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MOTION SIMULATION OF INDUSTRIAL SCREEN MESHES**

1. Introduction

The screening machine for which the simulation was run is a two-frequency screen consisting of two vibrators and a cuboidal sieve supported by spring mounting (Fig. 1).

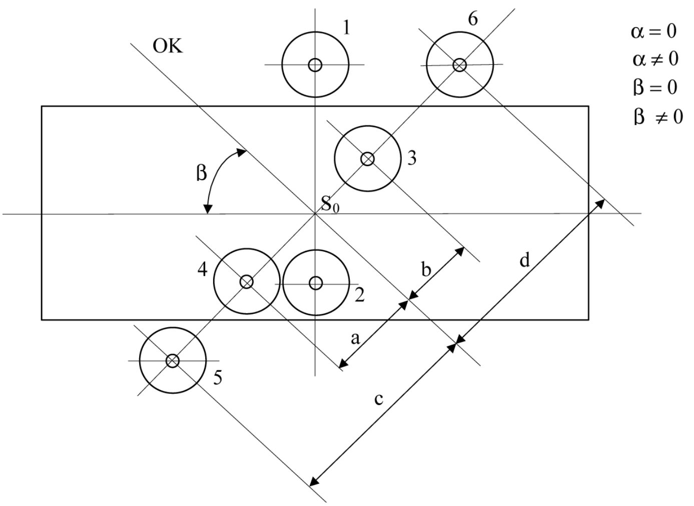

Fig. 1. Possible distribution of rotating vibrators' axes

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Moreover, screen vibrators can work under dynamic self-synchronizing conditions: concurrent and backward with any rotational speed: identical or different for both vibrators. It allows for obtaining very diversified trajectories of vibration.

It should be emphasized, however, that the presented method of simulation can be used for any screen, including machines with a greater number of vibrators and their different configuration, distribution, etc. The presented method was used for testing, for example, a cross-shaped screen, circling-and-revolving screen and linear-elliptic screen [3].

2. The method of lagrange

In the state of equilibrium of the screen solid, an *x*, *y*, *z* right-handed co-ordinate system was erected from the S_0 center of mass. The solid slung in such way has a freedom of movement of 6 degrees. However, due to absence of exciting forces acting in the direction of *z*, the tested system was limited to transverse plane for the *x*–*y* mesh. Damping in the tested system was omitted. Angles of rotation of the solid round the z transverse axis have been marked with φ_z.

The subject of the discussion is motion of the center of mass of screen's solid with the *M* mass and moment of inertia in relation to the *z* transverse axis amounting to I_z. The screen is slung by a set of four identical springs with a spring rate equal to *k*. Damping coefficients along particular axes amount to c_x and c_y.

In order to derive equations of motion the following Lagrange's equation was used:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}+\frac{\partial P}{\partial \dot{q}_i}=0 \tag{1}$$

where:

P — the dissipation of energy function.

L — the system's Lagrangian function:

$$L = T - U$$

T — the system's kinetic energy;

U — the system's potential energy;

In order to facilitate calculations the adopted co-ordinate system is concurrent with theoretical motion trajectories of the screen sieve (Fig. 2). Additionally, it was presumed that vibrators would move symmetrically in relation to the center of mass of the sieve (which means that the distance between axes of both vibrators from the center of mass of the sieve is always constant) and the settings of both vibrators would have the same values of exciting force.

