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A NOTE ON A FOURTH ORDER DISCRETE BOUNDARY VALUE PROBLEM

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Abstract. Using variational methods we investigate the existence of solutions and their dependence on parameters for certain fourth order difference equations.

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Mathematics Subject Classification: 39A10, 39A30.

1. INTRODUCTION

In this note we will study a Dirichlet boundary value problem for a fourth order discrete equation

$$\begin{aligned} &\Delta^2\left(p(k)\Delta^2 x(k-2)\right)+\Delta\left(q(k)\Delta x(k-1)\right)+f\left(k,x(k)\right)=g(k),\ k\in Z[2,T],\\ &x(0)=x(1)=x(T+1)=x(T+2)=0. \end{aligned}\tag{1.1}$$

For fixed $a,b\in\mathbb{N}$ we define $Z[a,b]=\{a,a+1,\dots,b-1,b\}$ as the so called discrete interval. Δ is the forward difference operator $\Delta x(k)=x(k+1)-x(k)$. By a solution of problem (1.1) we mean such a function $x:Z[0,T+2]\to\mathbb{R}$ which satisfies the difference equation on $Z[2,T]$ and the given boundary conditions. We note that since we do not assume anything about the sign condition of f near 0 our results may apply for both positone (i.e. when $f(k,0)\geq 0$, $k\in Z[2,T]$) and non-positone (i.e. when $f(k,0)<0$, $k\in Z[2,T]$) problems within one approach. This is not common within the boundary value problems, compare with [8].

Solutions are obtained in the space E of functions $x:\{0,T+2\}\to\mathbb{R}$ such that $x(0)=x(1)=x(T+1)=x(T+2)=0$ considered with a norm

$$\|x\|=\sqrt{\sum_{k=2}^{T+2}(\Delta^2 x(k-2))^2}.$$

All functions from E are defined on a finite set, and therefore these are continuous. The space E can be also considered with the following norms

$$\|x\|_1 = \sqrt{\sum_{k=2}^{T+1} (\Delta x(k-1))^2}$$

and

$$\|x\|_0 = \sqrt{\sum_{k=2}^{T} x^2(k)}.$$

Since E has finite dimension these norms are equivalent, thus

$$\begin{gathered} \beta\|x\| \leq \|x\|_1 \leq \beta_1\|x\|, \\ \|x\|_0 \leq \gamma\|x\| \end{gathered} \tag{1.2}$$

for a certain constants $\beta, \beta_1, \gamma > 0$ which do not depend on x. We assume that:

(A1) $f \in C(Z[2,T] \times \mathbb{R}, \mathbb{R})$, $p \in C(Z[2,T+3], \mathbb{R})$, $q \in C(Z[2,T+2], \mathbb{R})$, $g \in C(Z[2,T], \mathbb{R})$;
(A2) there exists a constant $\alpha > 0$ such that $xf(t,x) \leq 0$ for $|x| \geq \alpha$;
(A3) $M < N\beta^2$, where $M = \sup_{t\in\{2,3,\ldots,T+3\}} p(t)$, $N = \inf_{t\in\{2,3,\ldots,T+2\}} q(t)$.

Problems such as (1.1) arise when fourth order Dirichlet problems are being discretization and may be viewed as a discrete version of a simply supported elastic beam equation, see for example [2, 5]. The approach through symmetric Green's function is used in [6, 7], the Krein-Rutman Theorem is applied in [10], while in [9] the critical point theory is used with some other growth conditions. In fact the variational framework for problem (1.1) which we follow is descried in [9]. However, in the sources mentioned, the approach is somewhat different and with different set of assumptions. While in the literature mainly the problem of the existence of solutions and their multiplicity is considered we are going to go further and investigate also the dependence on a functional parameter.

