Dariusz Knez*, Eddie Megao*, Janusz Knez, Tomasz Śliwa***

INFLUENCE OF DRILLING DIRECTION ON WELLBORE STRESSES

1. INTRODUCTION

Bradley is acknowledging as a person that introduces Geomechanics to the drilling industries in seventies [2]. From that time, petroleum industry and services has rapidly developed. Maintaining a stable well is of primary importance during drilling and production of oil and gas. Hole collapse and solid particle influx must be prevented during production [5]. Wellbore Stability requires a proper balance between the uncontrollable factors of earth stress, rock strength, and pore pressure, and the controllable factors of wellbore fluid pressure and mud chemical composition [3]. There are many factors that influence the stress state of a wellbore. Some of these operational and technically induced factors can be regulated and controlled to obtain the desired results. Other factors like the formation strata and lithology of the environment are uncontrollable. During the initial operation stage, certain operating parameters take precedence over others and this paper will briefly outline the effect of two of these many parameters that influences the borehole stress regimes.

The two parameters in focus are the angle between σ_H and the projection of the borehole axis onto the horizontal plane β and the wellbore inclination α. Stress states were generated and graphed to see the behaviour of the wellbore stress regimes with varying β and α from 0–360^o and 0–90^o respectively with the increment interval of 10^o.

2. METHOD

In a given stress equations α, β angles where substituted and the stresses computed were tabulated and graphed with respect to varying α and β. In some stress equations that

* AGH University of Science and Technology, Krakow
** Kielce University of Technology

contains the two parameters, one is held constant while the stress state for the other is evaluated as the angles are increased in their given respective ranges, then the one held constant is evaluated while the previously used parameter held constant. Values held as constant in this type of situations are assumed: inclination α – 67°, drilling direction of borehole with respect to σ_H, β – 20°, vertical stress σ_v – 115 MPa, maximum horizontal stress σ_H – 99.78 MPa, minimum horizontal stress σ_h – 91.25 MPa, Poisson's ratio ν – 0.3, pore pressure P_o – 30.2 MPa, hydrostatic pressure P_h – 46.40 MPa, orientation angle, θ – 80°. Equation denotes in-situ stresses as S_1, S_2, S_3 (σ_1, σ_2, σ_3) and transforming these into stress components with the z axis aligned with the wellbore axis give stresses S_x, S_y, S_z (σ_x, σ_y, σ_z), τ_{xy}, τ_{xz} and τ_{yz}. Boundary conditions at the hole are $\sigma_r = P_w$ (the wellbore pressure) and $\tau_r\theta = \tau_{rz} = 0$ [6]. Impermeable mudcake was assumed.

It is commonly assumed that when a vertical well is drilled, the in-situ stress around the wellbore includes three mutually orthogonal principal stresses. That is the vertical stress σ_v the maximum horizontal stress σ_H and the minimum horizontal stress σ_h. However for the inclined borehole, the in-situ stress needs to be converted to a new co-ordinate system where one axis is in the borehole axial direction [4]. Therefore the in-situ stress (or far field stress) for an inclined wellbore can be expressed by eq. (1)–(3).

The stress acting alone the x axis in a borehole Cartesian coordinate system is given by:

$$\sigma_x = \cos^2\alpha\left(\sigma_H \cos^2\beta + \sigma_h \sin^2\beta\right) + \sigma_v \sin^2\alpha \tag{1}$$

where:

- σ_h – the minimum horizontal stress,
- β – the drilling direction of borehole with respect to σ_H,
- α – the borehole inclination,
- σ_v – the vertical stress.

The stress acting alone the y axis in a borehole Cartesian coordinate system is given by:

$$\sigma_y = \sigma_H \sin^2\beta + \sigma_h \cos^2\beta \tag{2}$$

The stress acting alone the z axis in a borehole Cartesian coordinate system is given by:

$$\sigma_z = \sin^2\alpha\left(\sigma_H \cos^2\beta + \sigma_h \sin^2\beta\right) + \sigma_v \cos^2\alpha \tag{3}$$

As represented in figure 1, the in-situ principal stresses obtained using eq. (1) and (3) are plotted. They are generated by holding β constant, while the α is varied from 0–90°.

