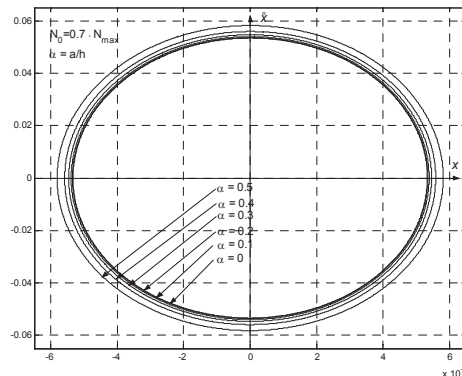


Fig. 7a. Damaged beam phase trajectories

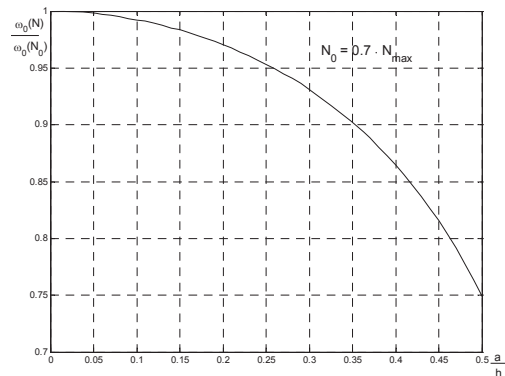


Fig. 7b. Natural frequency changing

Fig. 8a shows the phase trajectories changes and fig. 8b the change in the first natural frequency for the beam with prestress force $N_0 = 0.5 N_{max}$.

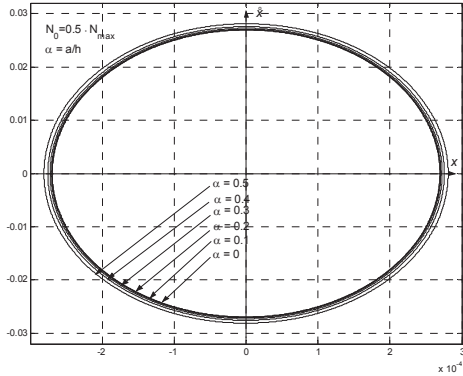


Fig. 8a. Damaged beam phase trajectories

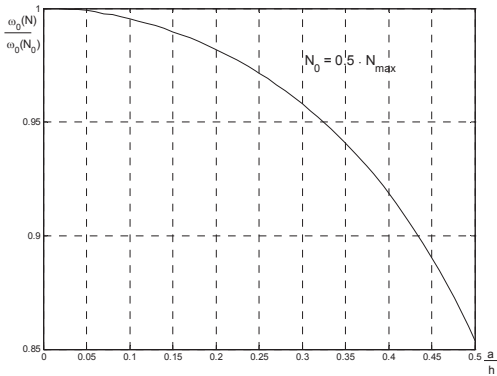


Fig. 8b. Natural frequency changing

Fig. 9a shows the phase trajectories changes and fig. 9b the change of the first natural frequency for the beam with prestress force $N_0 = 0.3 N_{max}$.

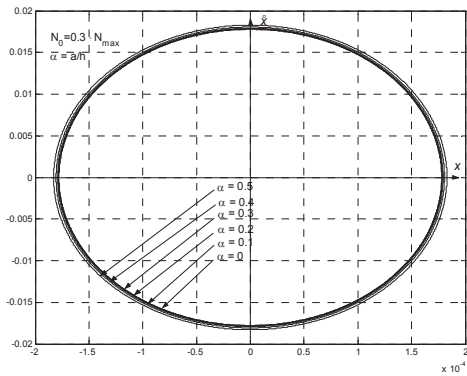


Fig. 9a. Damaged beam phase trajectories

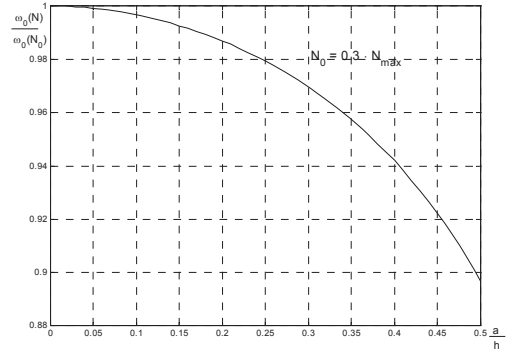


Fig. 9b. Natural frequency changing

The change of trajectories showed in figs 7, 8 and 9 leads to conclusion that the observation of phase trajectories permits to detect the crack the prestressed element.

