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WEAK SOLUTIONS FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS WITH INTEGRAL BOUNDARY CONDITIONS IN BANACH SPACES

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Abstract. The aim of this paper is to investigate a class of boundary value problems for fractional differential equations involving nonlinear integral conditions. The main tool used in our considerations is the technique associated with measures of weak noncompactness.

Keywords: boundary value problem, Caputo fractional derivative, measure of weak noncompactness, Pettis integrals, weak solution.

Mathematics Subject Classification: 26A33, 34B15, 34G20.

1. INTRODUCTION

The theory of fractional differential equations is an important branch of differential equation theory, which has an extensive physical, chemical, biological, and engineering background, and hence been emerging as an important area of investigation in the last few decades; see the monographs of Kilbas *et al.* [12], Miller and Ross [15], and the papers of Agarwal *et al.* [1], Benchohra *et al.* [4, 6, 7].

In this paper we investigate the existence of weak solutions, for the boundary value problem, for fractional differential equations with mixed boundary conditions of the form

$$^cD^\alpha x(t) = f(t, x(t)) \quad \text{for each} \quad t \in I = [0, T], \tag{1.1}$$

$$x(0) + \mu \int_0^T x(s)ds = x(T), \tag{1.2}$$

where $^cD^\alpha, 0 < \alpha \leq 1$ is the Caputo fractional derivative, $f : I \times E \to E$ is a given function satisfying some assumptions that will be specified later, E is a Banach space with norm $\|\cdot\|$ and $\mu \in \mathbb{R}^*$.

To investigate the existence of solutions of the problem above, we use Mönch's fixed point theorem combined with the technique of measures of weak noncompactness, which is an important method for seeking solutions of differential equations. This technique was mainly initiated in the monograph of Banaś and Goebel [2] and subsequently developed and used in many papers; see, for example, Banaś *et al.* [3], Guo *et al.* [11], Krzyska and Kubiaczyk [13], Lakshmikantham and Leela [14], Mönch [16], O'Regan [17,18], Szufla [21], Szufla and Szukała [22], and the references therein. As far as we know, there are very few results devoted to weak solutions of nonlinear fractional differential equations ([5, 20]). The present results complete and extend those considered in the scalar case [8].

2. PRELIMINARIES

This section is devoted to notation and results that will be used through this paper.

Let $I = [0, T]$, $L^1(I)$ denote the Banach space of real-valued Lebesgue integrable functions on the interval I and $L^\infty(I, E)$ denote the Banach space of real-valued essentially bounded and measurable functions defined over I with the norm $\|\cdot\|_{L^\infty}$.

Let E be the real Banach space with norm $\|\cdot\|$ and dual E^* also $(E, w) = (E, \sigma(E, E^*))$ denotes the space E with its weak topology. $C(I, E)$ is the Banach space of continuous functions $x : I \to E$, with the usual supremum norm

$$\|x\|_\infty = \sup\{\|x(t)\|,\ t \in I\}.$$

Definition 2.1. A function $h : E \to E$ is said to be weakly sequentially continuous if h takes each weakly convergent sequence in E to weakly convergent sequence in E (i.e. for any $(x_n)_n$ in E with $x_n \to x$ in (E, w) then $h(x_n) \to h(x)$ in (E, w) for each $t \in I$).

Definition 2.2 ([19])**.** The function $x : I \to E$ is said to be Pettis integrable on I if and only if there is an element $x_J \in E$ corresponding to each $J \subset I$ such that $\varphi(x_J) = \int_J \varphi(x(s))ds$ for all $\varphi \in E^*$, where the integral on the right is supposed to exist in the sense of Lebesgue. By definition, $x_J = \int_J x(s)ds$.

Let $P(I, E)$ be the space of all E-valued Pettis integrable functions in the interval I.

