AGH Journal of Mining and Geoengineering • Vol. 36 • No. 2 • 2012

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ACCURACY OF MINE TREMORS LOCATION WITH 1C/3C SEISMIC NETWORK

1. Introduction

The location of seismic emission is an important factor concerind geophysical mining survey workers. The location of sources of strong mining tremors (whose energy exceeds 1.0e3J) allows for the assessment of geomechanical quality of rock mass instability in areas of intensive mining exploitation to be measured. More difficult to pin point is the location of small energy seismic events. They are usually detected by a smaller number of seismic stations compared to the strong events. In spite of this location of hypocenters of the small events it is worth the effort because it can still provide crucial information for seismic hazard evaluation.

The problem in pin pointing seismic source location is not only limited to the proper formulation of the numerical algorithms for data processing or application of appropriate computer programs. It is indispensable to define physical models of media where seismic wave propagates. It is also helpful to model the wave field in such media. The quality of seismic location (e.a. location error) also depends on the accuracy of the measurement of particular parameters and optimal configuration of the seismic measurement network.

2. Location methods in 1C and 3C networks

The basic and most popular method used for seismic source location is the method that uses the onset times of direct longitudinal seismic waves (so called P waves) emitted from the source. The method needs seismograms to be registered on at least five seismic stations. For a smaller number of seismic stations the location technique can be treated as the one which supports the other methods. For example it can increase the accuracy of the directional location of seismic sources. Below the idea of the algorithm is outlined.

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When we analyze the network of seismic stations which consists of N single component (1C) detectors (usually oriented vertically). Seismometers, geophones or accelerometers of different sensitivity and frequency bandwidth can be used to. Record the vibrations of the rock mass. It is most important for the source of the location to register the precise times of P-wave onset at each station. By knowing these onsets and location of the stations (geographical coordinates) one can evaluate the location of the seismic event hypocenter.

Let's assume that the seismic source is the point in 3D and its predicted position has coordinates *x*, *y*, *z*. Let's assume also that the predicted time of the event is represented as *t*. The unknown, true source of the position is represented as (*x*, *y*, *z*) and the unknown true time of the seismic event in the source is represented as *t*. The quality of the approximation can be measured using formula (1):

$$f(x,y,z,t) = \sum_{j=1}^{N} [t_j - T_j]^2 = \sum_{j=1}^{N} [t_j - t - t_p(x_j, y_j, z_j; x, y, z)]^2 \tag{1}$$

where:

- Tj — is the predicted (evaluated) time of the P-wave onset on j-th sensor if source coordinates was assumed to be equal to (*x*, *y*, *z*),
- t_j — registered, real time of the P-wave onset on *j–th* detector,
- x_j, y_j, z_j — coordinates of the position of *j–th* detector ($j = 1,..., N$),
- t — predicted time of seismic event in the source,
- x, y, z — approximated coordinates of the hypocenter,
- t_p — time of the P-wave propagation between the detector and predicted hypocenter if the velocity of the P wave in the rock mass was assumed to be equal to v.

The solution for finding the location is point *x*, *y*, *z*, *t* for which the function is given in the formula (1) reaches a minimum, e. a. the point that fulfill the condition $f(x, y, z, t) = \min$.

The minimization of the function (1) can be achieved in different ways. The traditional method of the minimization of the function $f(x, y, z)$ is a linearization of the formula (1) and iterative solutions of the sets of linear equations [1]. The iteration starts from some initial solutions x^i, y^i, z^i, t^i and then is iteratively adjusted to minimize the functional (1).

The second algorithm uses the Monte Carlo simulation method [3]. For randomly chosen hypocenters from the location of possible locations the value of functional (1) is evaluated. To gain high accuracy the solution number of random draws has to be large (usually more than 1 000 000). Then we choose a solution which minimizes the functional (1) and from this point the local minimization procedure (e.a. Powell minimization methods [4]) starts.

The implementation of the three components detectors (3C) in some cases can increase accuracy of the source location [2]. The 3C sensor is usually more sensitive compared to 1C. On other hand 3C sensors allow for tracking the wavefront trajectory from the sensor to source. Only the polarization of the directional P-waves is used in location. The 3C sensors (A, B and C in Fig.1) register the three seismograms and use polarization analysis of the incoming wave direction which can be determined on each sensor. Tracking back the seismic rays the hypocenter H can be evaluated. The idea of the directional location method is presented in figure 1.