At $\alpha = 0$, σ_x is 99 MPa and σ_z is 115 MPa. The stress σ_z decreases while σ_x increases as α increase. They reach their turning point at 45° with the stress values of 107 MPa. From figure 1 it can be concluded that, by varying α, it affects the stress state of the wellbore. An optimal point (stress) for these stresses with varying α can be indentified prior to drilling for best stability result.

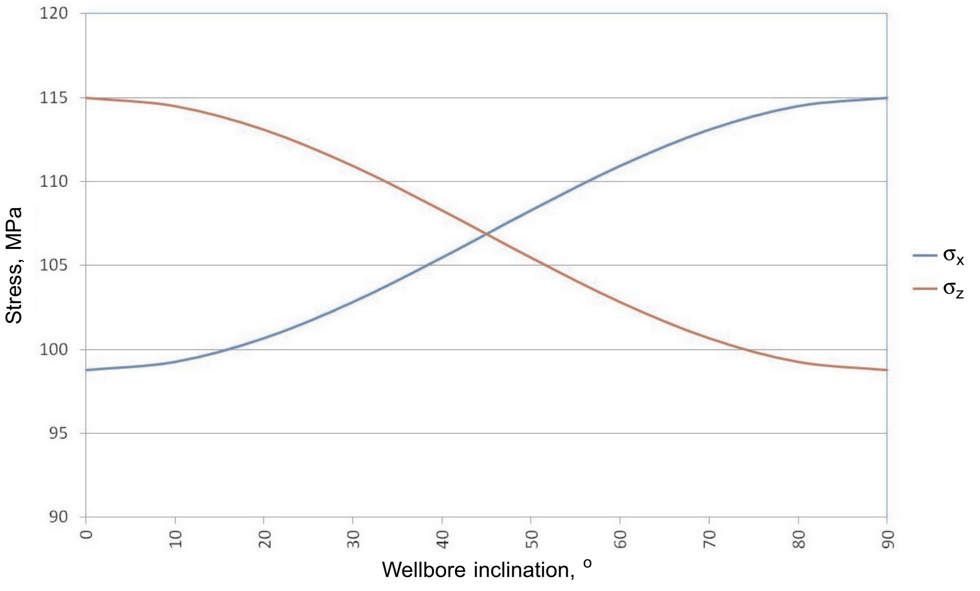

Fig. 1. Transformed in-situ stress vs. wellbore inclination (α) graph in Cartesian coordinate