Propostion 2.3 ([10, 19])**.** *If $x(\cdot)$ is Pettis integrable and $h(\cdot)$ is a measurable and essentially bounded real-valued function, then $x(\cdot)h(\cdot)$ is Pettis integrable.*

Definition 2.4 ([9])**.** Let E be a Banach space, Ω_E the bounded subsets of E and B_1 the unit ball of E. The De Blasi measure of weak noncompactness is the map $\beta : \Omega_E \to [0, \infty)$ defined by $\beta(X) = \inf\{\epsilon > 0 :$ there exists a weakly compact subset Ω of $E : X \subset \epsilon B_1 + \Omega\}$.

Properties. The De Blasi measure of noncompactness has the following properties:

(a) $A \subset B \Rightarrow \beta(A) \leq \beta(B)$,
(b) $\beta(A) = 0 \Leftrightarrow$ A is relatively compact,
(c) $\beta(A \cup B) = \max\{\beta(A), \beta(B)\}$,
(d) $\beta(\overline{A}^{\omega}) = \beta(A)$, ($\overline{A}^{\omega}$ denotes the weak closure of A),
(e) $\beta(A + B) \leq \beta(A) + \beta(B)$,
(f) $\beta(\lambda A) = |\lambda|\beta(A)$,
(g) $\beta(conv(A)) = \beta(A)$,
(h) $\beta(\cup_{|\lambda| \leq h} \lambda A) = h\beta(A)$.

The following result follows directly from the Hahn-Banach theorem.

Propostion 2.5. *Let E be a normed space with $x_0 \neq 0$. Then there exists $\varphi \in E^*$ with $\|\varphi\| = 1$ and $\varphi(x_0) = \|x_0\|$.*

For completeness we recall the definitions of the Pettis-integral and Caputo derivative of fractional order.

Definition 2.6 ([20]). Let $h : I \to E$ be a function. The fractional Pettis integral of the function h of order $\alpha \in \mathbb{R}_+$ is defined by

$$I^{\alpha} h(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} h(s) ds,$$

where the sign "$\int$" denotes the Pettis integral and Γ is the Gamma function.

Definition 2.7 ([12]). For a function $h : I \to E$, the Caputo fractional-order derivative of h, is defined by

$${}^c D^{\alpha} h(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{h^{(n)}(s) ds}{(t-s)^{1-n+\alpha}},$$

where $n = [\alpha] + 1$ and $[\alpha]$ denote the integer part of α.

Theorem 2.8 ([17]). *Let Q be a closed, convex and equicontinuous subset of a metrizable locally convex vector space $C(I, E)$ such that $0 \in Q$. Assume that $T : Q \to Q$ is weakly sequentially continuous. If the implication*

$$\overline{V} = \overline{conv}(\{0\} \cup T(V)) \Rightarrow V \text{ is relatively weakly compact}, \tag{2.1}$$

holds for every subset $V \subset Q$, then T has a fixed point.

3. MAIN RESULTS

Let us start by defining what we mean by a solution of the problem (1.1)–(1.2).

Definition 3.1. A function $x \in C(I, E_\omega)$ is said to be a solution of (1.1)–(1.2) if x satisfies the equation ${}^cD^\alpha x(t) = f(t, x(t))$ on I, and the condition (1.2).

For the existence results on the problem (1.1)–(1.2), we need the following auxiliary lemmas.

Lemma 3.2 ([23]). *Let $\alpha > 0$, then the differential equation*

$$ {}^cD^\alpha h(t) = 0 $$

has solutions $h(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_{n-1} t^{n-1}$, $c_i \in \mathbb{R}$, $i = 0, 1, 2, \ldots, n-1$, $n = [\alpha] + 1$.

Lemma 3.3 ([23]). *Let $\alpha > 0$, then*

$$ I^\alpha\, {}^cD^\alpha h(t) = I^\alpha h(t) + c_0 + c_1 t + c_2 t^2 + \ldots + c_{n-1} t^{n-1} $$

for some $c_i \in \mathbb{R}$, $i = 0, 1, 2, \ldots, n-1$, $n = [\alpha] + 1$.