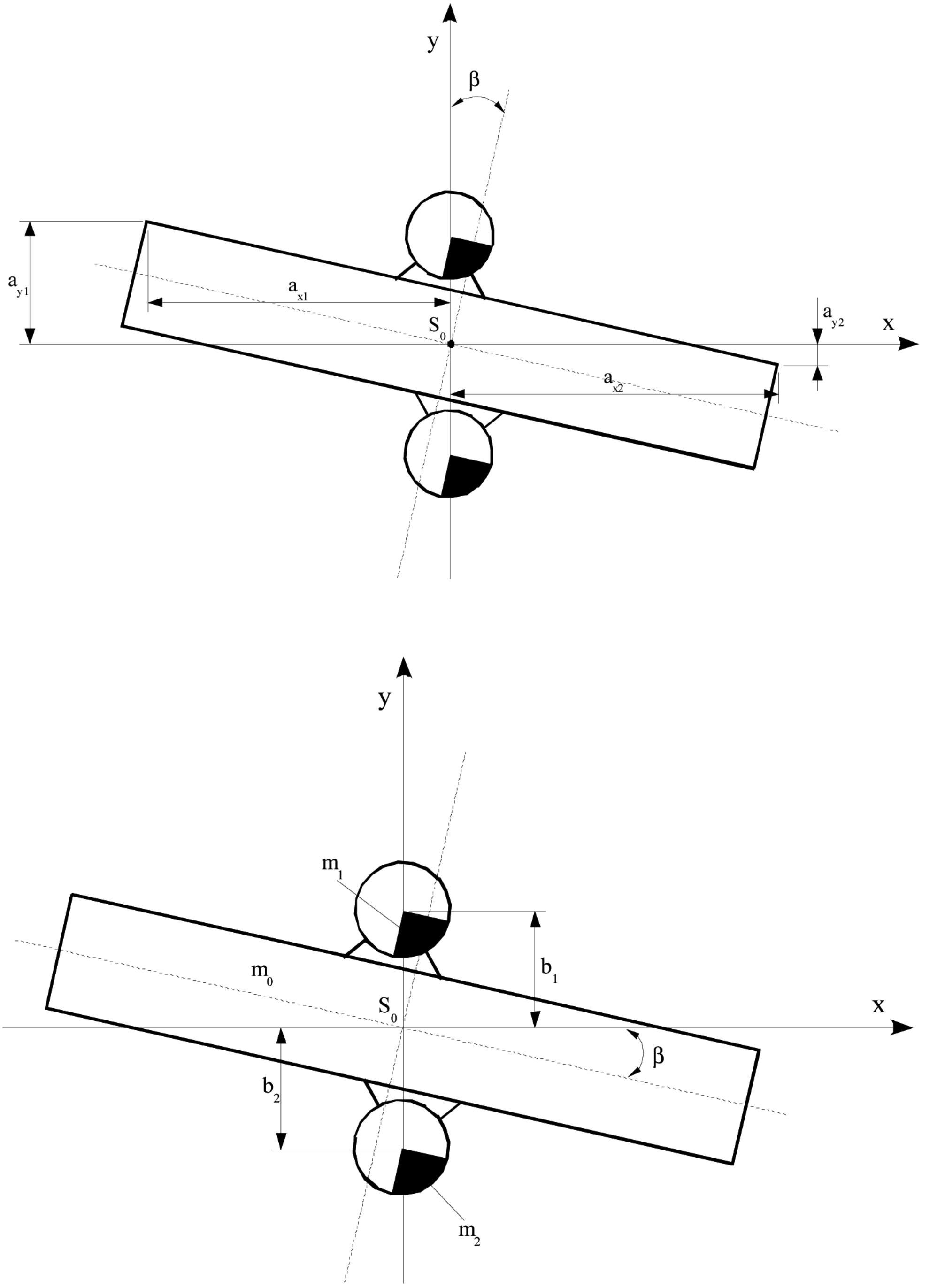

Fig. 2. Markings used for determining sieve motion equations

System's total kinetic energy can be described by the equation:

$$
\begin{aligned}
T = {} & \frac{1}{2} \cdot m \cdot \left(\dot{x}^2 + \dot{y}^2 \right) + \frac{1}{2} \cdot I_z \cdot \dot{\phi}^2 + \\
& + \frac{1}{2} \cdot m_0 \cdot \left(\dot{x} + b \cdot \dot{\phi} + \omega_1 \cdot r \cdot \sin\left(\omega_1 \cdot t \right) \right)^2 + \\
& + \frac{1}{2} \cdot m_0 \cdot \left(\dot{y} - \omega_1 \cdot r \cdot \cos\left(\omega_1 \cdot t \right) \right)^2 + \\
& + \frac{1}{2} \cdot m_0 \cdot \left(\dot{x} - b \cdot \dot{\phi} + \omega_2 \cdot r \cdot \sin\left(\omega_2 \cdot t \right) \right)^2 + \\
& + \frac{1}{2} \cdot I_1 \cdot \omega^2 + \frac{1}{2} \cdot m_0 \cdot \left(\dot{y} - \omega_2 \cdot r \cdot \cos\left(\omega_2 \cdot t \right) \right)^2 + \frac{1}{2} \cdot I_2 \cdot \omega^2
\end{aligned}
\tag{2}
$$