The paper is organized as follows. We are going first to apply a variational approach based on the so called direct variational method in order to get the existence result and next investigate the dependence of the solution on a functional parameter. We think that for our problem, as far as the existence is concerned, a lower-upper solution method introduced in [4] could also be applied. However the latter approach does not seem to allow for the investigations of the dependence on parameters due to the non-uniqueness of solutions and therefore we do not apply it.

For the sake of convenience, we now recall same basic tools used in our note, see [2]. A mapping J of a real Banach X space to $\mathbb{R}$ will be called a functional. A point x_0 where $J^{'}(x_0) = \theta$ is called a critical point of J, assuming that J is Gâteaux differentiable and that $J^{'}$ denotes the Gâteaux derivative.

J is weakly lower semi-continuous at $x \in X$ if

$$x_n \rightharpoonup x \Rightarrow \liminf_{n\to\infty} J(x_n) \geq J(x)$$

and J is coercive on X if

$$\lim_{\|x\|\to\infty} J(x) = +\infty,$$

where $\|x\|$ stands for a norm in X and "$\rightharpoonup$" denotes weak convergence in X.

Theorem 1.1. *Let E be a reflexive Banach space, $D \subset E$ be weakly closed, and $J : E \to \mathbb{R}$ be weakly lower semi-continuous and coercive, then J has a minimum over D.*

2. THE EXISTENCE OF SOLUTIONS

Let

$$F(k, y(k)) = \int_0^{y(k)} f(k,t)dt, \ y \in E.$$

The action functional $J : E \to \mathbb{R}$ corresponding to our problem is

$$\begin{aligned} J(y) = &\sum_{k=2}^{T+2} (\tfrac{-p(k)}{2} (\Delta^2 y(k-2))^2) + \\ &+ \sum_{k=2}^{T+1} \tfrac{q(k)}{2} (\Delta y(k-1))^2 + \sum_{k=2}^{T} (-F(k, y(k)) + g(k)y(k)). \end{aligned}$$

Lemma 2.1. *J is a Gâteaux differentiable functional; $y \in E$ is a critical point of J if and only if it is a solution to* (1.1).

Proof. We denote by $\varphi : \mathbb{R} \to \mathbb{R}$ the function $\varphi(\varepsilon) = J(y+\varepsilon h)$ for $y, h \in E$ and $\varepsilon \in \mathbb{R}$; here $y, h \in E$ are fixed. Then

$$\begin{aligned} \varphi(\varepsilon) = &\sum_{k=2}^{T+2} (\tfrac{-p(k)}{2} (\Delta^2 (y+\varepsilon h)(k-2))^2) + \sum_{k=2}^{T+1} \tfrac{q(k)}{2} (\Delta (y+\varepsilon h)(k-1))^2 + \\ &+ \sum_{k=2}^{T} (-F(k, y(k)+\varepsilon h(k)) + g(k)(y(k)+\varepsilon h(k))) \end{aligned}$$

and since φ is differentiable we get

$$\begin{aligned} \varphi'(0) = &\sum_{k=2}^{T+2} (-p(k)\Delta^2 y(k-2)\Delta^2 h(k-2)) + \sum_{k=2}^{T+1} q(k)\Delta y(k-1)\Delta h(k-1) + \\ &+ \sum_{k=2}^{T} (-f(k, y(k))h(k) + g(k)h(k)) = \sum_{k=2}^{T} \big(-\Delta^2 \big(p(k)\Delta^2 y(k-2)\big)h(k) - \\ &- \Delta\big(q(k)\Delta y(k-1)\big)h(k) - f(k, y(k))h(k) + g(k)h(k)\big). \end{aligned}$$

Thus $y \in E$ is critical point of J if and only if y satisfies equation (1.1). $\square$

Theorem 2.2. *Assume that* (A1), (A2), (A3) *hold. Then functional J is weakly lower semi-continuous and coercive on E.*

