Ryszard SOBCZAK
Przemysłowy Instytut Automatyki i Pomiarów, Warszawa
Mateusz TURKOWSKI
Politechnika Warszawska Instytut Metrologii i Systemów Pomiarowych
Andrzej BRATEK, Marcin SŁOWIKOWSKI
Przemysłowy Instytut Automatyki i Pomiarów, Warszawa

MATHEMATICAL MODELLING OF LIQUID FLOW DYNAMICS IN LONG RANGE TRANSFER PIPELINES. 1

Key words

Leak detection systems, leak localization systems, mathematic models of pipelines.

Abstract

This paper presents the mathematical model of liquid flow dynamics in long distance transfer pipelines. The model was formulated on the basis of a real pipeline installation. It was developed as a result of research concerning the leak detection and localisation algorithms. The model takes into account not only the liquid pressure and flow velocity evolution along the pipeline, but also the impact of the installation elements, i.e. a pump station, valves and a receiving tank. The first part of this paper describes the pipeline installation's elements, defines the model equations and finally the numerical computations scheme.

1. Introduction

The aim of this paper was to present the pipeline installation model that primarily was developed to determine the suitability and efficiency of leakage detection and localisation methods. However, the modelling of liquid flow dynamics for long distance transfer pipelines is essential for the analysing of physical phenomena that occur during the operating of the pipeline. The phenomena that liquid transportation always involves are extremely difficult to be detected and examined without being supported with mathematical and computational methods.

Above all, the mathematical model presented in the paper enables the simulation of various technological situations related to liquid transportation, such as start up or shut down the pumping, switching inlet or outlet of the pipeline from one tank to another, liquid transport in steady state conditions, stand-by conditions and others. Moreover, the model is a suitable tool to observe and analyse the effects of several media transportation, i.e. when two or more media of diverse physical characteristics are being transported subsequently, one bathe after another, at the same time, through the same pipeline.

Therefore, there is no doubt that, for the analytical methods [1, 2, 3] of the leak detection and localisation, an accurate mathematical model describing the real pipeline in all technological situations is of greatest importance.

This paper consists of two parts. The first one describes the elements of the system (the pipeline itself and tanks, valves, pumps), defines the mathematical model as the system of differential equations and presents the numerical computations scheme. In the second part of the paper, the computational results as well as the mathematical model verification vs. the real pipeline system behaviour will be presented. The authors will focus especially on typical technological situations. At the end, the final conclusions will be presented.

2. Short description of a pipeline transport system

Figure 1 shows the simplified scheme of the pipeline transport system [4]. A liquid is drawn from the supply tank T_A (one of the tanks from the set A of the tanks) by the main pump (2) which is supported by the auxiliary pump (1), and pumped through a pipeline to the receiving tank T_B (one of the set B of the tanks). Generally, the tanks are not only filled to different levels, but also with products of various physical characteristics. Remotely controlled gate valve stations are installed along a pipeline every several kilometres. Moreover, it was assumed that there are no branch pipes or storage tanks between the primary pump and the receiving tank T_B.

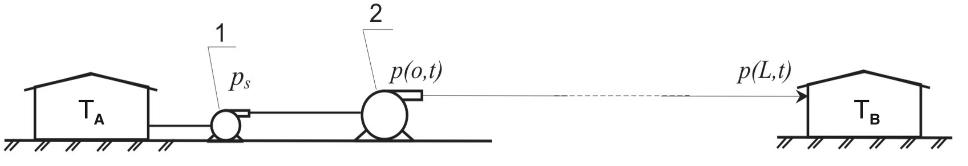

Fig. 1. Simplified scheme of a pipeline transport system

Each tank can be switched from one tank to another in the both supply A or receiving B tank set, which may result in a sudden disturbance of pressure at the pipe input or output. The change of the tank can also involve a change of a transported medium type. In the second case, when two or more media of different physical characteristics are being pumped through a pipeline sequentially, a distribution of pressure gradient along the pipeline changes radically at the region where the boundary of two various media is located.

