ARTIFICIAL INTELLIGENCE ALGORITHMS COMBINED WITH THE PIES IN IDENTIFICATION OF POLYGONAL BOUNDARY GEOMETRY

Eugeniusz ZIENIUK, Andrzej KUŻELEWSKI, Wiesław GABREL

University of Bialystok, Faculty of Mathematics and Physics, Institute of Computer Science
Sosnowa Street 64, 15-887 Białystok, Poland, e-mail: {ezieniuk, akuzel, wgabrel }@ii.uwb.edu.pl

Summary

Identification of a shape of a boundary belongs to a very interesting part of boundary problems called inverse problems. Various methods were used to solve these problems. Therefore in practice, there are two well-known methods widely applied to solve the problem: the FEM and the BEM. In this paper a competitive meshless and more effective method – the PIES combined with artificial intelligence (AI) methods is applied to solve the shape inverse problems. The aim of the paper is an examination of two popular AI algorithms (genetic algorithms and artificial immune systems) in identification of the shape of the boundary.

Keywords: the PIES, identification of a boundary shape, genetic algorithms, artificial immune systems.

ALGORYTMY SZTUCZNEJ INTELIGENCJI POŁĄCZONE Z PURC
W IDENTYFIKACJI KSZTAŁTU WIELOKĄTNEJ GEOMETRII BRZEGU

Streszczenie

Identyfikacja kształtu brzegu należy do bardzo interesującej grupy zagadnień brzegowych nazywanej zagadnieniami odwrotnymi. Istnieje liczna grupa metod służących rozwiązywaniu takich problemów. Jednakże w praktyce do rozwiązywania zagadnień odwrotnych szeroko wykorzystywane są dwie metody: MES i MEB. W niniejszej pracy zaproponowano zastosowanie alternatywnej bezelementowej i bardziej efektywnej metody – PURC połączonej z algorytmami sztucznej inteligencji (SI) do identyfikacji kształtu brzegu. Celem pracy jest zbadanie efektywności dwóch popularnych algorytmów SI (algorytmów genetycznych i sztucznych systemów immunologicznych) w identyfikacji kształtu brzegu.

Słowa kluczowe: PURC, identyfikacja kształtu brzegu, algorytmy genetyczne, sztuczne systemy immunologiczne.

INTRODUCTION

The problem of identification of a shape of a boundary consists in reproducing unknown part of the boundary using experimental values of measurements obtained in a few points of the domain or on the boundary. In practice, the problem is reduced to finding the whole domain, where solutions in a few points only are known. Identification of a shape of the boundary belongs to ill-posed problems [6], that is small fluctuations in input data might lead to obtaining significantly different results.

The commonly applied method of solving these kind of problems is based on minimization of a functional. From a practical point of view, it is equivalent to multiple solving of analysis problem with modification of a shape of the boundary geometry. Various numerical methods have been used to solving analysis problems, however the commonly applied methods are the Finite Element Method (FEM) [1, 12] and the Boundary Element Method (BEM) [2, 12]. These methods there is assumed, that the shape of the boundary geometry is considered using the discretization process. Effectiveness of the BEM or the FEM can be noticed in solving of analysis problems, that is in searching solutions of differential equations in given domains and known boundary conditions. However, use of discretization is very inconvenient when it should be repeated in every iteration of the identification process.

The alternative method of solving analysis problems is the Parametric Integral Equation System (the PIES) [8,10]. The shape of the boundary geometry is inserted into mathematical formalism of the PIES and continuously defined, that is the discretization process is not required. Definition of the boundary geometry is reduced to assigning a few corner points. Therefore, the identification process is reduced to finding only a few corner points.

Another part of identification process using above-mentioned method is an optimization algorithm applied to modifying the shape of the boundary geometry. In previous paper [11] Newton's iterative method was successfully adapted in the case of boundary problems described by Laplace's equation. However, this algorithm requires to start the identification process from close neighbourhood of a searching point. In practice, it is rarely occurred situation. This problem can be solved using heuristic methods of optimization. Artificial neural networks (ANN), genetic algorithms (GA) and artificial immune systems (AIS) belong to widely used AI methods of optimization.