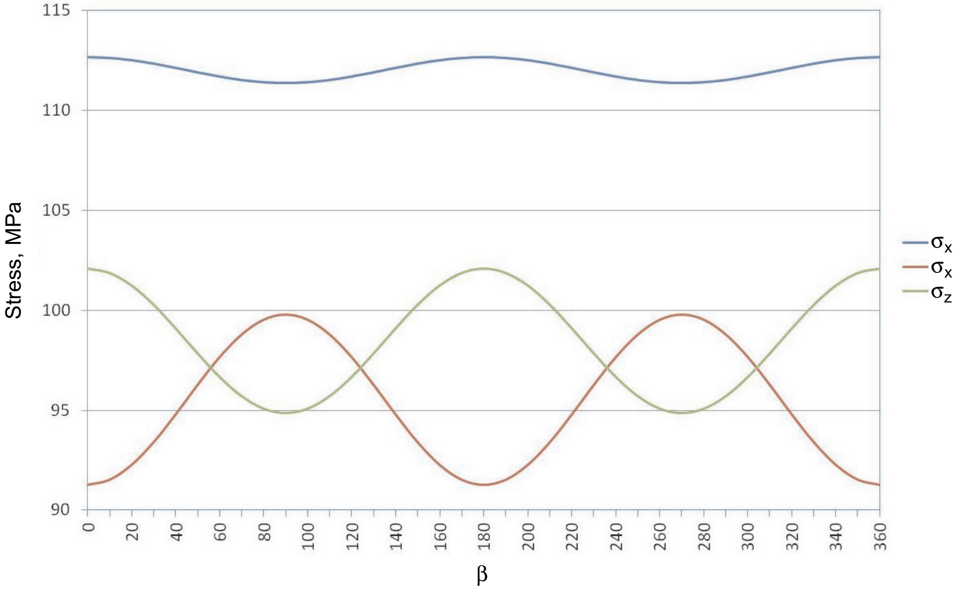

Fig. 2. Transformed in-situ Cartesian coordinate stress vs. β

The graph in figure 2 is generated when α is held constant and β is varied with an increment of 10^o from 0^o through to 360^o, the three Cartesian coordinate stresses σ_x, σ_y, σ_z all exhibits cyclic behavior. According to figure 2 the initial stress value when β = 0 are different for all three stresses with σ_y having the lowest stress of 91 MPa while σ_z has the highest stress, 113 MPa. As β is increased with an increment of 10^o, σ_x and σ_y displays similar but opposite behaviour while σ_z varies in between of 112 MPa and 113 MPa. As shown in graph, σ_x realises its lowest stress at $\beta = 90^o$ and 270^o, while σ_y is at 0^o, 180^o and 360^o. On the other hand σ_z has its maximum stresses throughout the cycle in the same angles compared to σ_x stresses.

Varying β has distinguishable effect on the borehole stress regime as shown in figure 2, thus it is vital to establish the optimal operating β prior to drilling operation.

In-situ shear stress component in a borehole Cartesian coordinate system that acts in the direction of *y* axis is given by:

$$\tau_{xy} = \cos\alpha \sin\beta \cos\beta \left(\sigma_h - \sigma_H\right) \tag{4}$$

where:

α – the wellbore inclination,

β – the drilling direction of borehole with respect to σ_H,

σ_H – the maximum horizontal stress,

σ_h – the minimum horizontal stress.

In-situ shear stress component in a borehole Cartesian coordinate system that acts in the *xz* plane and direction of *z* axis is given by:

$$\tau_{yz} = \sin\alpha \sin\beta \cos\beta \left(\sigma_h - \sigma_H\right) \tag{5}$$

In-situ shear stress component in a borehole Cartesian coordinate system that acts in the *yz* plane and direction of *z* axis is given by:

$$\tau_{xz} = \sin\alpha \cos\alpha \left(\sigma_H \cos^2\beta + \sigma_h \sin^2\beta - \sigma_v\right) \tag{6}$$

where σ_v is the vertical stress component.

The shear stress graph in figure 3 is plotted using eq. (4), (5) and (6). Holding β constant and varying α from $0–90^o$ with an increment of 10^o would cause τ_{xy} and τ_{yz} to display the same behaviour but with different phase. The shear stress component τ_{xz} is completely different from the two former. At $\alpha = 0$, τ_{xy} has the lowest stress value of –2.74 MPa and τ_{yz} and τ_{xz} with 0 MPa respectively. The shear stress component τ_{xz} has minimum value –8.1 MPa at 45^o and becomes 0 MPa at $\alpha = 90^o$ together with τ_{xy}.

Holding α constant and varying β from 0^o through to 360^o with the increment of 10^o would generate shear stress behaviour that has the pattern as shown in figure 4.