The tests of the pipeline mathematical model [8] were based on data measured on the pipeline equipped with instruments for continuous measurement and the recording of the following parameters:

- $p(0,t)$ - pressure at the pipe inlet,
- $p(L,t)$ - pressure at the pipe outlet,
- $p(x_i,t)$ - pressure at i-points, located mostly directly before the valves along the pipeline,
- $q(0,t)$ and $q(L,t)$ - volumetric flow rates at the pipe inlet and outlet,
- temperature $\tau_i(x_i,t)$ at the pressure recording points.

t denotes the time, L is the pipeline length, the subscript i denotes the measurement point number, and x_i is the distance from the pipeline starting point (i.e. pump location).

Unfortunately, some parameters were not measured: the density of the pumped medium ρ, the pressure at the primary pump inlet p_s and the pressure just before the outlet tank p_h. There is no doubt that these measurements would certainly increase the quality of the model.

The measurement results were transformed by means of the A/D converters to digital form with the sample time T_s by means of the distributed computer system and transmitted to a SCADA system located at the process control room.

3. The modelling of the liquid transport process

The mathematical model should reconstruct as accurately as possible the static and dynamic characteristics of all elements of a pipeline system such as:

- the pump station,
- the pipeline,
- the valves,
- the receiving tank,

as well as interactions between these elements.

3.1. The primary pump

The pipeline installation was equipped with a WORTHINGTON 6UHD 2 pump. Its static characteristics for petrol of the density 755 kg/m^3, estimated with reference to the WORTHINGTON catalogue, was approximated as

$$\Delta H(q) = -2{,}6499 \cdot 10^{-6} q^3 + 0{,}73238 \cdot 10^{-3} q^2 - 0{,}14757\, q + 340{,}95 \quad (1)$$

where $\Delta H(q) = H_T - H_S$ is the increment of a liquid column between pump suction and force sides (in m) and q is the numerical value of the volumetric flow rate (in m^3/h) downstream the pump.

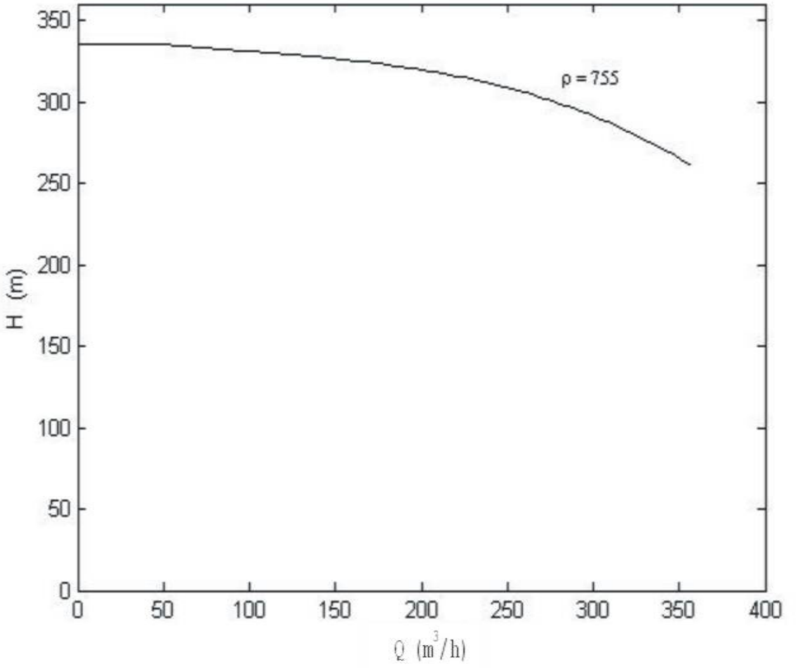

Fig. 2. Example of the pump characteristic

In the pump nominal working conditions ($200\ m^3/h < q < 300\ m^3/h$), the relative errors (differences between the approximation (1) and the catalogue data) are not greater than 0.2%. In any other (non-nominal) working conditions these errors are not greater than 0.63%.

Taking into account the equation (1), the pressure directly behind the pump is given by

$$p(0,t) = p_S(t) + \rho g \Delta H(q) \quad (2)$$

where $p_S(t)$ - the pressure at the auxiliary pump outlet (Pa), ρ - the density of the pumped medium (kg/m^3) and g - the acceleration due to the gravity (m/s^2).