Proof. Since J is continuous it is lower semi-continuous and since E is finite dimensional it is weakly lower semi-continuous. We have just demonstrated that J is Gâteaux differentiable. We show that J is coercive on E. To do this first notice that by (A2)

$$\sum_{k=2}^{T} F\left(k, y(k)\right) = \sum_{k=2}^{T} \int_{0}^{y(k)} f(k,t)dt \leq \sum_{k=2}^{T} \int_{-\alpha}^{\alpha} \mid f(k,t) \mid dt \leq C. \tag{2.1}$$

Further from (1.2), (2.1) and (A3) for any sequence $\{y_n\} \in E$

$$\begin{aligned} J(y_n) &= \sum_{k=2}^{T+2} \left(\tfrac{-p(k)}{2}(\Delta^2 y_n(k-2))^2\right) + \\ &\quad + \sum_{k=2}^{T+1} \tfrac{q(k)}{2}(\Delta y_n(k-1))^2 + \sum_{k=2}^{T}(g(k)y_n(k) - F(k, y_n(k))) \geq \\ &\geq \tfrac{-M}{2}\|y_n\|^2 + \tfrac{N}{2}\|y_n\|_1^2 - C - \sum_{k=2}^{T}|g(k)||y_n(k)| \geq \\ &\geq \tfrac{-M}{2}\|y_n\|^2 + \tfrac{N}{2}\beta^2\|y_n\|^2 - C - \sqrt{\sum_{k=2}^{T} g^2(k)}\|y_n\|_0 \geq \\ &\geq \left(\tfrac{-M}{2} + \tfrac{N}{2}\beta^2\right)\|y_n\|^2 - C - \gamma\sqrt{\sum_{k=2}^{T} g^2(k)}\|y_n\|. \end{aligned}$$

So $J(y_n) \to +\infty$ as $\|y_n\| \to \infty$. □

The main result of this section is contained in the next theorem.

Theorem 2.3. *Assume that* (A1), (A2), (A3) *hold. Then problem* (1.1) *has at least one solution $v \in E$ such that $J(v) = \inf_{y\in E} J(y)$.*

Proof. We use Theorem 1.1 and Lemma 2.1. Let $D = E$. Then D as a closed and convex set is weakly closed. By Theorem 2.2 J is weakly lower semi-continuous and coercive on D. So by Theorem 1.1 it has at least one argument for a minimum. Let us denote it by v. Since J is differentiable in the sense of Gâteaux, it follows that $J^{'}(v) = 0$ and the assertion follows by Lemma 2.1. □

3. THE DEPENDENCE ON PARAMETERS

The usage of a variational method allows us to consider a boundary value problem which is subject to some functional parameter and later to investigate the dependence of the solution on the parameter as it varies. We do not need to have uniqueness of solutions in order to investigate their dependence on parameters. In this section we will investigate the following Dirichlet problem

$$\begin{gathered}\Delta^2\left(p(k)\Delta^2 x(k-2)\right)+\Delta\left(q(k)\Delta x(k-1)\right)+f\left(k,x(k),u(k)\right)=g(k),\ k\in Z[2,T],\\ x(0)=x(1)=x(T+1)=x(T+2)=0\end{gathered}\tag{3.1}$$

subject to parameter $u\in L_D=\{u\in C(Z[2,T],\mathbb{R}):\|u\|_C\le D\}$, where $D>0$ is fixed and $\|u\|_C$ denotes classical maximum norm $\|u\|_C=\max_{k\in Z[2,T]}|u(k)|$.

Now we assume that:

(A4) $f\in C(Z[2,T]\times\mathbb{R}^2,\mathbb{R})$, $p\in C(Z[2,T+3],\mathbb{R})$, $q\in C(Z[2,T+2],\mathbb{R})$, $g\in C(Z[2,T],\mathbb{R})$;

(A5) there exists $\alpha>0$ such that $xf(t,x,u)\le 0$ for $|x|\ge\alpha, |u|\le D$.