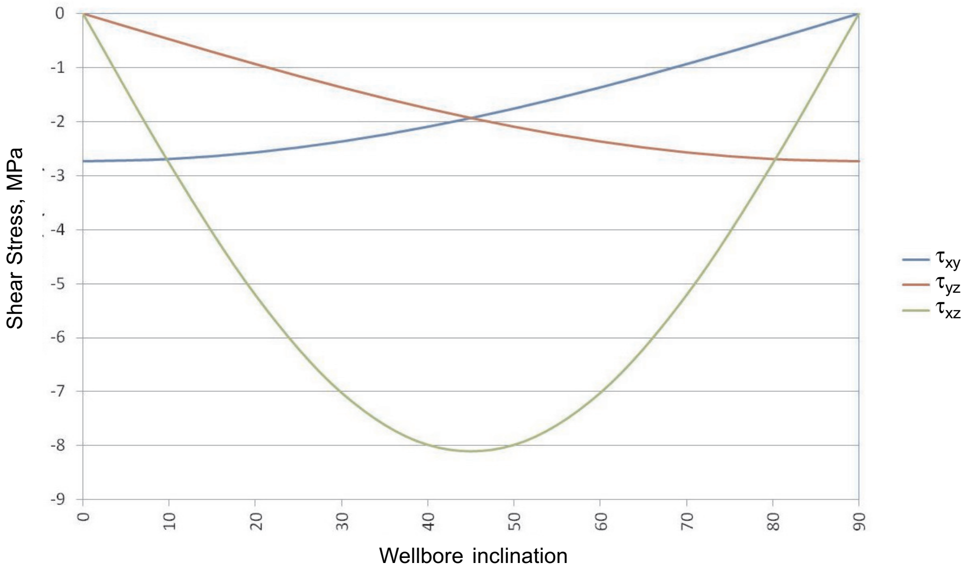

Fig. 3. Shear stress vs. wellbore inclination graph

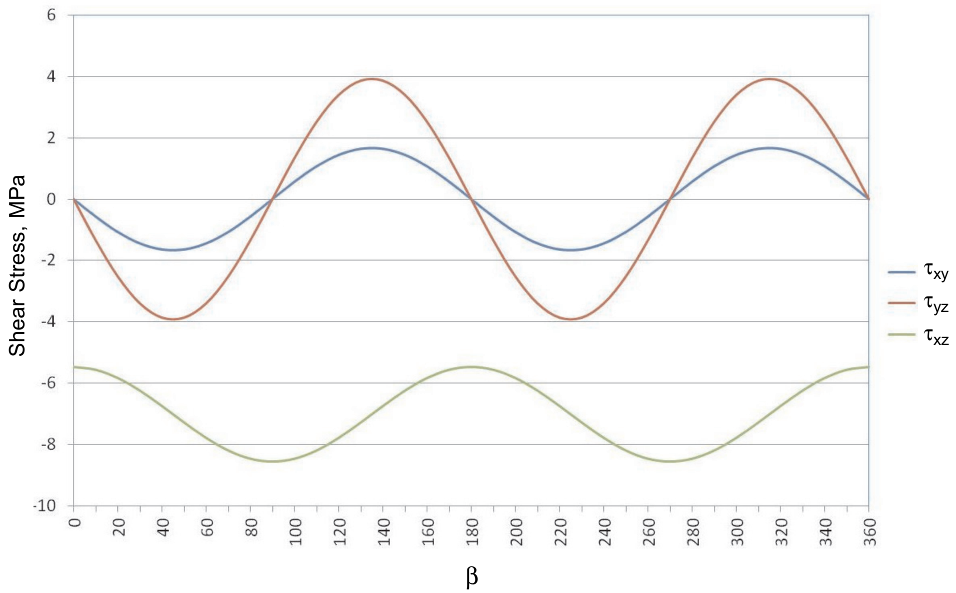

Fig. 4. Shear stress vs. β

As shown on the graph, they all have different behaviour though they are all cyclic in pattern. The shear stress component τ_{xy} and τ_{yz} starts of at 0 MPa when $\beta = 0^{o}$ then decreases together with different magnitude as β increases. According to the graph τ_{xy} and τ_{yz} has the highest stress values when β = 135° and 315°. The shear stress component τ_{xz} has the lowest stress values that varies between –8.5 MPa and –5.5 MPa when compared against τ_{xy} and τ_{yz}. The common thing about all these graphs is that, they have a cyclic behaviour pattern and reaches their minimum and maximum point at various respective interval of β. All the in-situ stress states with respect to borehole coordinate system in Cartesian coordinate system are used here to calculate the total stress state of the wellbore in the cylindrical coordinate system (r, θ, z) as shown in eq. (7)–(10). The total normal stresses and shear stresses at the wellbore wall for a deviated borehole in polar system are defined by the these equations, and then these total stress states are further used to establish the effective major and minor principal stresses as shown in eq. (11) and (12).