where:

m_0 — mass of element of unbalanced vibrator,
m — mass of screen sieve,
r — unbalanced mass eccentricity radius,
I, I_z — moments of inertia of unbalanced mass and sieve,
ω_1, ω_2 — vibrators' angular velocities,
b — distance of the center of vibrator's shaft from the center of mass of the screen.

System's total potential energy is the following:

$$
\begin{aligned}
U = {} & \frac{1}{2} k_y \cdot \left(y + a_{x1} \cdot \phi \right)^2 + \frac{1}{2} k_y \cdot \left(y + a_{x2} \cdot \phi \right)^2 + \\
& + \frac{1}{2} k_x \cdot \left(x + a_{y1} \cdot \phi \right)^2 + \frac{1}{2} k_x \cdot \left(x + a_{y1} \cdot \phi \right)^2
\end{aligned}
\tag{3}
$$

Dissipation of energy function is:

$$
\begin{aligned}
P = {} & \frac{1}{2} \cdot c_y \left(\dot{y} + a_{x1} \cdot \dot{\phi} \right)^2 + \frac{1}{2} \cdot c_y \left(\dot{y} + a_{x2} \cdot \dot{\phi} \right)^2 + \\
& + \frac{1}{2} \cdot c_x \left(\dot{x} - a_{yx1} \cdot \dot{\phi} \right)^2 + \frac{1}{2} \cdot c_x \left(\dot{x} - a_{y1} \cdot \dot{\phi} \right)^2
\end{aligned}
\tag{4}
$$

where:

a_{xi}, a_{yi} — distance of string attachment points from the sieve's center of gravity,
k_x, k_y — system's stiffness coefficient,
c_x, c_y — system's damping coefficients.

The Lagrangian function is therefore as follows:

$$
\begin{aligned}
L = & \frac{1}{2}\cdot m\cdot\left(\dot{x}^2+\dot{y}^2\right)+\frac{1}{2}\cdot I_z\cdot\dot{\phi}^2 + \\
& +\frac{1}{2}\cdot m_0\cdot\left(\dot{x}+b\cdot\dot{\phi}+\omega_1\cdot r\cdot\sin\left(\omega_1\cdot t\right)\right)^2 + \\
& +\frac{1}{2}\cdot m_0\cdot\left(\dot{y}-\omega_1\cdot r\cdot\cos\left(\omega_1\cdot t\right)\right)^2 + \\
& +\frac{1}{2}\cdot m_0\cdot\left(\dot{x}-b\cdot\dot{\phi}+\omega_2\cdot r\cdot\sin\left(\omega_2\cdot t\right)\right)^2 + \\
& +\frac{1}{2}\cdot I_1\cdot\omega^2+\frac{1}{2}\cdot m_0\cdot\left(\dot{y}-\omega_2\cdot r\cdot\cos\left(\omega_2\cdot t\right)\right)^2 + \\
& +\frac{1}{2}\cdot I_2\cdot\omega^2-\frac{1}{2}k_y\cdot\left(y+a_{x1}\cdot\phi\right)^2 - \\
& -\frac{1}{2}k_y\cdot\left(y+a_{x2}\cdot\phi\right)^2-\frac{1}{2}k_x\cdot\left(x+a_{y1}\cdot\phi\right)^2-\frac{1}{2}k_x\cdot\left(x+a_{y1}\cdot\phi\right)^2
\end{aligned}
\tag{5}
$$