The aim of this paper is to compare results obtained from two methods of identification of the shape of the boundary geometry. Both are based on the PIES, however different methods of heuristic optimization (GA and AIS) are applied.

1. FORMULATION OF THE PROBLEM AND PROPOSED METHOD OF SOLVING

The identification of unknown part of boundary geometry in 2D boundary problem described by Laplace's equation is considered. In practice, the problem is reduced to finding an unknown part of the boundary on the basis of experimentally measured values in a few measurement points (u_1, u_2, …, u_l) in domain Ω or on given part of the boundary.

In process of reproducing an unknown part of the boundary the PIES is applied. The manner of defining and modifying of polygonal boundary geometry in the PIES is presented in fig. 1.

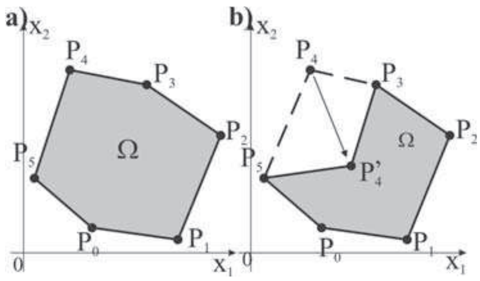

Fig.1. a) Defining of the shape of polygonal boundary geometry using corner points,
b) Modifying of the shape of polygonal boundary geometry by corner point P_4

Corner points P_i are designed for defining of a shape of the boundary geometry and, contrary to the traditional element methods, they are not making discretization of the boundary. After moving an arbitrary corner point P_i, great part of the boundary is changed. Using that point we can either modify or identify the shape of the boundary geometry. Therefore, in practice the identification process is reduced to finding coordinates of corner points.

The identification process is closely connected with experimental values of measurements obtained in measurement points on the boundary or in the domain. The commonly applied method of solving identification problems is based on minimization of the functional:

$$P(s,P_p)=\frac{1}{m}\sum_{i=1}^{m}\left[\tilde{u}_l(s_i,P_p)^*-u_l(s_i,P_p)\right]^2, \qquad (1)$$

where: $\tilde{u}_l(s_i,P_p)^*$ are results of experiments, $u_l(s_i,P_p)$ are results obtained from numerical method (i.e. the PIES), P_p – corner point.

The minimization is an iterative process and can be execute using various number of method. We take into consideration two heuristic methods: GA and AIS.

2. THE PIES FOR LAPLACE'S EQUATION

In order to solve the analysis problem the PIES for 2D Laplace's equation was applied. The system is presented by the following formula [8]:

$$0.5u_l(s_1)=\sum_{j=1}^{n}\int_{s_{j-1}}^{s_j}\Big\{\overline{U}_{lj}^{\,*}(s_1,s)p_j(s)- -\overline{P}_{lj}^{\,*}(s_1,s)u_j(s)\Big\}J_j(s)ds,\ \ l=1,2...n \qquad (2)$$

where: n – number of segments, s and s_1 – parametric variables $s_{j-1}<s<s_j$, $s_{l-1}\le s_1\le s_l$. Integrands are presented as follows:

$$\overline{U}_{lj}^{\,*}(s_1,s)=\frac{1}{2\pi}\ln\frac{1}{\left[\eta_1^2+\eta_2^2\right]^{0.5}},$$
$$\overline{P}_{lj}^{\,*}(s_1,s)=\frac{1}{2\pi}\frac{\eta_1 n_1^{(j)}(s)+\eta_2 n_2^{(j)}(s)}{\eta_1^2+\eta_2^2} \qquad (3)$$

where: S_k (k=j, l) – functions of any degree, which define boundary segments, $\eta_1=S_l^{(1)}(s_1)-S_j^{(1)}(s)$ and $\eta_2=S_l^{(2)}(s_1)-S_j^{(2)}(s)$. The boundary geometry in the PIES is inserted into integrands (3) and defining using curves (such as Beziér curves, B-spline, υ-spline) of arbitrary degree.

