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Dedicated to the memory of my father

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VAGUE BCK/BCI-ALGEBRAS

Abstract. In this note, by using the concept of vague sets, the notion of vague BCK/BCI-algebra is introduced. And the notions of α-cut and vague-cut are introduced and the relationships between these notions and crisp subalgebras are studied.

Keywords: vague sets, vague BCK/BCI-algebra, vague-cut.

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1. INTRODUCTION

Processing of certain information, especially inferences based on certain information, is based on classical two-valued logic. Thus, it is natural and necessary to attempt to establish some rational logic system as the logical foundation for uncertain information processing. This kind of logic cannot be two-valued logic itself but might form a certain extension of two-valued logic.

As it is well known, BCK/BCI-algebras are two classes of algebras of logic. They were introduced by Imai and Iseki [6, 7]. BCI-algebras are generalizations of BCK-algebras. Most of the algebras related to the t-norm based logic, such as MTL-algebras, BL-algebras [3, 4], hoop, MV-algebras and Boolean algebras et al., are extensions of BCK-algebras.

The notion of vague set theory introduced by W. L. Gau and D. J. Buehrer [2], as a generalizations of Zadeh's fuzzy set theory [10]. In [1], R. Biswas applied these notion to group theory and introduced vague groups.

Now, in this note we use the notion of vague set to establish the notions of vague BCK/BCI-algebras and then we obtain some related results which have been mentioned in the abstract.

2. PRELIMINARIES

In this section, we present now some preliminaries on the theory of vague sets (VS). In his pioneer work [10], Zadeh proposed the theory of fuzzy sets. Since then it has been applied in wide varieties of fields like Computer Science, Management Science, Medical Sciences, Engineering problems etc. to list a few only.

Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universe of discourse. The membership function for fuzzy sets can take any value from the closed interval $[0; 1]$. Fuzzy set A is defined as the set of ordered pairs $A = \{(u; \mu_A(u)) \mid u \in U\}$ where $\mu_A(u)$ is the grade of membership of element u in set A. The greater $\mu_A(u)$, the greater is the truth of the statement that 'the element u belongs to the set A'. But Gau and Buehrer [2] pointed out that this single value combines the 'evidence for u' and the 'evidence against u'. It does not indicate the 'evidence for u' and the 'evidence against u', and it does not also indicate how much there is of each. Consequently, there is a genuine necessity of a different kind of fuzzy sets which could be treated as a generalization of Zadeh's fuzzy sets [10].

Definition 2.1. [1] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(1) a truth membership function

$$t_A : U \to [0, 1]$$

and

(2) a false membership function

$$f_A : U \to [0, 1],$$

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the 'evidence for u', and $f_A(u)$ is a lower bound of the negation of u derived from the 'evidence against u' and

$$t_A(u) + f_A(u) \leq 1.$$

Thus the grade of membership of u in the vague set A is bounded by a sub interval $[t_A(u), 1 - f_A(u)]$ of $[0, 1]$. This indicates that if the actual grade of membership is $\mu(u)$, then

$$t_A(u) \leq \mu(u) \leq 1 - f_A(u).$$

The vague set A is written as

$$A = \{(u, [t_A(u), f_A(u)]) \mid u \in U\},$$

where the interval $[t_A(u), 1 - f_A(u)]$ is called the 'vague value' of u in A and is denoted by $V_A(u)$.

It is worth to mention here that interval-valued fuzzy sets (i-v fuzzy sets) [11] are not vague sets. In i-v fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the 'evidence for u' only, without considering 'evidence against u'. In vague sets both are independently proposed by the decision maker. This makes a major difference in the judgment about the grade of membership.

Definition 2.2. [1] A vague set A of a set U is called:

(1) the zero vague set of U if $t_A(u) = 0$ and $f_A(u) = 1$ for all $u \in U$,
(2) the unit vague set of U if $t_A(u) = 1$ and $f_A(u) = 0$ for all $u \in U$,
(3) the α-vague set of U if $t_A(u) = \alpha$ and $f_A(u) = 1 - \alpha$ for all $u \in U$, where $\alpha \in (0, 1)$.