The verification of the equation (2) was based on several of real pipeline data sets of $q(0,t)$ and $p(0,t)$, recorded during steady media flow. The uncertainty of the $p(0,t)$ measurements were not greater than 0.5%.

Considering the liquid delivery, the pump can be treated as a non-inertial object. The pump engine reaches its nominal rotational speed in 5-7 s while the volumetric flow rate $q(0,t)$ achieves more than 80 % of its steady state value. Later variation of $q(0,t)$ is implied by the pipeline dynamics, and according to the investigation (described in the second part of this paper), considerable back coupling delay in flow influence $q(0,t)$ on the pressure $p(0,t)$ behind the pump takes place. Hence, disregarding the delay reason, the transmittance obtained experimentally is given by

$$G(s)=(Ts+1)^{-2} \tag{4}$$

where T is the time constant and s is the Laplace operator. The equation (4) was included into the model to relate $q(0,t)$ with $\Delta H(q)$ in the equation (1).

The relationship between the volumetric flow rate $q(t)$ in m^3/h and the flow velocity $w(t)$ in m/s is given by the commonly known formula

$$q(t) = 3600 \frac{\pi D^2}{4} w(t) \tag{5}$$

where D is the pipe diameter in m.

3.2. The pipeline

In order to describe the pipeline dynamics for the modelling purposes [5] the pipeline was arbitrarily divided into sections at x_i points where measuring transmitters were installed on the real pipeline. It enabled the direct comparison of the variables values obtained by simulation with the real ones, recorded in the real pipeline. Additionally, each pipeline section between x_i and x_{i+1} points was divided into shorter, equal parts Δx_j, (together $K = 131$ segments).

Every segment fulfils a set of partial differential equations as a result of the law of mass and momentum conservation. In the case of a leak-proof pipeline (that is a pipeline for which neither mass decrement, nor momentum decrement are observed) and taking into account that $w(x,t) << c$, where c is the sound speed, these equations, according to [8] can be written as

$$\frac{\partial w(x,t)}{\partial x} + \frac{1}{E}\frac{\partial p(x,t)}{\partial t} = 0 \tag{6}$$

$$\frac{\partial p(x,t)}{\partial x} + \rho(x)\frac{\partial w(x,t)}{\partial t} = -\rho(x) g \sin\alpha - \frac{\lambda(x)\rho(x)}{2d} w(t)|w(t)| \tag{7}$$

and the sound speed of a liquid in a pipeline is given by formula commonly known [6] as Żukowski-Allevy formula:

$$c = \sqrt{\frac{E_C}{\rho(x)\left(1+\frac{D}{b}\frac{E_C}{E_R}\right)}} \tag{8}$$

where

- E_C – the elasticity coefficient of a liquid (Pa),
- E_R – the direct elasticity coefficient of a pipeline material (Young's modulus) (Pa),
- b – the pipeline wall thickness (m),
- D – the pipeline inner diameter (m),
- $\lambda(x)$ – the pipe friction factor,
- α – the angle of inclination of a pipeline segment.

The liquid density is a function of pressure p and temperature τ. Taking into account maximum differences Δp and $\Delta\tau$ along a pipeline, the values of the elasticity coefficient of a liquid-pipeline system and the liquid cubical expansion coefficient, it is possible to demonstrate that the change of the liquid density with respect to pressure is not greater than 0.3%, and with respect to temperature is not greater than 4.4%.

The pressure and flow velocity partial derivatives with respect to the time coordinate t can be rewritten from (6) and (7) as follows

$$\frac{\partial p(x,t)}{\partial t} = -E\frac{\partial w(x,t)}{\partial x} \tag{9}$$

$$\frac{\partial w(x,t)}{\partial t} = -\frac{1}{\rho(x)}\frac{\partial p(x,t)}{\partial x} - g\sin\alpha - \frac{\lambda(x)}{2d}w(x,t)\left|w(x,t)\right| \tag{10}$$