The solution in the domain $\boldsymbol{x}\in\Omega$ can be obtained using new integral identity [9] presented by the following relation:

$$u(\boldsymbol{x})=\sum_{j=1}^{n}\int_{s_{j-1}}^{s_j}\Big\{\hat{\overline{U}}_j^{\,*}(\boldsymbol{x},s)p_j(s)- -\hat{\overline{P}}_j^{\,*}(\boldsymbol{x},s)u_j(s)\Big\}J_j(s)ds,\qquad \boldsymbol{x}\in\Omega \qquad (4)$$

Integrands $\hat{\overline{U}}_j^{*}$, $\hat{\overline{P}}_j^{*}$ are presented by formulas:

$$\hat{\overline{U}}_j^{*}(\mathrm{x},s)=\frac{1}{2\pi}\ln\frac{1}{\left[\vec{r}_1^{\,2}+\vec{r}_2^{\,2}\right]^{0.5}},$$
$$\hat{\overline{P}}_j^{*}(\mathrm{x},s)=\frac{1}{2\pi}\frac{\vec{r}_1 n_1^{(j)}(s)+\vec{r}_2 n_2^{(j)}(s)}{\vec{r}_1^{\,2}+\vec{r}_2^{\,2}} \qquad (5)$$

where $\vec{r}_1=x_1-S_j^{(1)}(s)$ and $\vec{r}_2=x_2-S_j^{(2)}(s)$.

3. PROPOSED ALGORITHMS OF OPTIMIZATION

3.1 Application of GA

GA consist in the emulation of nature [5, 6]. The elementary processes occurring in nature (enabling individual species to survive and continuously adapt themselves to the environment, such as natural selection and reproduction) are finding application in GA. The idea of GA can be described as follows:

1. *Create the initial population randomly.*

2. *Calculate the fitness of each individual from the population (using the PIES).*
3. *At each iteration, do*:
   - 3.1 *Select individuals from population depending on their fitness (selection).*
   - 3.2 *Recombination (cross-over and mutation).*
   - 3.3 *Calculate the fitness (using the PIES).*
   - 3.4 *If solutions are close enough to measured values or fixed number of iterations are achieved stop the iteration process.*

The algorithm applied in this paper differs from traditional genetic algorithm described in [5]. Proposed algorithm uses the floating-point representation, ranking selection, simple and arithmetical crossover, and uniform and non-uniform mutation [6].

3.2 Application of AIS

Formal definition of AIS is presented in [3] and reads as follows: "Artificial immune systems are adaptive systems, inspired by theoretical immunology and observed immune functions, principles and models, which are applied to problem solving".

In practice, optimization algorithms based on AIS exploit two processes from theoretical immunology: cloning and changing of the shape of antibodies (hypermutation). Combination of these mechanisms is applied in clonal selection algorithm [4]. Application of this algorithm combined with the PIES can be described as follows:

1. *Create a random initial population in given domain of searching and calculate the fitness of each individual from the population (using the PIES).*
2. *At each iteration, do*:
   - 2.1 *Reproduce the whole population, that is create a number of clones of the each individual from the population.*
   - 2.2 *Perform hypermutation on the each clone.*
   - 2.3 *Calculate fitness for the each clone (using the PIES) and find only the best members from all groups of the clones.*
   - 2.4 *If solutions are close enough to measured values or fixed number of iterations are achieved stop the iteration process.*

4. RESULTS OF EXPERIMENTS

From practical point of view, we may consider any physical problem, which can be modelled by Laplace's equation.

The first problem of identification of the shape is presented in figure 2. Coordinates of two points P_0 and P_3 are unknown. Two domain of searching are considered: a=4 and a=12.

All experiments were repeated 100 times and mean values were calculated. In order to compare two methods of heuristic optimizations the number of calculations of fitness function in one iteration was fixed and equals 80. Thus, the size of population in GA was 80, and in AIS was 20 with 4 clones for an individual. In order to assess the quality of searching the distance between founded and real point was calculated. It was assumed that maximum number of iterations equals 200. Measurement of time of executing of one iteration was carried out on the same PC and applications were compiled by the same compiler. Results of experiments are presented in the table 1.