Let $D[0, 1]$ denote the family of all closed sub-intervals of $[0, 1]$. Now we define the refined minimum (briefly, $rmin$) and an order "$\leq$" on elements $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2]$ of $D[0, 1]$ as:

$$rmin(D_1, D_2) = [min\{a_1, a_2\}, min\{b_1, b_2\}],$$

$$D_1 \leq D_2 \Longleftrightarrow a_1 \leq a_2 \quad \wedge \quad b_1 \leq b_2.$$

Similarly we can define $\geq$, $=$ and $rmax$. Then the concept of $rmin$ and $rmax$ could be extended to define $rinf$ and $rsup$ of infinite number of elements of $D[0, 1]$.

It is a known fact that $L = \{D[0, 1], rinf, rsup, \leq\}$ is a lattice with universal bounds $[0, 0]$ and $[1, 1]$.

For $\alpha, \beta \in [0, 1]$ we now define (α, β)-cut and α-cut of a vague set.

Definition 2.3. [1] Let A be a vague set of a universe X with the true-membership function t_A and false-membership function f_A. The (α, β)-cut of the vague set A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by

$$A_{(\alpha,\beta)} = \{x \in X \mid V_A(x) \geq [\alpha, \beta]\},$$

where $\alpha \leq \beta$.

Clearly $A(0, 0) = X$. The (α, β)-cuts are also called vague-cuts of the vague set A.

Definition 2.4. [1] The α-cut of the vague set A is a crisp subset A_α of the set X given by $A_\alpha = A_{(\alpha,\alpha)}$.

Note that $A_0 = X$ and if $\alpha \geq \beta$ then $A_\beta \subseteq A_\alpha$ and $A_{(\beta,\alpha)} = A_\alpha$. Equivalently, we can define the α-cut as

$$A_\alpha = \{x \in X \mid t_A(x) \geq \alpha\}.$$

Definition 2.5. Let f be a mapping from the set X to the set Y and let B be a vague set of Y. The inverse image of B, denoted by $f^{-1}(B)$, is a vague set of X which is defined by $V_{f^{-1}(B)}(x) = V_B(f(x))$ for all $x \in X$.

Conversely, let A be a vague set of X. Then the image of A, denoted by $f(A)$, is a vague set of Y such that:

$$V_{f(A)}(y) = \begin{cases} rsup_{z \in f^{-1}(y)} V_A(z) & \text{if } f^{-1}(y) = \{x : f(x) = y\} \neq \emptyset, \\ [0,0] & \text{otherwise.} \end{cases}$$

Definition 2.6. A vague set A of BCK/BCI-algebra X is said to have the sup property if for any subset $T \subseteq X$ there exists $x_0 \in T$ such that

$$V_A(x_0) = rsup_{t \in T} V_A(t).$$

Definition 2.7. [6] Let X be a non-empty set with a binary operation "$*$" and a constant "0". Then $(X, *, 0)$ is called a BCI-algebra if it satisfies the following conditions:
(i) $((x * y) * (x * z)) * (z * y) = 0$,
(ii) $(x * (x * y)) * y = 0$,
(iii) $x * x = 0$,
(iv) $x * y = 0$ and $y * x = 0$ imply $x = y$,
for all $x, y, z \in X$.

We can define a partial ordering $\leq$ by $x \leq y$ if and only if $x * y = 0$.

If a BCI-algebra X satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a BCK-algebra.

A nonempty subset S of X is called a subalgebra of X if $x * y \in S$ for all $x, y \in S$.

We refer the reader to the books [5, 9] for further information regarding BCK/BCI-algebras.

Definition 2.8. [9] Let μ be a fuzzy set in a BCK/BCI-algebra X. Then μ is called a fuzzy BCK/BCI-subalgebra of X if

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$$

for all $x, y \in X$.

3. VAGUE BCK/BCI-ALGEBRAS

From now on $(X, *, 0)$ is a BCK/BCI-algebra, unless otherwise is stated.

