MULTIOBJECTIVE FUZZY APPROACH TO THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

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Summary

The paper presents a model of the vehicle routing problem with flexible (fuzzy) constraints. This kind of model allows a decision maker to explore a set of alternatives with diverse cost and constraint satisfaction levels. The model is tested on well-known instances of the vehicle routing problem with time windows adjusted to the fuzzy case. They are solved by a multiobjective Pareto Memetic Algorithm. The obtained results indicate that the introduction of fuzzy constraints leads to exploration of new alternatives which may be interesting to a decision maker.

Keywords: fuzzy vehicle routing, flexible constraints, multiobjective optimization.

WIELOKRYTERIALNY PROBLEM PLANOWANIA TRAS Z OKNAMI CZASOWYMI W WERSJI ROZMYTEJ

Streszczenie

Artykuł prezentuje model problemu planowania tras z elastycznymi (rozmytymi) ograniczeniami. Model takiego rodzaju pozwala decydentowi na wybór rozwiązania spośród zbioru alternatyw ze zróżnicowanym kosztem i stopniem spełnienia ograniczeń. Ten model został przetestowany na klasycznym zestawie instancji problemu planowania tras z oknami czasowymi dostosowanych do przypadku rozmytych ograniczeń. Rozwiązania są uzyskiwane przez użycie wielokryterialnego algorytmu memetycznego. Uzyskane wyniki wskazują na to, że wprowadzenie elastycznych ograniczeń prowadzi do odkrycia rozwiązań, które mogą być interesujące z punktu widzenia decydenta

Słowa kluczowe: problem planowania tras, elastyczne ograniczenia, optymalizacja wielokryterialna.

1. INTRODUCTION

Models of the Vehicle Routing Problem (VRP) proposed in literature mainly define it as single objective one (with minimization of cost of a whole routing plan), with hard constraints (e.g. related to capacity of vehicles or time of delivery) and deterministic parameters (like travel time). However, decision makers (DMs) involved in the planning process (e.g. the owner or executive staff of a transportation company) usually know that there is more variability connected with parameters of such problems: travel times vary depending on circumstances, times of delivery may be renegotiated with customers. Therefore, even optimal solutions of the mentioned models may not be enough for DMs to make up their minds, because there are scarcely any options (variants) generated from these models which could be a basis for 'what-if' analysis. Moreover, the optimal or nearly optimal solution might be regarded by DMs as a 'magic' one. Even if there are multiple solutions generated, they are related only to the defined objective function, constraints and parameters, so they do not reveal new possibilities to DMs, which could arise with slight changes to e.g. travel times and time windows.

Some models of the VRP try to address the issues of changes to parameters of the problem by introducing a level of constraint satisfaction. The model developed by Zimmermann [11] allows to define flexible constraints with fuzzy values. The original objective is converted into an additional constraint with its upper limit being a fuzzy number called aspiration level. The new objective is to maximize satisfaction of all constraints.

This model has major drawbacks: a decision maker may not understand the meaning of satisfaction value (it can be either a not fully satisfied constraint or not fully satisfied aspiration level). Also, the aspiration level has to be set in advance, forcing a DM to make a guess (potentially incorrect).

In [10] the authors introduced soft time windows, the violations of which result in a penalty. Value of the penalty depends on the type of customer and is a component of the objective function.

The aforementioned models do not provide a DM with a set of alternatives.

This drawback is not present in the model described in [8], which is a multiobjective one and allows constraints to be violated. One of the objectives is the weighted number of not fulfilled constraints. However, the extent to which they are violated is not measured. Due to this fact, the amount of violation may be either negligible or huge, thus unusable for a DM.

Another aspect of real-world situations which is not addressed by these models is the uncertainty of travel times. There are models which try to handle this aspect, but they are usually limited to stochastic approaches, rich in strong assumptions (e.g. independency of travel times or the type of their distribution) [3].

This paper describes a new model for the VRP with time windows (VRPTW) [9]. On one hand, the model allows for generation of multiple alternatives and, on the other hand, provides means of measuring the level of constraints satisfaction. Additionally, in contrast to the stochastic approach, the fuzzy model of travel times is proposed.

The purpose of this paper is to demonstrate that introduction of flexibility into constraints does not decrease the quality of results. Another goal is to show that the acceptance of small violations of constraints can lead to solutions with lower cost, which may be valuable to DMs.

2. FUZZY VEHICLE ROUTING PROBLEM WITH TIME WINDOWS (FVRPTW)

Let N be the set of customers. Each customer i has some demand (the amount of goods) and must be served within the time window $[e_i, l_i]$. Customers are visited by vehicles from the set K, each having the capacity q. Some time is required to travel between a pair of customers. All vehicles start and end their travel in the same depot. The objective is to find a set of routes (i.e. the sequences of customers) with minimal cost, such that:

- each customer is placed on exactly one route
- the time at which the customer i is visited is between e_i and l_i
- the sum of customers' demands on each route is not greater than q

The above model assumes hard time windows. To allow small violation of time constraints, the time windows must be presented in a different way. Previous works ([1], [4]) show that fuzzy intervals are a good model to achieve this requirement.