Numerating succeeded segments with the discrete parameter k, where $k = 1, 2, \ldots K$, and assuming that the lengths of segments are short enough, the partial derivatives with respect to the pipeline length can be approximated by the differences

$$\frac{\partial p(x_k,t)}{\partial x} \cong \frac{\Delta p(x_k,t)}{\Delta x_k} = \frac{p_k(t)-p_{k-1}(t)}{x_k - x_{k-1}} \tag{11}$$

$$\frac{\partial w(x_k,t)}{\partial x} \cong \frac{\Delta w(x_k,t)}{\Delta x_k^{'}} = \frac{w_k(t)-w_{k+1}(t)}{x_k - x_{k+1}} = \frac{w_{k+1}(t)-w_k(t)}{\Delta x_{k+1}} \tag{12}$$

From the equation pairs (9, 10) and (11, 12), after some evident algebraic transformations, the following relations are obtained

$$\frac{\partial p(x_k,t)}{\partial t} = \frac{dp_k(t)}{dt} \cong \frac{E}{\Delta x_{k+1}}\left[w_{k+1}(t) - w_k(t)\right] \tag{13}$$

$$\frac{\partial w(x_k,t)}{\partial t} = \frac{dw_k(t)}{dt} \cong -\frac{\lambda(x_k)}{2D} w_k(t)\left|w_k(t)\right| - g \sin\alpha_k - \\ - \frac{1}{\rho(x_k)\Delta x_k}\left[p_k(t) - p_{k-1}(t)\right] \tag{14}$$

Note that the set of partial differential equations (6), (7) has been converted to the set of 2K coupled ordinary differential equations (13), (14).

To resolve the sequence of equations (13, 14) MATLAB-SIMULINK software can easily be used. It is worth pointing out that the applied computational discretization scheme enables to obtain the pressures $p(x,t)$ strictly at the points of pipeline division into segments, while the flow velocities $w(x,t)$ are obtained for all the sections between these points, which is illustrated in figure 3.

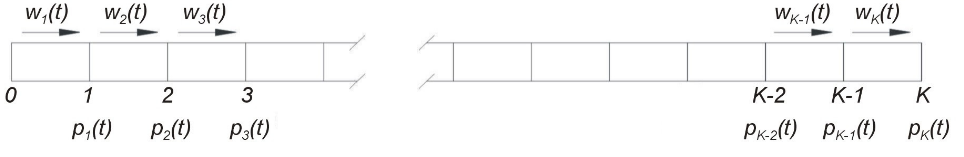

Fig. 3. Applied scheme of the computational discretization

3.3. The receiving tank

The real installation has been equipped with the outlet tank of diameter D_T = 56 m, height H_T =13 m with a floating roof causing small constant overpressure above the liquid surface. The liquid flows to the tank or from the tank through manifolds installed in the bottom part of the tank.

Note that a change of the liquid level in any tank connected to the pipeline results in a change of the pressure at either the input or the output of the pipeline, introducing in this way a disturbance to a transportation process. The change of the liquid level rate is given by

$$\frac{dH_T}{dt} = 3600\left(\frac{D}{D_T}\right)^2 \cdot \int w_K(t)dt \tag{15}$$

and the pressure variation rate at the pipeline output is given by

$$\frac{dp_K(t)}{dt} = \rho(x_K, t) g \frac{dH_T}{dt}. \tag{16}$$

Knowing that $w(t)$, the liquid velocity in a pipeline, is less than 1.1 m/s, then the maximum change rate of the liquid level in a tank is smaller than 0.12 m/h, while the maximum change rate of the pressure in the pipeline output is smaller than 1000 Pa/h.

However, switching the fully filled tank to the empty one causes strong disturbances, and it generates a sudden pressure jump at the output side of the system. This sudden change can achieve the magnitude even of $\Delta p_T = \rho g \Delta H_T \approx 0.11$ MPa.

3.4. The gate valve

The pins of the real pipeline valves are being moved by electrical drives with constant linear movement speed. The average time period of switching between one to another extreme position was about 150 s.