Definition 3.1. A vague set A of X is called a vague BCK/BCI-algebra of X if it satisfies the following condition:

$$V_A(x * y) \geq rmin\{V_A(x), V_A(y)\}$$

for all $x, y \in X$, that is

$$t_A(x * y) \geq \min\{t_A(x), t_A(y)\},$$
$$1 - f_A(x * y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}.$$

Example 3.2. Consider a BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let A be a vague set in X. Define

$$t_A(x) = \begin{cases} 0.7 & \text{if } x = 0, \\ 0.3 & \text{if } x \neq 0, \end{cases}$$

and

$$f_A(x) = \begin{cases} 0.2 & \text{if } x = 0, \\ 0.4 & \text{if } x \neq 0. \end{cases}$$

It is routine to verify that $A = \{(x, [t_A(x), f_A(x)]) \mid x \in X\}$ is a vague BCI-algebra of X.

Lemma 3.3. *If A is a vague BCK/BCI-algebra of X, then $V_A(0) \geq V_A(x)$, for all $x \in X$.*

Proof. For all $x \in X$, we have $x * x = 0$, hence

$$V_A(0) = V_A(x * x) \geq rmin\{V_A(x), V_A(x)\} = V_A(x). \qquad \square$$

If $\{x_n\}$ is a sequence in X, then $V_A(x_n)$ is an interval $[t_A(x_n), 1 - f_A(x_n)]$, for any positive integer n. We define

$$\lim_{n\to\infty} V_A(x_n) = [\lim_{n\to\infty} t_A(x_n), 1 - \lim_{n\to\infty} f_A(x_n)].$$

Theorem 3.4. *Let A be a vague BCK/BCI-algebra of X. If there exists a sequence $\{x_n\}$ in X, such that*

$$\lim_{n\to\infty} V_A(x_n) = [1, 1].$$

Then $V_A(0) = [1, 1]$.

Proof. By Lemma 3.3, we have $V_A(0) \geq V_A(x)$, for all $x \in X$, thus $V_A(0) \geq V_A(x_n)$, for every positive integer n. Since $t_A(0) \leq 1$ and $1 - f_A(0) \leq 1$, then we have $V_A(0) = [t_A(0), 1 - f_A(0)] \leq [1, 1]$. Consider

$$V_A(0) \geq \lim_{n\to\infty} V_A(x_n) = [1, 1].$$

Hence $V_A(0) = [1, 1]$. □

For vague sets we can define the intersection of two vague sets A and B of X by

$$V_{A\cap B}(x) := rmin\{V_A(x), V_B(x)\}.$$

Theorem 3.5. *Let A_1 and A_2 be vague BCK/BCI-algebras of X. Then $A_1 \cap A_2$ is a vague BCK/BCI-algebra of X.*

Proof. Let $(x, [t_A(x), f_A(x)]), (y, [t_A(y), f_A(y)]) \in A_1 \cap A_2$. Since A_1 and A_2 are vague BCK/BCI-algebras of X therefore we have:

$$\begin{aligned} V_{A_1\cap A_2}(x * y) &= rmin\{V_{A_1}(x * y), V_{A_2}(x * y)\} \geq \\ &\geq rmin\{rmin(V_{A_1}(x), V_{A_1}(y)), rmin(V_{A_2}(x), V_{A_2}(y))\} = \\ &= rmin\{V_{A_1\cap A_2}(x), V_{A_1\cap A_2}(y)\}, \end{aligned}$$

which proves the theorem. □

Corollary 3.6. *Let $\{A_i | i \in \Lambda\}$ be a family of vague BCK/BCI-algebras of X. Then $\bigcap_{i\in\Lambda} A_i$ is also a vague BCK/BCI-algebra of X.*

Proposition 3.7. *A vague set $A = \{(u, [t_A(u_), f_A(u)]) \mid u \in X\}$ of X is a vague BCK/BCI-algebra of X if and only if t_A and $1-f_A$ are fuzzy BCK/BCI-subalgebras of X.*

Proof. The proof is straightforward. □

Proposition 3.8. *Zero vague set, unit vague set and α-vague set of X are trivial vague BCK/BCI-algebras of X.*