Another step is to calculate proper derivatives:

$$\frac{\partial L}{\partial \dot{x}}=\left(m+2\cdot m_0\right)\cdot\dot{x}+m_0\cdot\omega_1\cdot r\cdot\sin\left(\omega_1\cdot t\right)+m_0\cdot\omega_2\cdot r\cdot\sin\left(\omega_2\cdot t\right) \tag{6}$$

$$\frac{\partial L}{\partial \dot{y}}=\left(m+2\cdot m_0\right)\cdot\dot{y}-m_0\cdot\omega_1\cdot r\cdot\cos\left(\omega_1\cdot t\right)-m_0\cdot\omega_2\cdot r\cdot\cos\left(\omega_2\cdot t\right) \tag{7}$$

$$\frac{\partial L}{\partial \dot{\phi}}=\left(I_z+2\cdot m_0\cdot b^2\right)\cdot\dot{\phi}+b\cdot m_0\cdot\omega_1\cdot r\cdot\sin\left(\omega_1\cdot t\right)-b\cdot m_0\cdot\omega_2\cdot r\cdot\sin\left(\omega_2\cdot t\right) \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=M\cdot\ddot{x}+m_0\cdot\omega_1{}^2\cdot r\cdot\cos\left(\omega_1\cdot t\right)+m_0\cdot\omega_2{}^2\cdot r\cdot\cos\left(\omega_2\cdot t\right) \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)=M\cdot\ddot{y}+m_0\cdot\omega_1{}^2\cdot r\cdot\sin\left(\omega_1\cdot t\right)+m_0\cdot\omega_2{}^2\cdot r\cdot\sin\left(\omega_2\cdot t\right) \tag{10}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right)=\left(I+m_0\cdot b^2\right)\cdot\ddot{\phi}+b\cdot m_0\cdot\omega_1{}^2\cdot r\cdot\cos\left(\omega_1\cdot t\right)-b\cdot m_0\cdot\omega_2{}^2\cdot r\cdot\cos\left(\omega_2\cdot t\right) \tag{11}$$

$$\frac{\partial L}{\partial x} = -k_x\left(x + a_{y1}\cdot\phi\right) - k_x\left(x + a_{y2}\cdot\phi\right) \tag{12}$$

$$\frac{\partial L}{\partial y} = -k_y\left(y + a_{x1}\cdot\phi\right) - k_y\left(y + a_{x2}\cdot\phi\right) \tag{13}$$

$$\begin{aligned}\frac{\partial L}{\partial \phi} = &-k_y\left(y + a_{x1}\cdot\phi\right)\cdot a_{x1} - k_y\left(y + a_{x2}\cdot\phi\right)\cdot a_{x2} - \\ &-k_x\left(x + a_{y1}\cdot\phi\right)\cdot a_{y1} - k_x\left(x + a_{y2}\cdot\phi\right)\cdot a_{y2}\end{aligned} \tag{14}$$

$$\frac{\partial P}{\partial x} = c_x\left(\dot{x} + a_{y1}\cdot\dot{\phi}\right) + c_x\left(\dot{x} + a_{y2}\cdot\dot{\phi}\right) \tag{15}$$

$$\frac{\partial P}{\partial x} = c_y\left(\dot{x} + a_{x1}\cdot\dot{\phi}\right) + c_y\left(\dot{x} + a_{x2}\cdot\dot{\phi}\right) \tag{16}$$

$$\begin{aligned}\frac{\partial P}{\partial x} = &c_y\left(\dot{x} + a_{x1}\cdot\dot{\phi}\right)\cdot a_{x1} + c_y\left(\dot{x} - a_{x2}\cdot\dot{\phi}\right)\cdot a_{x2} + \\ &+c_x\left(\dot{x} + a_{y1}\cdot\dot{\phi}\right)\cdot a_{y1} + c_x\left(\dot{x} + a_{y2}\cdot\dot{\phi}\right)\cdot a_{y2}\end{aligned} \tag{17}$$