Lemma 3.4. *Let $0 < \alpha \le 1$ and let $h \in C(I, E)$ be a given function, then the boundary value problem*

$$ {}^cD^\alpha x(t) = h(t), \quad t \in I, \tag{3.1} $$

$$ x(0) + \mu \int_0^T x(s)ds = x(T), \quad \mu \in \mathbb{R}^* \tag{3.2} $$

has a unique solution given by

$$ x(t) = \int_0^T G(t,s)h(s)ds, \tag{3.3} $$

where $G(t,s)$ is the Green's function defined by the formula

$$ G(t,s) = \begin{cases} \frac{-(T-s)^\alpha + \alpha T (t-s)^{\alpha-1}}{T\Gamma(\alpha+1)} + \frac{(T-s)^{\alpha-1}}{T\mu\Gamma(\alpha)}, & \text{if } \ 0 \le s < t, \\ \frac{-(T-s)^\alpha}{T\Gamma(\alpha+1)} + \frac{(T-s)^{\alpha-1}}{T\mu\Gamma(\alpha)}, & \text{if } \ t \le s < T. \end{cases} \tag{3.4} $$

Proof. By Lemma 3.3, we can reduce the problem (3.1)–(3.2) to an equivalent integral equation

$$ x(t) = I^\alpha h(t) - c_0 = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} h(s)ds - c_0, $$

for some constant $c_0 \in \mathbb{R}$. We have by integration (using Fubini's integral theorem)

$$\int_0^T x(s)ds = \int_0^T \left(\int_0^s \frac{(s-\tau)^{\alpha-1}}{\Gamma(\alpha)} h(\tau)d\tau - c_0 \right) ds =$$

$$= \int_0^T \left(\int_\tau^T \frac{(s-\tau)^{\alpha-1}}{\Gamma(\alpha)} ds \right) h(\tau)d\tau - c_0 T = \int_0^T \frac{(T-\tau)^{\alpha}}{\alpha\Gamma(\alpha)} h(\tau)d\tau - c_0 T.$$

Applying the boundary condition (3.2), we get

$$x(0) = -c_0,$$

$$x(T) = \int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)} h(s)ds - c_0,$$

that is

$$c_0 = \frac{1}{T} \int_0^T \left(-\frac{(T-s)^{\alpha-1}}{\mu\Gamma(\alpha)} + \frac{(T-s)^{\alpha}}{\Gamma(\alpha+1)} \right) h(s)ds.$$

Therefore, the unique solution of (3.1)–(3.2) has the form

$$x(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} h(s)ds + \frac{1}{T} \int_0^T \left(\frac{(T-s)^{\alpha-1}}{\mu\Gamma(\alpha)} - \frac{(T-s)^{\alpha}}{\Gamma(\alpha+1)} \right) h(s)ds =$$

$$= \int_0^t \left(\frac{-(T-s)^{\alpha} + \alpha T(t-s)^{\alpha-1}}{T\Gamma(\alpha+1)} + \frac{(T-s)^{\alpha-1}}{T\mu\Gamma(\alpha)} \right) h(s)ds +$$

$$+ \int_t^T \left(-\frac{(T-s)^{\alpha}}{T\Gamma(\alpha+1)} + \frac{(T-s)^{\alpha-1}}{T\mu\Gamma(\alpha)} \right) h(s)ds = \int_0^T G(t,s)h(s)ds,$$

which completes the proof. □

Remark 3.5. The function $t \mapsto \int_0^T |G(t,s)|ds$ is continuous on I, and hence is bounded. Let

$$\tilde{G} = \sup\left\{ \int_0^T |G(t,s)|ds : \ t \in I \right\}.$$

To establish our main result concerning the existence of solutions (1.1)–(1.2), we list the following hypotheses:

(H1) For each $t \in I$, the function $f(t, \cdot)$ is weakly sequentially continuous.
(H2) For each $x \in C(I, E)$, the function $f(\cdot, x(\cdot))$ is Pettis integrable on I.