Well known formula defines the valve coefficient of a pressure loss z as the ratio of the pressure drop across the valve and the total kinetic energy of a flowing medium as shown below

$$z = \frac{\Delta p}{\rho \frac{w^2}{2}} \tag{17}$$

For numerical simulation the following relation, based on the above formula, was used

$$w = \sqrt{\frac{2}{z}} \cdot \sqrt{\frac{\Delta p}{\rho}} = K(x) \cdot \sqrt{\frac{\Delta p}{\rho}} \tag{18}$$

where z is the coefficient of a pressure loss, Δp is the pressure drop across the valve (Pa), w is the flow velocity of the liquid (m/s). $K(x)$ is the coefficient related to the valve's aperture $x=H/D$, H is the linear shift of the valve pin and D is the pipeline inner diameter.

It was also assumed that

$$\begin{aligned} &\text{when } x = 1, && \text{then} && K(x) = K_z, \\ &\text{when } x = 0, && \text{then} && K(x) = 0, \\ &\text{when } 0 < x < 1, && \text{then} && K(x) = K_z \cdot \varphi(x), \end{aligned} \tag{19}$$

where $\varphi(x)$ – is the ratio of a valve's clearance surface to the cross section of the pipe.

It may be shown (through analogy to the segmental orifice, [7]) that

$$\varphi(x) = \frac{1}{\pi}\arccos(1-2x) - \frac{2}{\pi}(1-2x)\sqrt{x-x^2} \tag{20}$$

Based on the pressure balance along a pipeline, the parameter K_Z was estimated as $K_Z = 0.45$. The clearance x of the valve is a function of time t as shown below

$$x = \begin{cases} 0 & \text{if } t_S \le t \\ \dfrac{t-t_S}{T_Z} & \text{if } t_S < t < t_S + T_Z \\ 1 & \text{if } t_S + T_Z \le t \end{cases} \tag{21}$$

where t_S is the start time of valve switching and T_Z is the time interval of entire switching.

3.5. Numerical computations

The computational scheme of the model, described in section 3, is shown in figure 4. This scheme only presents the model's blocks of the pump, the outlet tank, the first two pipeline's segments and the last one, while all other pipeline's segments are skipped.