Radial stress is given by:

$$\sigma_r = P_h - P_o = P_w \tag{7}$$

where:

P_w – the wellbore pressure,
P_o – the pore pressure,
P_h – the hydrostatic pressure.

The tangential stress (hoop stress) is given by:

$$\sigma_\theta = \left(\sigma_x + \sigma_y\right) - 2\left(\sigma_x - \sigma_y\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta - \left(P_h - P_o\right) \tag{8}$$

Where P_o is the pore pressure, θ is the orientation angle around the wellbore. Components σ_x, σ_y and τ_{xy} are transformed in-situ principal stress and shear stress in a Cartesian co-ordinate system. The axial stress in a cylindrical coordinate system is given by:

$$\sigma_{z'} = \sigma_z - 2\nu\left(\left(\sigma_x - \sigma_y\right)\cos 2\theta + 2\tau_{xy}\sin 2\theta\right) \tag{9}$$

where σ_z is the axial stress in the Cartesian coordinate system and ν is the Poisson's ratio.

The effective shear stress component is given by:

$$\sigma_{z'} = \sigma_z - 2\nu\left(\left(\sigma_x - \sigma_y\right)\cos 2\theta + 2\tau_{xy}\sin 2\theta\right) \tag{10}$$

Effective principal stresses are the ultimate average stresses, all the stresses in eq. (1) through to eq. (10) [1]. Thus they, more or less gives the average summary of the stress state of the wellbore as a whole. In figures 5 and 6, is the typical representation of stress behaviour as β is changed. The in-situ stresses and shear stresses contribute to give this effective maximum principal stress thus it resembles the cyclic behaviour of theses stresses.

Effective major principal stress is given by:

$$\sigma_1 = \frac{\left(\sigma_\theta + \sigma_{z'}\right)}{2} + \frac{1}{2}\sqrt{\left(\sigma_\theta - \sigma_{z'}\right)^2 + 4\tau_{\theta z'}{}^2} \tag{11}$$