With these assumptions and with (A3) the action functional $J_u:E\to\mathbb{R}$ corresponding to (3.1) with a fixed function $u\in L_D$ reads

$$\begin{aligned}J_u(y)=&\sum_{k=2}^{T+2}\left(\tfrac{-p(k)}{2}(\Delta^2 y(k-2))^2\right)+\\&+\sum_{k=2}^{T+1}\tfrac{q(k)}{2}(\Delta y(k-1))^2+\sum_{k=2}^{T}(g(k)y(k)-F(k,y(k),u(k))),\end{aligned}$$

where $F(k,y(k),u(k))=\int_0^{y(k)}f(k,t,u(k))dt,\ y\in E$.

Reasoning as in the proof of Theorem 2.2 we obtain

Theorem 3.1. *Assume that* (A3), (A4), (A5) *hold. Then for any fixed* $u\in L_D$ *the problem* (3.1) *has at least one solution in* V_u.

Let for any fixed $u\in L_D$

$$V_u=\{y\in E: J_u(y)=\inf_{v\in E}J_u(v)\quad\text{and}\quad J_u'(y)=0\}$$

be the set which consists of the arguments of a minimum to J_u. Due to Theorem 3.1 $V_u\neq\emptyset$. We will investigate the behavior of the sequence $\{y_n\}$ of solutions to (3.1) depending on convergence of the sequence of parameters $\{u_n\}$.

Theorem 3.2. *Assume that* (A3), (A4), (A5) *hold. For any fixed* $u\in L_D$ *there exists at least one solution* $y\in V_u$ *to problem* (3.1). *Let* $\{u_n\}\subset L_D$ *be a convergent sequence of parameters, where* $\lim_{n\to\infty}u_n=\bar{u}\in L_D$. *For any sequence* $\{y_n\}$ *of solutions* $y_n\in V_n$ *to the problem* (3.1) *corresponding to* u_n, *there exist a subsequence* $\{y_{n_i}\}\subset E$ *and an element* $\overline{y}\in E$ *such that* $\lim_{i\to\infty}y_{n_i}=\overline{y}$ *and* $J_{\bar{u}}(\overline{y})=\inf_{y\in E}J_{\bar{u}}(y)$. *Moreover* $\overline{y}\in V_{\bar{u}}$, *i.e.* $\overline{y}$ *satisfies*

$$\begin{gathered}\Delta^2\left(p(k)\Delta^2\overline{y}(k-2)\right)+\Delta\big(q(k)\Delta\overline{y}(k-1)\big)+f\big(k,\overline{y}(k),\bar{u}(k)\big)=g(k),\\ \overline{y}(0)=\overline{y}(1)=\overline{y}(T+1)=\overline{y}(T+2)=0.\end{gathered}$$

Proof. By Theorem 3.1 we get for $n \in \mathbb{N}$ the existence of solution $y_n \in V_{u_n}$ to (3.1). Notice, that for $n \in \mathbb{N}$,

$$y_n \in V_{u_n} \subset \{y : J_{u_n} \leq J_{u_n}(0)\}.$$

By (A5) we get for some constant $C > 0$

$$\begin{aligned}\sum_{k=2}^{T} F\big(k, y_n(k), u_n(k)\big) &= \sum_{k=2}^{T} \int_0^{y_n(k)} f(k,t,u_n(k))dt \leq \\ &\leq \sum_{k=2}^{T} \int_{-\alpha}^{\alpha} \mid f(k,t,u_n(k)) \mid dt \leq C.\end{aligned} \tag{3.2}$$