Proof. Let A be a α-vague set of X. For $x, y \in X$ we have

$$t_A(x * y) = \alpha = \min\{\alpha, \alpha\} = \min\{t_A(x), t_A(y)\},$$
$$1 - f_A(x * y) = \alpha = \min\{\alpha, \alpha\} = \min\{1 - f_A(x), 1 - f_A(y)\}.$$

By the above proposition it is clear that A is a vague BCK/BCI-algebra of X. The proof of other cases is similar. □

Theorem 3.9. *Let A be a vague BCK/BCI-algebra of X. Then for $\alpha \in [0, 1]$, the α-cut A_α is a crisp subalgebra of X.*

Proof. Let $x, y \in A_\alpha$. Then $t_A(x), t_A(y) \geq \alpha$, and so $t_A(x*y) \geq \min\{t_A(x), t_A(y)\} \geq \alpha$. Thus $x * y \in A_\alpha$. □

Theorem 3.10. *Let A be a vague BCK/BCI-algebra of X. Then for all $\alpha, \beta \in [0, 1]$, the vague-cut $A_{(\alpha,\beta)}$ is a (crisp) subalgebra of X.*

Proof. Let $x, y \in A_{(\alpha,\beta)}$. Then $V_A(x), V_A(y) \geq [\alpha, \beta]$, and so $t_A(x), t_A(y) \geq \alpha$ and $1-f_A(x), 1-f_A(y) \geq \beta$. Then $t_A(x*y) \geq \min\{t_A(x), t_A(y)\} \geq \alpha$, and $1-f_A(x*y) \geq \min\{1 - f_A(x), 1 - f_A(y)\} \geq \beta$. Thus $x * y \in A_{(\alpha,\beta)}$. □

The subalgebra $A_{(\alpha,\beta)}$ is called a vague-cut subalgebra of X.

Theorem 3.11. *Let A be a vague BCK/BCI-algebra of X. Two vague-cut subalgebras $A_{(\alpha,\beta)}$ and $A_{(\delta,\epsilon)}$ with $[\alpha, \beta] < [\delta, \epsilon]$ are equal if and only if there is no $x \in X$ such that*

$$[\alpha, \beta] \leq V_A(x) \leq [\delta, \epsilon].$$

Proof. Suppose that $A_{(\alpha,\beta)} = A_{(\delta,\epsilon)}$ where $[\alpha, \beta] < [\delta, \epsilon]$ and there exists $x \in X$ such that $[\alpha, \beta] \leq V_A(x) \leq [\delta, \epsilon]$. Then $A_{(\delta,\epsilon)}$ is a proper subset of $A_{(\alpha,\beta)}$, which is a contradiction.

Conversely, suppose that there is no $x \in X$ such that $[\alpha, \beta] \leq V_A(x) \leq [\delta, \epsilon]$. Since $[\alpha, \beta] < [\delta, \epsilon]$, then $A_{(\delta,\epsilon)} \subseteq A_{(\alpha,\beta)}$. If $x \in A_{(\alpha,\beta)}$, then $V_A(x) \geq [\alpha, \beta]$ by the hypotheses we get that $V_A(x) \geq [\delta, \epsilon]$. Therefore $x \in A_{(\delta,\epsilon)}$, then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\epsilon)}$. Hence $A_{(\delta,\epsilon)} = A_{(\alpha,\beta)}$. □

Theorem 3.12. *Let* $| X | < \infty$ *and let* A *be a vague* BCK/BCI*-algebra of* X*. Consider the set* $V(A)$ *given by*

$$V(A) := \{V_A(x) \mid x \in X\}.$$

Then $A_{(\alpha,\beta)}$ *are the only vague-cut subalgebras of* X*, where* $(\alpha, \beta) \in V(A)$*.*