(H3) There exist $p \in L^\infty(I, E)$ and a continuous nondecreasing function $\psi : [0, \infty) \to (0, \infty)$ such that

$$\|f(t, x)\| \leq p(t)\psi(\|x\|).$$

(H4) There exists a constant $R > 0$ such that

$$\frac{R}{\|p\|_{L^\infty}\psi(R)\tilde{G}} > 1.$$

(H5) For each bounded set $Q \subset E$, and each $t \in I$, the following inequality holds

$$\beta(f(t, Q)) \leq p(t)\beta(Q).$$

Theorem 3.6. *Let E be a Banach space, and assume assumptions* (H1)–(H5) *are satisfied. If*

$$\|p\|_{L^\infty}\tilde{G} < 1, \tag{3.5}$$

then the boundary value problem (1.1)–(1.2) *has at least one solution.*

Proof. We shall reduce the existence of solutions of the boundary value problem (1.1)–(1.2) to a fixed point problem. To this end we consider the operator $T : C(I, E) \to C(I, E)$ defined by

$$(Tx)(t) = \int_0^T G(t, s)f(s, x(s))ds, \tag{3.6}$$

where $G(\cdot, \cdot)$ is the Green's function defined by (3.4). Clearly the fixed points of the operator T are solutions of the problem (1.1)–(1.2).

First notice that, for $x \in C(I, E)$, we have $f(\cdot, x(\cdot)) \in P(I, E)$ (assumption (H2)). Since, $s \mapsto G(t, s) \in L^\infty(I)$ then $G(t, \cdot)f(\cdot, x(\cdot))$ for all $t \in I$ is Pettis integrable (Proposition 2.3) and thus, the operator T makes sense. Let $R \in \mathbb{R}_+^*$, and consider the set

$$Q = \Bigg\{ x \in C(I, E) : \|x\|_\infty \leq R \quad \text{and}$$
$$\|x(t_1) - x(t_2)\| \leq \|p\|_{L^\infty}\psi(R) \int_0^T |G(t_2, s) - G(t_1, s)|ds \quad \text{for} \quad t_1, t_2 \in I \Bigg\}.$$

Clearly, the subset Q is closed, convex and equicontinuous. We shall show that T satisfies the assumptions of Theorem 2.8.

Step 1. T maps Q into itself. Take $x \in Q, t \in [0, T]$ and assume that $Tx(t) \neq 0$. Then there exists $\varphi \in E^*$ such that $\|Tx(t)\| = \varphi(Tx(t))$. Thus

$$\|Tx(t)\| = \varphi(Tx(t)) = \varphi\left(\int_0^T G(t, s)f(s, x(s))ds\right) \leq \int_0^T |G(t, s)|\varphi(f(s, x(s)))ds \leq$$
$$\leq \int_0^T |G(t, s)|p(s)\psi(\|x\|)ds \leq \|p\|_{L^\infty}\psi(R)\tilde{G} \leq R.$$

Let $t_1, t_2 \in I, t_1 < t_2, x \in Q$, so $Tx(t_2) - Tx(t_1) \neq 0$. Then there exist $\varphi \in E^*$ such that

$$\|Tx(t_2) - Tx(t_1)\| = \varphi(Tx(t_2) - Tx(t_1)).$$

Thus

$$\begin{aligned}\|Tx(t_2) - Tx(t_1)\| &= \varphi\Big(\int_0^T G(t_2, s)f(s, x(s))ds - \int_0^T G(t_1, s)f(s, x(s))ds\Big) \leq \\ &\leq \int_0^T |G(t_2, s) - G(t_1, s)| \|f(s, x(s)\| ds \leq \\ &\leq \|p\|_{L^\infty}\psi(R) \int_0^T |G(t_2, s) - G(t_1, s)| ds.\end{aligned}$$

Hence $T(Q) \subset Q$.

Step 2. T is weakly sequentially continuous. Let (x_n) be a sequence in Q and let $(x_n(t)) \to x(t)$ in (E, w) for each $t \in I$. Fix $t \in I$. Since f satisfies assumptions (H1), we have $f(t, x_n(t))$, converging weakly uniformly to $f(t, x(t))$. Hence the Lebesgue Dominated Convergence theorem for Pettis integral implies $Tx_n(t)$ converging weakly uniformly to $Tx(t)$ in E_w. We do it for each $t \in I$ so $Tx_n \to Tx$. Then $T : Q \to Q$ is weakly sequentially continuous.