By substitution of equation (1) for appropriate derivatives the following equations of motion for the screen's center of gravity were obtained:

$$\begin{aligned}0 = &M\cdot\ddot{x} + mr\cdot\omega_1^2\cdot\cos\left(\omega_1\cdot t\right) + mr\cdot\omega_2^2\cdot\cos\left(\omega_2\cdot t\right) + k_x\left(x + a_{y1}\cdot\phi\right) + \\ &+k_x\left(x + a_{y2}\cdot\phi\right) + c_x\left(\dot{x} + a_{y1}\cdot\dot{\phi}\right) + c_x\left(\dot{x} + a_{y2}\cdot\dot{\phi}\right)\end{aligned} \tag{18}$$

$$\begin{aligned}0 = &M\cdot\ddot{y} + mr\cdot\omega_1^2\cdot\sin\left(\omega_1\cdot t\right) + mr\cdot\omega_2^2\cdot\sin\left(\omega_2\cdot t\right) + k_y\left(y + a_{x1}\cdot\phi\right) + \\ &+k_y\left(y + a_{x2}\cdot\phi\right) + c_y\left(\dot{x} + a_{x1}\cdot\dot{\phi}\right) + c_y\left(\dot{x} + a_{x2}\cdot\dot{\phi}\right)\end{aligned} \tag{19}$$

$$\begin{aligned}0 = &\left(I_z + 2\cdot m_0\cdot b^2\right)\cdot\ddot{\phi} + mr\cdot b\cdot\omega_1^2\cdot\cos\left(\omega_1\cdot t\right) - mr\cdot b\cdot\omega_2^2\cdot\cos\left(\omega_2\cdot t\right) + \\ &+k_y\left(y + a_{x1}\cdot\phi\right)\cdot a_{x1} + k_y\left(y + a_{x2}\cdot\phi\right)\cdot a_{x2} + k_x\left(x + a_{y1}\cdot\phi\right)\cdot a_{y1} + \\ &+k_x\left(x + a_{y2}\cdot\phi\right)\cdot a_{y2} + c_y\left(\dot{x} + a_{x1}\cdot\dot{\phi}\right)\cdot a_{x1} + c_y\left(\dot{x} - a_{x2}\cdot\dot{\phi}\right)\cdot a_{x2} + \\ &+c_x\left(\dot{x} + a_{y1}\cdot\dot{\phi}\right)\cdot a_{y1} + c_x\left(\dot{x} + a_{y2}\cdot\dot{\phi}\right)\cdot a_{y2}\end{aligned} \tag{20}$$

where M — screen's total mass ($M = m + 2\ m_0$).

3. Solution of equations of motion

Equations of motion (18–20) were solved numerically using parameters of an experimental screen. The ongoing calculations require the insertion to a program of data containing geometrical parameters, masses and screen's spring system rates. The Figures show projections of momentary positions of the screen's centre of mass on the system's plane in accordance with observation time periods. A set of momentary positions of solid in motion reveals the projection of motion trajectory.

Data used for calculations comes from parameters of a screen mounted at an experimental stand. Its basic parameters include:

- screen's total mass: $M = 244.73$ kg;
- mass of a single vibrator: $m_w = 56$ kg;
- mass of electric vibrator's backing plate: $m_{pm} = 10.12$ kg;
- spring rate at each end of the sieve: $k = 1.11 \cdot 10^5$ N/m;
- mass-moment of inertia of the sieve in relation to the z axis: $I_p = 50.38$ kg m^2;
- mass-moment of inertia of the electric vibrator: $I_w = 0.25$ kg m^2;
- mass-moment of inertia of the backing plate: I_{pm}=0.06 kg m^2;
- angular velocities of vibrators: $\omega_1 = \omega_2 = 146$ rds/s.

Based on the above data and Steiner theorem, mass-moments of inertia for the screen were determined for analyzed configurations of co-ordinate system:

- setting 1 ($\beta = 12°48'$) $I_z = 57.03$ kg m^2,
- setting 2 ($\beta = 42°18'$) $I_z = 61.99$ kg m^2,
- setting 3 ($\beta = 61°12'$) $I_z = 77.89$ kg m^2.

With regard to an inclination angle of the screen sieve and different position-dependent spring rates, the co-ordinate system (2) was calculated based on the following dependencies:

$$k_x = k \cdot \sin(\alpha + \beta) \tag{21}$$

$$k_x = k \cdot \cos(\alpha + \beta) \tag{22}$$

where:

α — inclination angle of the screen sieve,
β — theoretical angle of screen sieve trajectories.