Proof. Let $[a_1, a_2] \notin V(A)$, where $[a_1, a_2] \in D[0, 1]$. If $[\alpha, \beta] < [a_1, a_2] < [\delta, \epsilon]$, where $[\alpha, \beta], [\delta, \epsilon] \in V(A)$, then $A_{(\alpha,\beta)} = A_{(a_1,a_2)} = A_{(\delta,\epsilon)}$. If $[a_1, a_2] < [a_1, b]$ where

$$[a_1, b] = rmin\{(x, y) \mid (x, y) \in V(A)\},$$

then $A_{(a_1,a_2)} = X = A_{(a_1,b)}$. Hence for any $[a_1, a_2] \in D[0, 1]$, the vague-cut subalgebra $A_{(a_1,b)}$ is one of the $A_{(\alpha,\beta)}$ for $(\alpha, \beta) \in V(A)$. □

Theorem 3.13. *Any subalgebra* S *of* X *is a vague-cut subalgebra of some vague* BCK/BCI*-algebra of* X*.*

Proof. Define

$$t_A(x) = \begin{cases} \alpha & \text{if } x \in S, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f_A(x) = \begin{cases} 1 - \alpha & \text{if } x \in S, \\ 1 & \text{otherwise,} \end{cases}$$

It is clear that

$$V_A(x) = \begin{cases} [\alpha, \alpha] & \text{if } x \in S, \\ [0, 0] & \text{otherwise,} \end{cases}$$

where $\alpha \in (0, 1)$. It is clear that $S = A_{(\alpha,\alpha)}$. Let $x, y \in X$. We consider the following cases.

Case 1. If $x, y \in S$, then $x * y \in S$ therefore

$$V_A(x * y) = [\alpha, \alpha] = rmin\{V_A(x), V_A(y)\}.$$

Case 2. If $x, y \notin S$, then $V_A(x) = [0, 0] = V_A(y)$ and so

$$V_A(x * y) \geq [0, 0] = rmin\{V_A(x), V_A(y)\}.$$

Case 3. If $x \in S$ and $y \notin S$, then $V_A(x) = [\alpha, \alpha]$ and $V_A(y) = [0, 0]$. Thus

$$V_A(x * y) \geq [0, 0] = rmin\{[\alpha, \alpha], [0, 0]\} = rmin\{V_A(x), V_A(y)\}.$$

Therefore A is a vague BCK/BCI-algebra of X. □

Theorem 3.14. *Let* S *be a subset of* X *and let* A *be a vague set of* X *which is given in the proof of the above theorem. If* A *is a vague* BCK/BCI*-algebra of* X*, then* S *is a (crisp) subalgebra of* X*.*

Proof. Let A be a vague BCK/BCI-algebra of X and $x, y \in S$. Then $V_A(x) = [\alpha, \alpha] = V_A(y)$, thus

$$V_A(x * y) \geq rmin\{V_A(x), V_A(y)\} = rmin\{[\alpha, \alpha], [\alpha, \alpha]\} = [\alpha, \alpha],$$

which implies that $x * y \in S$. □

Theorem 3.15. *Let A be a vague BCK/BCI-algebra of X. Then the set*

$$X_{V_A} := \{x \in X \mid V_A(x) = V_A(0)\}$$

is a (crisp) subalgebra of X.

Proof. Let $a, b \in X_{V_A}$. Then $V_A(a) = V_A(b) = V_A(0)$, and so

$$V(a * b) \geq rmin\{V_A(a), V_A(b)\} = V_A(0).$$

Then X_{V_A} is a subalgebra of X. □

Theorem 3.16. *Let N be the vague set of X which is defined by:*

$$V_N(x) = \begin{cases} [\alpha, \alpha] & \text{if } x \in N, \\ [\beta, \beta] & \text{otherwise,} \end{cases}$$

for $\alpha, \beta \in [0, 1]$ with $\alpha \geq \beta$. Then N is a vague BCK/BCI-algebra of X if and only if N is a (crisp) subalgebra of X. Moreover, in this case $X_{V_N} = N$.

Proof. Let N be a vague BCK/BCI-algebra of X. Let $x, y \in X$ be such that $x, y \in N$. Then

$$V_N(x * y) \geq rmin\{V_N(x), V_N(y)\} = rmin\{[\alpha, \alpha], [\alpha, \alpha]\} = [\alpha, \alpha]$$

and so $x * y \in N$.