For the customer i his time window is fuzzified (see Fig. 1 for an example). Only the right part of the time window (i.e. the latest service time, l_i) is fuzzified, because the left part plays role only when the vehicle arrival takes place before the e_i and has nothing to do with the customer's satisfaction.

Formally, soft time window of customer i is represented by a fuzzy set of service times satisfying the customer, with its membership function $\mu_i(t)$ equal to the level of customer's satisfaction when the service starts at time t_i.

The original time constraints are no longer present in the flexible model. Instead, the satisfaction levels assigned to time windows are part of solutions' quality. They constitute an additional objective: to minimize the global satisfaction level (i.e. the satisfaction level of least satisfied constraint). This makes the problem multiobjective.

As mentioned in previous sections, the assumption that travel times are fixed numbers known a priori is very strong. Therefore, fuzzy travel times are introduced. Each travel time T_i is modeled as a 3-point fuzzy number. This representation requires least assumptions and is easy to build with a human expert, e.g. by asking him questions in the form: "What is the expected, smallest and greatest travel time between X and Y".

The arithmetic of fuzzy values is simple, compared to the one of stochastic variables, and the concept of satisfying flexible constraints with uncertain values is often used ([1]). An example of fuzzy time value satisfying flexible constraint is shown on Fig. 1. The concepts used here have roots in the possibility theory [2].

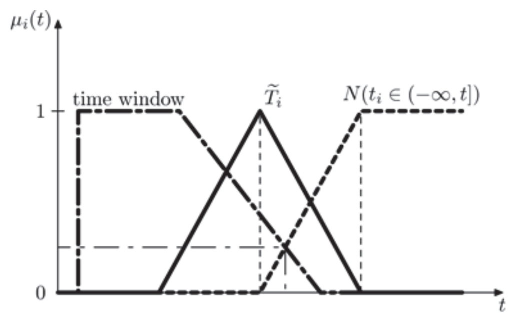

Fig. 1. Fulfillment of a flexible constraint by uncertain value

3. THE PROPOSED ALGORITHM

The model described above is solved by a multiobjective metaheuristic algorithm called Pareto Memetic Algorithm (PMA) [5]. Its goal is to find a set of approximately Pareto-optimal solutions. The algorithm uses a mechanism of scalarizing functions to drive the search in the direction of the Pareto-optimal set (Pareto set, PS). In this work the achievement scalarizing functions are used [5].

The main loop of the PMA, which starts right after generation of initial solutions by a randomized heuristic, is presented in Fig. .

The general idea of this algorithm is to generate new approximately Pareto-optimal solutions by recombination of already found solutions which are good with respect to a randomly chosen scalarizing function. The offspring resulting from this recombination is then improved by a local search heuristic optimizing the same scalarizing function. The rationale behind this procedure is based on the assumption that recombination of good solutions is likely to produce good starting points for local search.

repeat

Draw at random a weight wector λ

From the current set CS of solutions draw with uniform probability a sample T of solutions

Recombine the best and the second best solution on s($\mathbf{z}$,…, λ) from T obtaining $\mathbf{x_1}$

Apply a local search heuristic optimizing s($\mathbf{z}$,…, λ) to $\mathbf{x_1}$ obtaining $\mathbf{x_1}$'

If $\mathbf{x_1}$' is better on s($\mathbf{z}$,…, λ) then the second best solution in T **then**

Add $\mathbf{x_1}$' to the current set CS of solutions

Update the set of approximately Pareto-optimal solutions with $\mathbf{x_1}$'

until the stopping criterion is met

Fig. 2. Main loop of the PMA. s($\mathbf{z}$,…, λ) stands for a scalarizing function value with weight vector λ for a solution with objective values $\mathbf{z}$

The Pareto Memetic Algorithm was adapted to the FVRPTW by means of problem-specific randomized heuristic procedure, local search and recombination operators. There is no explicit mutation operator defined.

The randomized heuristic procedure generates solutions using the insertion method I1 described in [9], with parameter values drawn randomly with uniform probability. This heuristic was chosen because it generates good quality solutions. During construction of routes I1 requires that all hard constraints are met, so it had to be adjusted to the case of flexible time windows. If time constraints were removed totally, it could result in all initial solutions having satisfaction equal to zero, especially in problems with very tight time windows. Thus, the modified I1 heuristic requires that constraints are not fully violated (they have non-zero satisfaction level).

The local search uses steepest approach to find the best neighbor of the current solution. The operators used to construct the neighborhood are moving a customer from one route to any position on another one or switching two customers between routes.

Two recombination operators were implemented: the route-based crossover (RBX) [7] and the common-edges-preserving crossover (CEPX) [6]. RBX generates an offspring by first copying a randomly chosen set of routes from one parent. Then, the offspring is completed with routes from the other parent, omitting the customers already inserted when necessary. Consequently, RBX contains a random component. CEPX is, on the contrary, a fully deterministic crossover. Routes of its offspring are composed only of edges which are common to both parents.