The string rates data allowed for the calculation of natural frequencies for *x* and *y*:

$$\omega_{0x} = \sqrt{\frac{2 \cdot k_x}{M}} \tag{23}$$

$$\omega_{0y} = \sqrt{\frac{2 \cdot k_y}{M}} \tag{24}$$

Another step was to calculate system's damping coefficients:

$$c_x = \xi \frac{k_x}{\omega_{0x}} \tag{25}$$

$$c_y = \xi \frac{k_y}{\omega_{0y}} \tag{26}$$

where ξ — damping coefficient (for steel springs $\xi = 0.005$).

Phase displacement λ between masses of vibrators at the same rotational speed was taken into account by introducing the $\omega \cdot t + \lambda$ expression to appropriate terms of equations (18–20).

Equations of motion modified in this way were integrated numerically, which allowed for obtaining the results which showed the position of center of gravity for the screen's solid and the value of torsional vibration angle. Figure 3 is a comparison of results calculated for five values of phase displacement angle ($\lambda = 0°, 45°, 90°, 135°$ i $180°$) at the same rotational frequency of both vibrators.

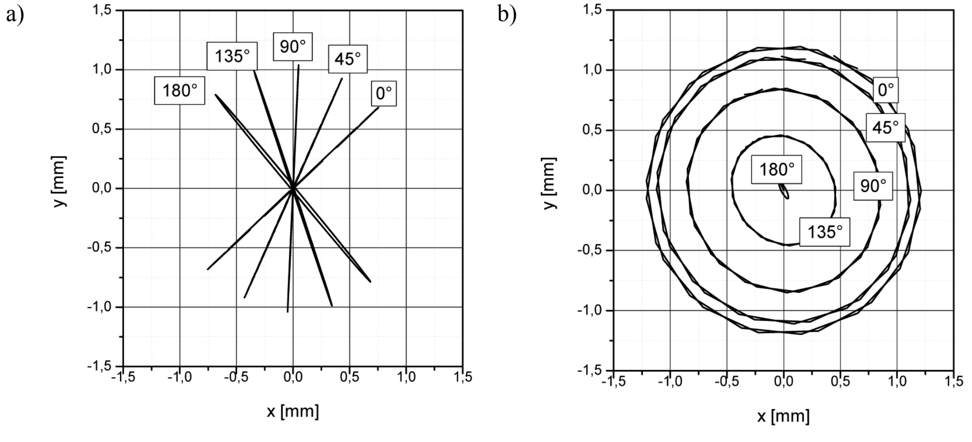

Fig. 3. Motion trajectories of the sieve's center of mass for:
a) backward and b) concurrent synchronization

System of equations (18–20) was also used for simulation of the sieve's motion at different rotational speed of vibrators. Figures 4 and 5 show projections of momentary positions of the sieve's center of mass on the system's plane in accordance with observation time periods.