Conversely, suppose that N is a (crisp) subalgebra of X, let $x, y \in X$.

(i) If $x, y \in N$ then $x * y \in N$, thus

$$V_N(x * y) = [\alpha, \alpha] = rmin\{V_N(x), V_N(y)\}.$$

(ii) If $x \notin N$ or $y \notin N$, then

$$V_N(x * y) \geq [\beta, \beta] = rmin\{V_N(x), V_N(y)\}.$$

This shows that N is a vague BCK/BCI-algebra of X. Moreover, we have

$$X_{V_N} := \{x \in X \mid V_N(x) = V_N(0)\} = \{x \in X \mid V_N(x) = [\alpha, \alpha]\} = N.$$ □

Proposition 3.17. *Let X and Y be BCK/BCI-algebras and f a BCK/BCI-homomorphism from X into Y and let G be a vague BCK/BCI-algebra of Y. Then the inverse image $f^{-1}(G)$ of G is a vague BCK/BCI-algebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} V_{f^{-1}(G)}(x * y) &= V_G(f(x * y)) = \\ &= V_G(f(x) * f(y)) \geq \\ &\geq rmin\{V_G(f(x)), V_G(f(y))\} = \\ &= rmin\{V_{f^{-1}(G)}(x), V_{f^{-1}(G)}(y)\}. \end{aligned}$$ □

Proposition 3.18. *Let X and Y be BCK/BCI-algebras and f a BCK/BCI-homomorphism from X onto Y and let D be a vague BCK/BCI-algebra of X with the* sup *property. Then the image $f(D)$ of D is a vague BCK/BCI-algebra of Y.*

Proof. Let $a, b \in Y$, let $x_0 \in f^{-1}(a), y_0 \in f^{-1}(b)$ such that

$$V_D(x_0) = rsup_{t \in f^{-1}(a)} V_D(t), \quad V_D(y_0) = rsup_{t \in f^{-1}(b)} V_D(t).$$

Then by the definition of $V_{f(D)}$, we have

$$\begin{aligned} V_{f(D)}(x * y) &= rsup_{t \in f^{-1}(a*b)} V_D(t) \geq \\ &\geq V_D(x_0 * y_0) \geq \\ &\geq rmin\{V_D(x_0), V_D(y_0) = \\ &= rmin\{rsup_{t \in f^{-1}(a)} V_D(t), rsup_{t \in f^{-1}(b)} V_D(t)\} = \\ &= rmin\{V_{f(D)}(a), V_{f(D)}(b)\}. \end{aligned}$$ □

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REFERENCES

[1] R. Biswas, *Vague groups*, Internat. J. Comput. Cognition 4 (2006) 2, 20–23.

[2] W.L. Gau, D.J. Buehrer, *Vague sets*, IEEE Transactions on Systems, Man and Cybernetics 23 (1993), 610–614.

[3] P. Hajek, *Metamathematics of Fuzzy Logic*, Kluwer Academic Publishers, Dordrecht, 1998.

[4] M. Haveshki, A. Borumand Saeid, E. Eslami, *Some types of filters in BL-algebras*, Soft Computing **10** (2006), 657–664.

[5] Y.S. Huang, *BCI-algebra,* Science Press, China (2006).

[6] Y. Imai, K. Iseki, *On axiom systems of propositional calculi*, XIV Proc. Japan Academy 42 (1966), 19–22.

[7] K. Iseki, *An algebra related with a propositional calculus*, XIV proc. Japan Academy 42 (1966), 26–29.

[8] K. Iseki, S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japonica 23 (1978) 1, 1–26.

[9] J. Meng, Y. B. Jun, *BCK-algebras,* Kyungmoon Sa Co., Korea (1994).

[10] L.A. Zadeh, *Fuzzy sets,* Inform. Contr. **8** (1965), 338–353.

[11] L.A., Zadeh, *The concept of a linguistic variable and its application to approximate reasoning-I*, Inform. Contr. 8 (1975), 199–249.

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