The PMA was implemented in Java.

4. COMPUTATIONAL EXPERIMENT

In order to generate approximations of Pareto sets and to assess the efficiency of the proposed algorithm (especially of the two crossover operators) a computational experiment was designed. Well-known Solomon's instances of VRPTW [9] were used. From the whole set of Solomon's instances we selected 3 groups (called R, C, RC in short) with the largest number of customers (100); 56 instances in total. All these instances were augmented with information about fuzzy time windows and travel times. Each time window was enlarged by 10% of its original width, with customer's satisfaction decreasing linearly to zero. Travel times were fuzzyfied by 5% in either direction (compare Fig. 1).

Two versions of the PMA were run: one with RBX only, the other with CEPX. They were executed 15 times for each instance, 420 seconds each run, on PCs with Intel Pentium 4 processor 3.2 GHz and 1GB RAM, running Windows XP.

Evaluation of results of a multiobjective algorithm requires that specialized measures of quality of approximations of PS are employed. In this paper two such measures are used: coverage C(A,B) of approximation A of PS by approximation B [12] and the average best value of the weighted Tchebycheff scalarizing function R(A) for one approximation A [5].

C(A,B) is a percentage of solutions in A which are dominated (covered) by some solutions from B. It is normalized and not symmetric. C(A,B)=1 indicates that A is completely dominated by B. If C(A,B)=0 then the approximations A and B of PS are incomparable.

R(A), shows a quality of an approximation A with respect to a reference point, usually being an ideal point. This measure is nonnegative and not normalized. The closer it is to 0, the better the approximation A is (being generally closer to the ideal point).

Table 1 presents values of the coverage measure C for the two employed algorithms. Values of the R measure are shown in Fig. 3.

Table 1. Average values of the C measure between approximations generated by two algorithms in groups of instances

Group of instances	C(RBX,CEPX)	C(CEPX,RBX)
C	16,7%	63,2%
R	0,8%	52,9%
RC	0,8%	53,0%
All	5,6%	56,0%

As can be seen from the Table 1 and Fig. 4, the solutions obtained from the computations using

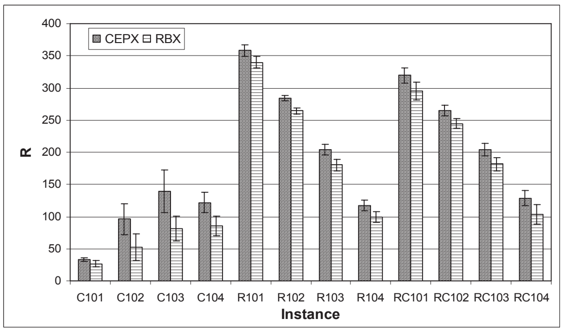

Fig. 3. Values of the R measure for approximations of PS generated by different algorithms

CEPX operator are usually dominated by the solutions resulting from computations using RBX. It may be due to the fact that in the case of CEPX the whole procedure of recombination and local search is deterministic. Thus, the only random component of the algorithm determining the structure of solutions is randomized initial heuristic, which might be not enough to achieve diversified solutions.

One approximation of a Pareto front which we obtained from the experiment is shown in Fig. 4. The horizontal line represents the cost of the optimal solution to the presented problem.

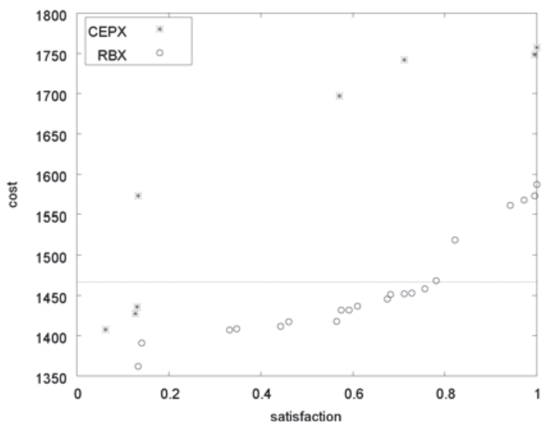

Fig. 4. An example of a Pareto front approximation for instance R101

In this figure it can be seen that the cost of a solution can be greatly lowered in exchange for a satisfaction decrease. This fact gives a DM a chance to choose between two interesting, but highly different solutions: one with reasonable cost and fully satisfied constraints, the other with smaller satisfaction, but less expensive The knowledge about such solutions can be the basis for renegotiation of time windows with customers.

5. CONCLUSIONS AND FURTHER RESEARCH

The constructed FVRPTW is valuable if a DM should have the possibility to choose a solution from a set o good and diversified ones. Using the proposed approach, a DM is able to choose time windows, which, when altered, can result in much less expensive routes.

The comparison of crossover operators shows that there is a need for randomness in the PMA: the use of randomized RBX operator lead to better results than the use of deterministic CPEX.

The main direction for further research is the improvement of heuristics and operators, which are suitable to the model of flexible constraints.

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