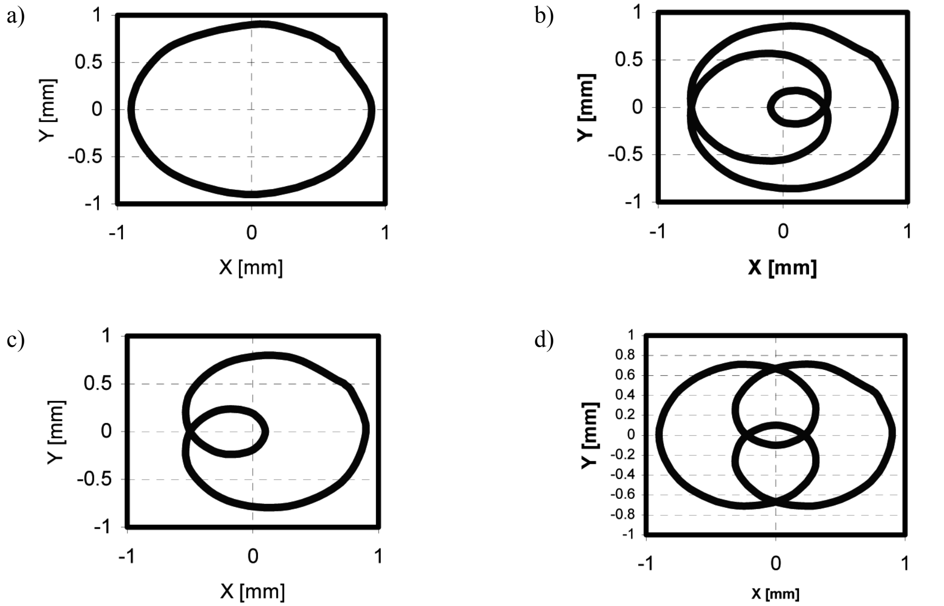

Fig. 4. Motion trajectories for compatible synchronization. Vibrators' rotational frequency ratio ω_1/ω_2:
a) $\omega_1/\omega_2 = 1$; b) $\omega_1/\omega_2 = 2/3$; c) $\omega_1/\omega_2 = ½$; d) $\omega_1/\omega_2 = 1/3$

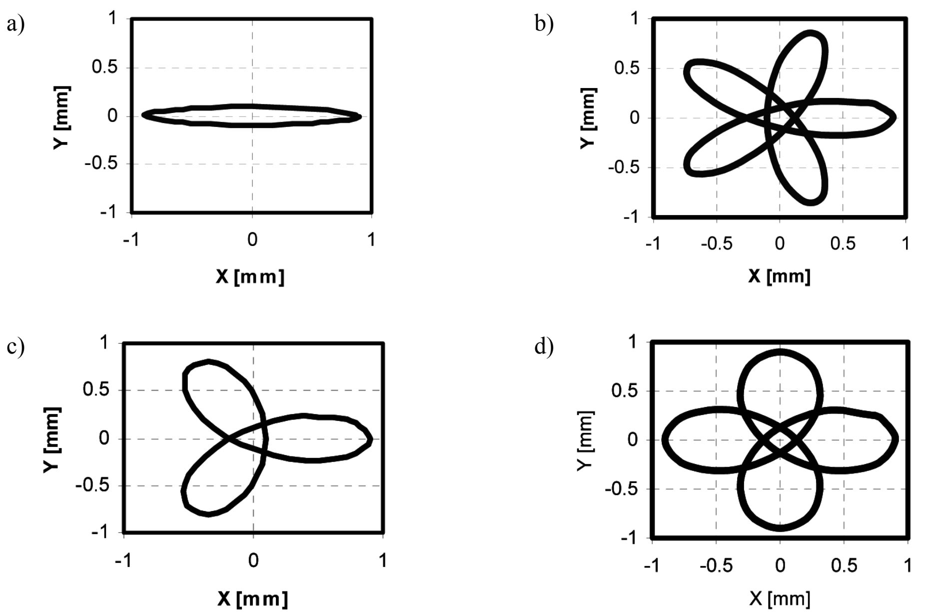

Fig. 5. Motion trajectories for reverse synchronization. Vibrators' rotational frequency ratio ω_1/ω_2:
a) $\omega_1/\omega_2 = -1$; b) $\omega_1/\omega_2 = -2/3$; c) $\omega_1/\omega_2 = -1/2$; d) $\omega_1/\omega_2 = -1/3$

4. Conclusions

Simulations showed and further kinematic tests confirmed that a phase displacement between electric vibrators has a crucial influence on the sieve's motion for the same rotational speed of both vibrators. The influence consists in a transition from circular motion to torsional vibration of the sieve in the case of concurrent performance of vibrators as well as a change to the angle of the screen's centre of mass motion trajectory and to the motion of sieve's tips in the case of sieve's backward movement. The above changes have a significant influence on basic processing parameters of the screen such as K throwing index and speed of carrying the material upon the screen sieve surface.

It is obvious that at different rotational speed of vibrators the phase displacement losses the physical meaning and it is not accounted for in numerical calculations. Nonetheless, it this case theoretically anticipated motion trajectories were fully compatible with the results of further kinematic tests for a ready two-frequency screen [4].

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