Then (3.2) and (1.2) imply

$$\begin{aligned}J_u(y_n) &= \sum_{k=2}^{T+2} (\tfrac{-p(k)}{2} (\Delta^2 y_n(k-2))^2) + \sum_{k=2}^{T+1} \tfrac{q(k)}{2} (\Delta y_n(k-1))^2 + \\ &+ \sum_{k=2}^{T} (g(k) y_n(k) - F(k, y_n(k), u(k))) \geq \\ &\geq \tfrac{-M}{2} \|y_n\|^2 + \tfrac{N}{2} \|y_n\|_1^2 - C - \sum_{k=2}^{T} |g(k)||y_n(k)| \geq \\ &\geq \tfrac{-M}{2} \|y_n\|^2 + \tfrac{N}{2}\beta^2 \|y_n\|^2 - C - \sqrt{\sum_{k=2}^{T} g^2(k)} \|y_n\|_0 \geq \\ &\geq (\tfrac{-M}{2} + \tfrac{N}{2}\beta^2) \|y_n\|^2 - C - \gamma\sqrt{\sum_{k=2}^{T} g^2(k)} \|y_n\|.\end{aligned}$$

We also know that $F(k, 0, u_n(k)) = 0$, so $J_{u_n}(0) = 0$. As a consequence for $y_n \in V_{u_n}$ we see that

$$(\tfrac{-M}{2} + \tfrac{N}{2}\beta^2) \|y_n\|^2 - \gamma\sqrt{\sum_{k=2}^{T} g^2(k)} \|y_n\| \leq C \tag{3.3}$$

so $\{y_n\}$ is bounded in E and hence it has a convergent subsequence $\{y_{n_i}\}$. We denote its limit by $\overline{y}$.

In order to demonstrate that $\overline{y}$ satisfies (3.1) corresponding to $\bar{u}$ we follow the same steps as in the proof of Theorem 1 in [3]. However, we proceed with the reasoning for the reader's convenience and slightly simplify the approach of [3]. Observe that by Theorem 3.1 there exists $y_0 \in E$ such that y_0 solves (3.1) with $\overline{u}$ and $J_{\overline{u}}(y_0) = \inf_{y \in E} J_{\overline{u}}(y)$ and either $J_{\overline{u}}(y_0) < J_{\overline{u}}(\overline{y})$ or $J_{\overline{u}}(y_0) = J_{\overline{u}}(\overline{y})$. Suppose that $J_{\overline{u}}(y_0) < J_{\overline{u}}(\overline{y})$. Then, for some constant $\delta > 0$ we have

$$\delta < \big(J_{u_{n_i}}(y_{n_i}) - J_{\overline{u}}(y_0)\big) - \big(J_{u_{n_i}}(y_{n_i}) - J_{\overline{u}}(\overline{y})\big). \tag{3.4}$$

By continuity, it follows that

$$\lim_{i\to\infty}\left(J_{u_{n_i}}(y_{n_i}) - J_{\overline{u}}(\overline{y})\right) = 0. \tag{3.5}$$

Since y_{n_i} minimizes $J_{u_{n_i}}$ over E we see that $J_{u_{n_i}}(y_{n_i}) \le J_{u_{n_i}}(y_0)$ for any n_i. Therefore, we get

$$\lim_{i\to\infty}\left(J_{u_{n_i}}(y_{n_i}) - J_{\overline{u}}(y_0)\right) \le \lim_{i\to\infty}\left(J_{u_{n_i}}(y_0) - J_{\overline{u}}(y_0)\right) = 0. \tag{3.6}$$

Using (3.5) and (3.6) we obtain $\delta \le 0$ in (3.4), which is a contradiction. Thus $J_{\overline{u}}(\overline{y}) = \inf_{y\in E} J_{\overline{u}}(y)$ and since $J_{\overline{u}}$ is differentiable in the sense of Gâteaux we have $\overline{y} \in V_{\overline{u}}$. Hence $\overline{y}$ necessarily satisfies (3.1). On the other hand, if we have $J_{\overline{u}}(y_0) = J_{\overline{u}}(\overline{y})$ then the result readily follows. □

4. FURTHER EXISTENCE RESULTS AND EXAMPLES

In this section we will also investigate (1.1) but our assumptions will be somewhat different. This change forces us to use a different action functional.