Step 3. The implication (2.1) holds. Now let V be a subset of Q such that $V = \overline{conv}(T(V) \cup \{0\})$. Obviously $V(t) \subset \overline{conv}(T(V) \cup \{0\})$ for all $t \in I$. $TV(t) \subset TQ(t), t \in J$ is bounded in E. By assumption (H5), and the properties of the measure β we have for each $t \in I$

$$\begin{aligned}\upsilon(t) \leq \beta(T(V(t)) \cup \{0\}) \leq \beta(T(V(t))) &\leq \int_0^T |G(t, s)| p(s)\beta(V(s))ds \leq \\ \leq \|p\|_{L^\infty} \int_0^T |G(t, s)|\upsilon(s)ds &\leq \|p\|_{L^\infty}\|\upsilon\|_\infty \tilde{G}.\end{aligned}$$

This means that

$$\|\upsilon\|_\infty (1 - \|p\|_{L^\infty}\tilde{G}) \leq 0.$$

By (3.5) it follows that $\|\upsilon\|_\infty = 0$, that is $\upsilon(t) = 0$ for each $t \in I$, and then $V(t)$ is relatively weakly compact in E. Applying Theorem 2.8 we conclude that T has a fixed point which is a solution of the problem (1.1)–(1.2). $\square$

4. AN EXAMPLE

As an application of our results we consider the following partial hyperbolic fractional differential equations of the form

$$({}^cD_0^\alpha x_n)(t) = \frac{1}{3e^{t+4}}(1+|x_n(t)|), \quad \text{if} \quad t \in I = [0,1], \tag{4.1}$$

$$x_n(0) + \int_0^1 x_n(s)ds = x_n(1). \tag{4.2}$$

Let

$$E = l^1 = \left\{ x = (x_1, x_2, \dots, x_n, \dots) : \sum_{n=1}^{\infty} |x_n| < \infty \right\}$$

with the norm

$$\|x\|_E = \sum_{n=1}^{\infty} |x_n|.$$

Set

$$x = (x_1, x_2, \dots, x_n, \dots) \quad \text{and} \quad f = (f_1, f_2, \dots, f_n, \dots),$$
$$f_n(t, x_n) = \frac{1}{3e^{t+4}}(1+|x_n|), \quad t \in [0,1].$$

For each $x_n \in \mathbb{R}$ and $t \in [0,1]$ we have

$$|f_n(t,x_n)| \leq \frac{1}{3e^{t+4}}(1+|x_n|). \tag{4.3}$$

Hence conditions (H1), (H2) and (H3) are satisfied with

$$p(t) = \frac{1}{3e^{t+4}}, \quad t \in [0,1], \quad \text{and}$$
$$\psi(u) = 1 + u, \quad u \in [0, \infty).$$

By (4.3), for any bounded set $B \subset l^1$, we have

$$\beta(f(t,B)) \leq \frac{1}{3e^{t+4}}\beta(B), \quad \text{for each} \quad t \in [0,1].$$

Hence (H5) is satisfied.

From (3.4) we have

$$G(t,s) = \begin{cases} \frac{-(1-s)^\alpha + \alpha(t-s)^{\alpha-1}}{\Gamma(\alpha+1)} + \frac{(1-s)^{\alpha-1}}{\Gamma(\alpha)}, & \text{if} \quad 0 \leq s < t, \\ \frac{-(1-s)^\alpha}{\Gamma(\alpha+1)} + \frac{(1-s)^{\alpha-1}}{\Gamma(\alpha)}, & \text{if} \quad t \leq s < 1. \end{cases} \tag{4.4}$$

Using (4.4) and the fact that $\int_0^1 G(t,s)ds = \int_0^t G(t,s)ds + \int_t^1 G(t,s)ds$ we get $\tilde{G} < 7$ and (H4) is satisfied for $R > \frac{7}{3e^4-7}$. It is also clear to see that (3.5) is satisfied.

Consequently, Theorem 3.6 implies that problem (4.1)–(4.2) has a solution defined on $[0,1]$.