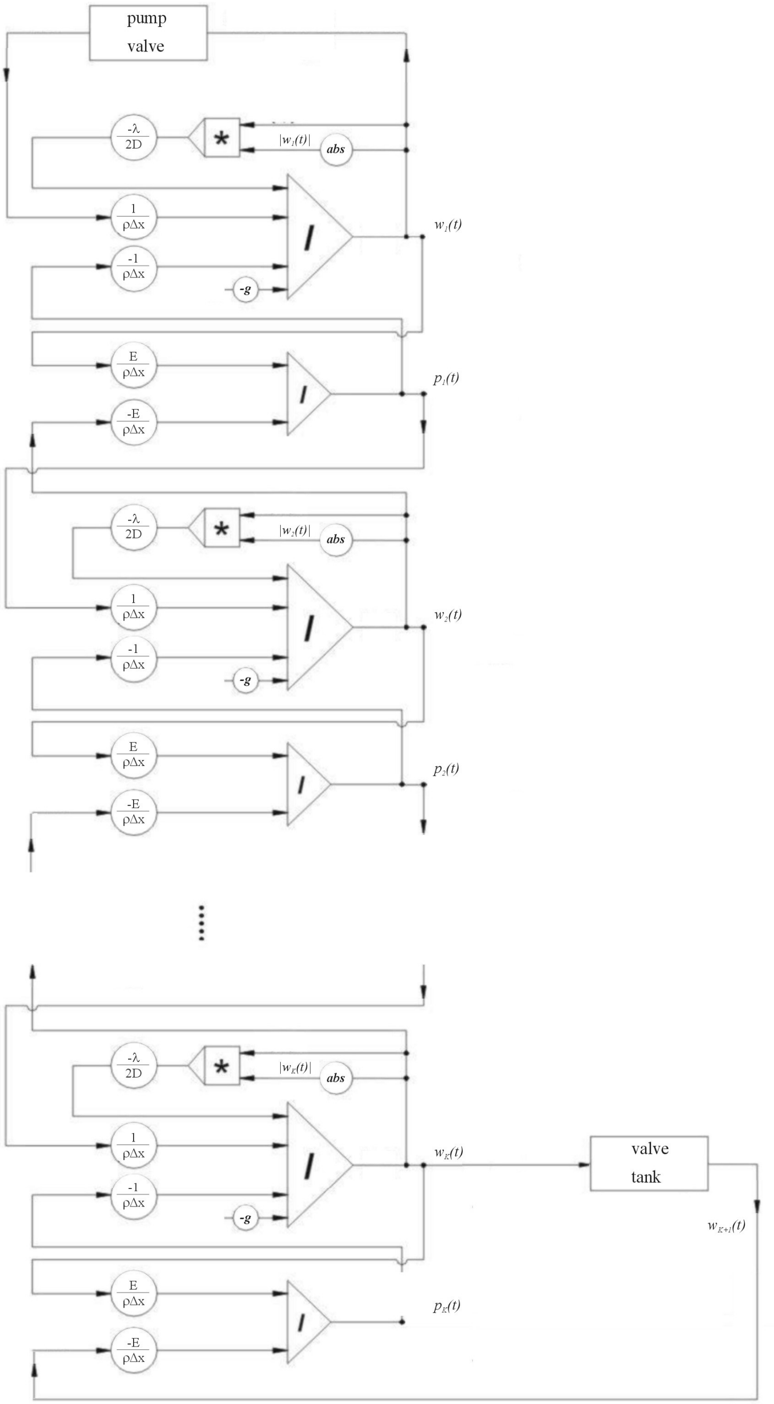

Fig. 4. Simplified computational scheme of the model

Acknowledgements

The paper is a result of carried out scientific investigations, financed with the national budget founds destined for education in 2004–2006, and with the Education Minister founds as a research project being realised within the scope of the Multi-Year Programme PW-004 entitled “Development of innovativeness systems of manufacturing and maintenance 2004-2008”.

Praca naukowa finansowana ze środków Ministra Nauki i Szkolnictwa Wyższego, wykonana w ramach realizacji Programu Wieloletniego pn. „Doskonalenie systemów rozwoju innowacyjności w produkcji i eksploatacji w latach 2004–2008”.

References

1. Bilman, L. Isermann R.: „Leak detection methods for pipelines”, Automatica, vol. 23, 1987, 3, 381–385.
2. Gunawikrama K.: „Leak detection for pipelines”, Msc Dissertation TU Gdańsk, Faculty of Electronics, Telekom. and Computer Science, nr PG/KSA/ZK34D/02/96,1996.
3. Kowalczuk Z., Gunawikrama K.: „Detekcja i lokalizacja wycieków w rurociągach przemysłowych”, rozdział 21 pracy zbiorowej pod redakcją J. Korbicza i J. Kościelnego: „ Diagnostyka procesów. Modele. Metody sztucznej inteligencji. Zastosowania”, WNT, Warszawa 2002.
4. Michałowski Witold S., Trzop S.: „Rurociągi dalekiego zasięgu”, wydanie IV, Fundacja Odysseum, Warszawa 2005.
5. Malczewski J., Piekarski M.: „Modele procesów transportu masy, pędu i energii”, WNT, Warszawa 1992.
6. Mitosek M.: „Mechanika płynów w inżynierii środowiska”. Oficyna Wydawnicza PW, Waszawa 1997.
7. VDI/VDE 2041 – Drosselgeraete fuer Sonderfaelle.
8. Sobczak R: „Detekcja wycieków z rurociągów magistralnych cieczy”, Nafta, Gaz, 2/01, 97–107.

Recenzent:
Mariusz SŁAWOMIRSKI

Modelowanie matematyczne dynamiki przepływu cieczy w rurociągach dalekiego zasięgu

Słowa kluczowe

Systemy detekcji nieszczelności, systemy lokalizacji nieszczelności, modele matematyczne rurociągów.

Streszczenie

W artykule przedstawiono model matematyczny dynamiki przepływu cieczy przez rurociąg transportowy dalekiego zasięgu. Model został opracowany w oparciu o rzeczywisty układ rurociągu. Model opracowano w wyniku badań nad algorytmami detekcji i lokalizacji nieszczelności. Model uwzględnia nie tylko przebieg ciśnienia i prędkości wzdłuż rurociągu, ale także wpływ najważniejszych elementów systemu, tj. stacji pomp, zaworów i zbiorników. Pierwsza część artykułu przedstawia opis elementów systemu, równania modelu i schemat obliczeń numerycznych.