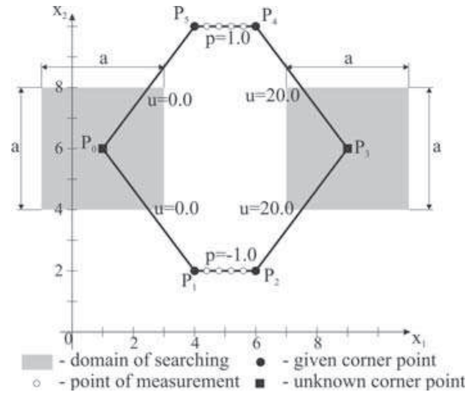

Fig. 2. Identification of the shape of the boundary (searching corner points do not belong to one segment)

Table 1. Results of the first identification problem

a	GA + the PIES		AIS + the PIES	
	min	mean	min	mean
	number of generations			
4	21	35.56	24	51.76
12	45	66.00	35	99.26
	value of fitness function			
4	5.1e-6	8.3e-6	5e-6	8.7e-6
12	4.9e-6	7.2e-6	5.1e-6	7.6e-6
	distance between founded and real point			
4	0.0018	0.012	0.0008	0.011
12	0.0022	0.010	0.0002	0.0097
	time of executing of one iteration [s]			
-	15.76		2.8	

As can be seen results obtained using GA and AIS are very similar. Only the application with AIS is about four times faster then the one with GA.

In the second example more complicated problem is considered (fig. 3). Two searching points are neighbours, that is the points create one segment.

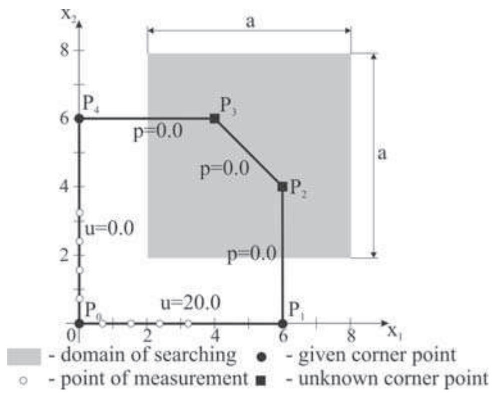

Fig.3. Identification of the shape of the boundary (searching corner points belong to one segment)

All experiments were repeated 100 times and mean values were calculated. The size of population in GA was 200, and in AIS was 50 with 4 clones for an individual. It was assumed that maximum number of iterations equals 1000. Results obtained in the second example are presented in the table 2.

Table 2. Results of the second identification problem

a	**GA + the PIES**		**AIS + the PIES**	
	min	**mean**	**min**	**mean**
	number of generations			
6	863	964.2	1000	1000
	value of fitness function			
6	6.1e-6	8.9e-6	4.1e-6	4.2e-6
	distance between founded and real point			
6	0.0217	0.182	0.0146	0.106
	time of executing of one iteration [s]			
-	28.46		4.5	

As can be seen similarity between results obtained using GA and AIS is much less than in first example. The application with AIS is about six times faster and more accurate then the one with GA.

5. CONCLUSIONS

Combination of the PIES and two methods of heuristic optimization in order to identify boundary geometry were presented in the paper.

Advantages of the PIES (in comparison with the BEM or the FEM) allow to reduce either the size of input data or the number of searching points. Hence, it increases the effectiveness of the presented algorithms (comparing to the traditional mesh methods).

Application of AIS seems to be more effective and accurate than GA, therefore it is not a big difference. Accuracy of both methods might be insufficient in more complicated problems. Therefore, in the authors opinion, the next researches should be oriented to searching hybrid methods, which combine heuristic algorithms of optimization with traditional ones.

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Eugeniusz ZIENIUK is an associated professor in Institute of Computer Science, University of Bialystok. His main research interests include computer methods in boundary problems. He is the author of the parametric integral equations system.

Andrzej KUŻELEWSKI is an assistant in Institute of Computer Science, University of Bialystok. His research interests include interval and fuzzy arithmetic, artificial intelligence methods in connection with boundary problems.

Wiesław GABREL is an assistant in Institute of Computer Science, University of Bialystok. His main research interests include artificial intelligence methods, especially genetic and evolutionary algorithms, in connection with boundary problems.