7. SUMMARY

Analysis of the diagrams showed in figs 4-9 leads to conclusion that the observation of phase trajectories permits to detect the damage of prestressed element. The comparison of the trajectory changes (figs "a") and first natural frequency (figs "b") permits to affirm that the proposed method of diagnostics is more sensitive.

The observation of crack element trajectories (figs 7-9) could lead to wrong conclusion that this method of diagnostics is not suitable to detect this kind of damages. The change of phase trajectories strongly depend on this, how "far" the frequency of excitation is from the beam natural frequency. For example if we want to monitor crack in beam with prestress force $N_0 = 0.3 N_{max}$, we may "better" choose the frequency of excitation in diagnostic experiment. Results in such case were showed in fig. 10.

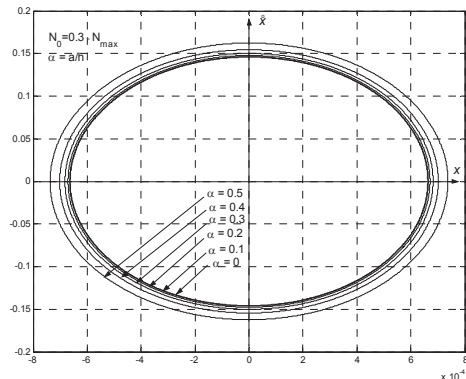


Fig. 10. Damaged beam phase trajectories

LITERATURE

- [1] Batko W.: *Stateczność techniczna – nowa perspektywa modelowa dla budowy rozwiązań systemów monitorujących zmiany stanu maszyn*. Mat.XXXIV Ogólnopolskiego Sympozjum Diagnostyka Maszyn. Wyd. Wydz.Transportu Politechnika Śląska CD-Węgierska Górka 2007.
- [2] Bogusz W.: *Stateczność techniczna*, PWN, Warszawa 1972.
- [3] Carpinteri A., Spagnoli A., Vantadori S.: *Mechanical damage of ordinary or prestressed reinforced concrete beams under cyclic bending*. Engineering Fracture Mechanics 72(2005) pp. 1313-1328.
- [4] Hamed E., Frosting Y.: *Free vibrations of cracked prestressed concrete beams*. Engineering Structures 26(2004) pp. 1611-1621.
- [5] Kaufman S., Olszak W., Eimer Cz.: *Budownictwo betonowe*, tom III Konstrukcje sprężone, Arkady, Warszawa 1965.
- [6] Murakami Y.: *Stress Intensity Factors Handbook*, Pergamon Press Oxford, 1987.
- [7] Majkut L.: *Identyfikacja pęknięcia w belkach na podstawie pomiaru częstości własnych kwartalnik AGH Mechanika 24 (2005)*.
- [8] Majkut L.: *Identyfikacja pęknięcia w belkach o znanych warunkach brzegowych*. Diagnostyka, 2004, 32 str. 107-116.
- [9] Majkut L.: *Modelowanie pęknięcia w wibroakustycznej diagnostyce według modelu*. Diagnostyka, 34 (2005), str. 15-22.
- [10] Majkut L.: *Wibroakustyczne symptomy pęknięcia belki*, Zagadnienia Eksploatacji Maszyn, 40 z.4, str. 165-181.
- [11] Neimitz A.: *Mechanika pęknięcia*, Warszawa PWN 1998.
- [12] Olszak W.: *Konstrukcje wstępnie sprężone*. Tom 1, PWN Warszawa 1955.



Prof. dr hab. inż. **Wojciech BATKO**, prof. zw. AGH ur. 1949. Kierownik Katedry Mechaniki i Wibroakustyki AGH, autor i współautor ponad 230 publikacji, w tym 12 książkowych. Zajmuje się zagadnieniami dynamiki i wibroakustyki maszyn oraz zagadnieniami diagnostyki technicznej.



Dr inż. **Leszek MAJKUT** (ur. 1970), absolwent Wydziału Elektroniki, Automatyki, Informatyki i Elektrotechniki AGH (1995). Pracę doktorską dotyczącą wpływu lokalnej zmiany sztywności na amplitudę drgań i widmo częstości obronił w 1999 na Wydziale Inżynierii Mechanicznej i Robotyki AGH w Krakowie. Obecnie prace badawcze dotyczące diagnostyki wibroakustycznej, problemów związanych z odwracaniem modeli diagnostycznych oraz szeroko pojętej teorii drgań (drżania, wibroizolacja, hałas), ze szczególnym uwzględnieniem układów ciągłych i dyskretno-ciągłych prowadzi w Katedrze Mechaniki i Wibroakustyki AGH.

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