(A6) there exists $\alpha_1 > 0$ such that $xf(t,x) \ge 0$ for $|x| \ge \alpha_1$;
(A7) $M_1 > N_1\beta_1^2$, where $M_1 = \inf_{t\in\{2,3,\ldots,T+3\}} p(t)$, $N_1 = \sup_{t\in\{2,3,\ldots,T+2\}} q(t)$.

Now we consider the following action functional

$$J_1(y) = \sum_{k=2}^{T+2} \tfrac{p(k)}{2}(\Delta^2 y(k-2))^2 - \sum_{k=2}^{T+1} \tfrac{q(k)}{2}(\Delta y(k-1)) + \sum_{k=2}^{T}(F(k,y(k)) - g(k)y(k)).$$

Theorem 4.1. *Assume that* (A1), (A6), (A7) *hold. Then functional J_1 is weakly lower semi-continuous and coercive on E.*

Proof. The fact that J_1 is lower semi-continuous is obvious. We show that J_1 is coercive on E. To do this first notice that by (A6) (similar to proof of Theorem 2.2) we get for any $y \in E$

$$\sum_{k=2}^{T} F(k, y(k)) \ge -C \tag{4.1}$$

for some constant $C > 0$. Further from (1.2), (4.1) for any sequence $\{y_n\} \in E$

$$
\begin{aligned}
J_1(y_n) = & \sum_{k=2}^{T+2} \tfrac{p(k)}{2} (\Delta^2 y_n(k-2))^2 - \\
& - \sum_{k=2}^{T+1} \tfrac{q(k)}{2} (\Delta y_n(k-1))^2 + \sum_{k=2}^{T} (F(k, y_n(k)) - g(k) y_n(k)) \geq \\
& \geq \tfrac{M_1}{2} \|y_n\|^2 - \tfrac{N_1}{2} \|y_n\|_1^2 - C - \sum_{k=2}^{T} |g(k)||y_n(k)| \geq \\
& \geq \tfrac{M_1}{2} \|y_n\|^2 - \tfrac{N_1}{2} \beta_1^2 \|y_n\|^2 - C - \sqrt{\sum_{k=2}^{T} g^2(k)} \, \|y_n\|_0 \geq \\
& \geq \left(\tfrac{M_1}{2} - \tfrac{N_1}{2} \beta_1^2\right) \|y_n\|^2 - C - \gamma \sqrt{\sum_{k=2}^{T} g^2(k)} \, \|y_n\| .
\end{aligned}
$$

So $J_1(y_n) \to +\infty$ as $\|y_n\| \to \infty$. □

The existence of a minimum for J_1 follows from Theorem 1.1 (with $D = E$) and Theorem 4.1. So we get the following theorem.

Theorem 4.2. *Assume that* (A1), (A6), (A7) *hold. Then the problem* (1.1) *has at least one solution.*

Example 4.3. Let l be any natural number and let $r \in C(\mathbb{R}, \mathbb{R})$ be bounded. Define function $f(t, x) = r(t)h(x)$ with

$$
h(x) = \begin{cases} x^{2l}, & x < 0, \\ -x^{2l}, & x \geq 0. \end{cases}
$$

For $r \in C(\mathbb{R}, \mathbb{R}_+)$ function f satisfy (A2), but it does not satisfy (A6). For $r \in C(\mathbb{R}, \mathbb{R}_-)$ it does not satisfy (A2), but it satisfies (A6).

Example 4.4. Let $r \in C(\mathbb{R}, \mathbb{R})$ be bounded. Define function $f(t, x) = r(t)h(x)$ with

$$
h(x) = \arctan x.
$$

For $r \in C(\mathbb{R}, \mathbb{R}_+)$ function f satisfies (A6), but it does not satisfy (A2). For $r \in C(\mathbb{R}, \mathbb{R}_-)$ it does not satisfy (A6), but it satisfies (A2).

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