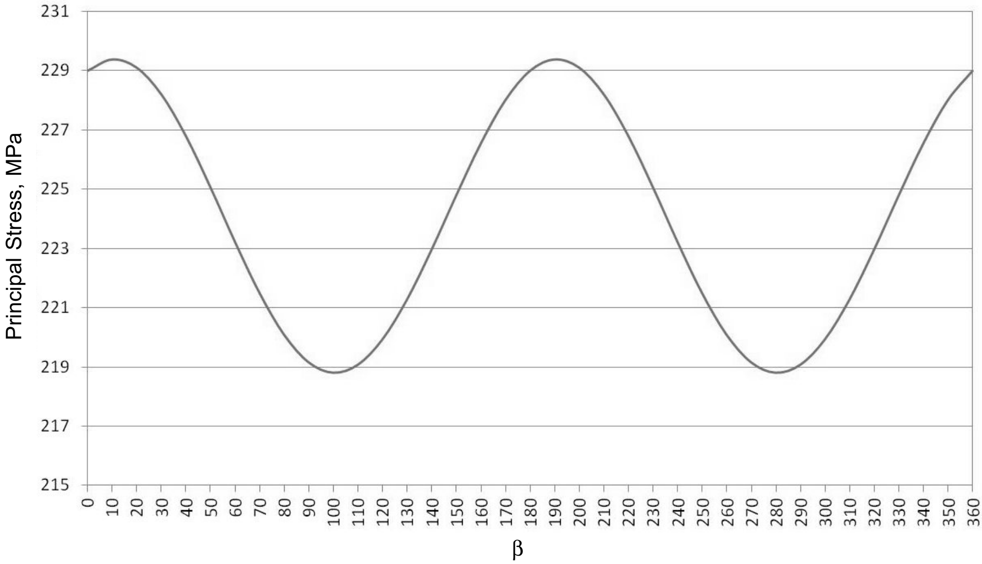

Fig. 5. Effective principal stress σ_1 vs. β

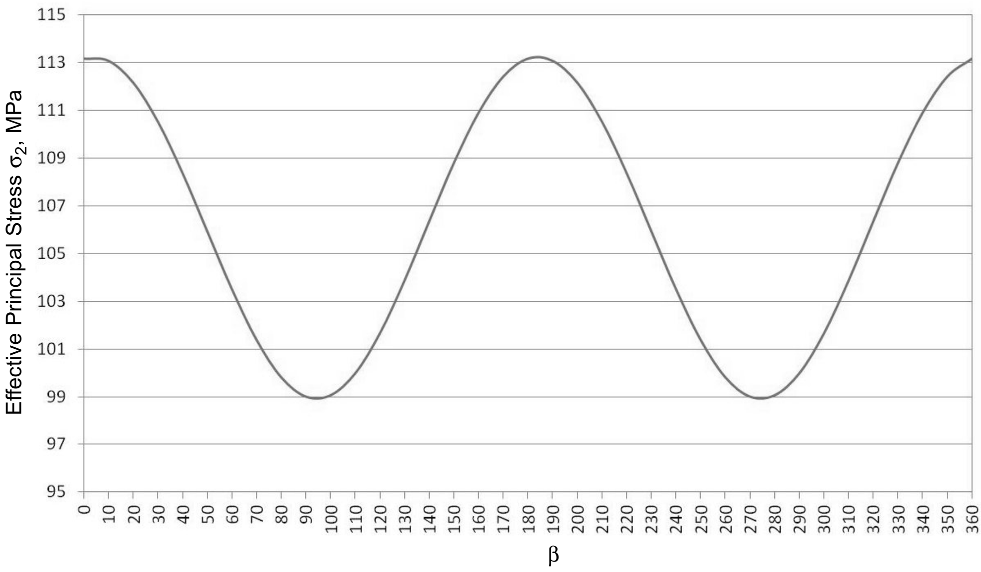

Fig. 6. Effective principal stress σ_2 vs. β

According to the graph σ_1 has a cyclic behaviour and realises its maximum stress value 229.4 MPa when $\beta = 10^o$ and 190^o and the minimum stress 218.8 MPa was realised at $\beta = 100^o$ and 280^o.

Effective minor principal stress is given by:

$$\sigma_2 = \frac{(\sigma_\theta + \sigma_{z'})}{2} - \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_{z'})^2 + 4\tau_{\theta z'}^2} \tag{12}$$

The effective minor principal stress have the similar behaviour as the σ_1 but shifted slightly. Its minimum stress value is observed at β = 95° and 275° with the stress value of 99 MPa. The maximum stress value is realised at 10° and 190° with a stress value of 113.1 MPa

3. CONCLUSIONS

1) Altering inclination or azimuth will have influence on the overall wellbore stress regime. At different values of α and β there is specific characteristic stress and would always affect overall wellbore stress regime.
2) The effective stress is important from wellbore stability point of view.
3) The highest and the lowest stress angles can be clearly determined graphically.
4) Plotting all the in-situ stress, shear stress and stresses related to borehole cylindrical coordinate system, the optimal wellbore stress can be established. Therefore prior to drilling, it would be wise to establish optimal points in advance before the actual drilling is commenced.
5) Wellbore inclination and azimuth can be usually controlled and as such variables will have important influence on hydraulic fracturing performance.

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