REFERENCES

[1] R.P. Agarwal, M. Benchohra, S. Hamani, *A survey on existence result for boundary value problems of nonlinear fractional differential equations and inclusions*, Acta Appl. Math. **109** (2010), 973–1033.

[2] J. Banaś, K. Goebel, *Measures of Noncompactness in Banach Spaces*, Marcel Dekker, New York, 1980.

[3] J. Banaś, K. Sadarangani, *On some measures of noncompactness in the space of continuous functions*, Nonlinear Anal. **68** (2008), 377–383.

[4] M. Benchohra, J.R. Graef, S. Hamani, *Existence results for boundary value problems with non-linear fractional differential equations*, Appl. Anal. **87** (2008) 7, 851–863.

[5] M. Benchohra, J.R. Graef, F. Mostefai, *Weak solutions for nonlinear fractional differential equations on reflexive Banach spaces*, Electron. J. Qual. Theory Differ. Equ. **54** (2010), 1–10.

[6] M. Benchohra, S. Hamani, S.K. Ntouyas, *Boundary value problems for differential equations with fractional order*, Surv. Math. Appl. **3** (2008), 1–12.

[7] M. Benchohra, S. Hamani, S.K. Ntouyas, *Boundary value problems for differential equations with fractional order and nonlocal conditions*, Nonlinear Anal., TMA **71** (2009), 2391–2396.

[8] M. Benchohra, F. Ouaar, *Existence results for nonlinear fractional differential equations with integral boundary conditions*, Bull. Math. Anal. Appl. **2** (2010) 2, 7–15.

[9] F.S. De Blasi, *On the property of the unit sphere in a Banach space*, Bull. Math. Soc. Sci. Math. R.S. Roumanie **21** (1977), 259–262.

[10] J. Diestel, J.J. Uhl (Jr.), *Vector Measures*, [in:] Math. Surveys, vol. 15, Amer. Math. Soc., Providence, R.I., 1977.

[11] D. Guo, V. Lakshmikantham, X. Liu, *Nonlinear Integral Equations in Abstract Spaces*, Mathematics and Its Applications, Kluwer, Dordrecht, 1996.

[12] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North Holland Mathematics Studies, 204, Elsevier Science B.V., Amsterdam, 2006.

[13] S. Krzyska, I. Kubiaczyk, *On bounded pseudo and weak solutions of a nonlinear differential equation in Banach spaces*, Demonstratio Math. **32** (1999), 323–330.

[14] V. Lakshmikantham, S. Leela, J. Vasundhara, *Theory of Fractional Dynamic Systems*, Cambridge Academic Publishers, Cambridge, 2009.

[15] K.S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Differential Equations*, John Wiley, New York, 1993.

[16] H. Mönch, *Boundary value problems for nonlinear ordinary differential equations of second order in Banach spaces*, Nonlinear Anal. **4** (1980), 985–999.

[17] D. O'Regan, *Fixed point theory for weakly sequentially continuous mapping*, Math. Comput. Modelling **27** (1998) 5, 1–14.

[18] D. O'Regan, *Weak solutions of ordinary differential equations in Banach spaces*, Appl. Math. Lett. **12** (1999), 101–105.

[19] B.J. Pettis, *On integration in vector spaces*, Trans. Amer. Math. Soc. **44** (1938), 277–304.

[20] H.A.H. Salem, A.M.A. El-Sayed, O.L. Moustafa, *A note on the fractional calculus in Banach spaces*, Studia Sci. Math. Hungar. **42** (2005) 2, 115–130.

[21] S. Szufla, *On the application of measure of noncompactness to existence theorems*, Rend. Semin. Mat. Univ. Padova **75** (1986), 1–14.

[22] S. Szufla, A. Szukała, *Existence theorems for weak solutions of n-th order differential equations in Banach spaces*, Funct. Approx. Comment. Math. **26** (1998), 313–319.

[23] S. Zhang, *Positive solutions for boundary-value problems of nonlinear fractional diffrential equations*, Electron. J. Differential Equations **36** (2006), 1–12.

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