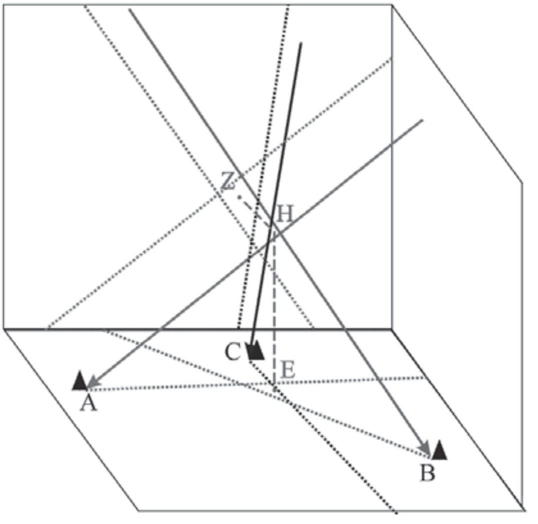

Fig. 1. The idea of the directional method of source location. Each of 3C sensors (A, B and C) register three seismograms and by means of polarization analysis the incoming wave direction can be determined. Tracking back the seismic rays the hypocenter H can be evaluated (details in text)

Using only two seismic rays it is possible to find the point located at the least distance from both rays. That point can be regarded as hypocenter H. For more than two rays the hypocenter is the point for which the functional (2) is minimized:

$$H = \sum_{j=1}^{N} \| P(x,y,z) - L_j(\phi,\varphi) \| = \min \tag{2}$$

where:

N — number of seismic stations,
L_j — straight line ray from *j–th* detector,
ϕ, φ — azimuth and inclination of straight line ray L_j,
$P(x, y, z)$ — approximated hypocenter with coordinates x, y, z.

The projection of the hypocenter on a horizontal plane (e.g. Earth surface) gives the epicenter of the seismic event (E).

In stratified or heterogeneous media seismic rays are no longer a straight line. They change their direction according to Snell's low. To find the hypocenter it is necessary to perform a minimization of the functional (2) for the case when the rays L_j are not straight lines. To perform ray tracing from each sensor a detailed model of the geological medium is essential. The model has to precisely define the particular geometry of the geological strata and its petrophysical parameters, like density and velocity of waves' propagation. Using that model it is possible to render the ray trajectory from sensor towards hypocenter and then perform the optimization defined with formula (2).

3. Location accuracy for 1C and 3C detector nets

Other ways we could assess the accuracy of hypocenter location in different nets depends on the type of sensors that they consists of, measurable parameters used for the location and geological model of the media where the location is worked out. Analytical formulas that quantitatively estimate the location error exist only for the homogeneous media. In the present article we will analyze error evaluation only for such cases.

For the seismic net that consist of 1C detectors we can evaluate the total derivative of formula (1) and obtain the following matrix equation as a result:

$$\left[\frac{\delta f_j}{\delta X_j}\right] \cdot \Delta X + \left[\frac{\delta f_j}{\delta d_j}\right] \cdot \Delta d = 0 \quad j = 1,\ldots,N \quad i = 1,\ldots,4 \tag{3}$$

where ΔX is a vector of perturbed unknowns $[\delta X_i] = [\delta x, \delta y, \delta z, \delta t]$, Δd is perturbation of data vector $[\delta d_j] = [\delta x_j, \delta y_j, \delta z_j, \delta t_j, \delta v_j]$ and N is number of detectors (parameter's description like on formula (1)). Substituting $[\delta f_j/\delta X_i] = \mathbf{A}$ and $[\delta f_j/\delta d_j] = \mathbf{B}$ after simple calculation we obtain components of the vector of unknowns:

$$\Delta X = -\left(\mathbf{A}^T \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \cdot \mathbf{B} \cdot \Delta d \tag{4}$$

or assuming that components of data vector have different variances:

$$\Delta X = -\left(\mathbf{A}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{B} \cdot \Delta d \tag{5}$$

where **V** is the covariance matrix of perturbed data vector.

A similar approach can be applied to evaluate the distribution of the error of the directional location method which uses 3C detectors. The parametrical equation of the straight line in 3D space has a form:

$$\begin{aligned} &x - x_j - t \cdot \cos\phi_j \cdot \sin\varphi_j = 0 \\ &y - y_j - t \cdot \sin\phi_j \cdot \sin\varphi_j = 0 \quad j = 1,\ldots,N \\ &z - z_j - t \cdot \cos\varphi_j = 0 \end{aligned} \tag{6}$$

As previously, the total derivative can be evaluated and ones the separation of unknowns and the measured parameter on different sides of equation the following matrix equation can be obtain:

$$\mathbf{A} \cdot \Delta X = \mathbf{I} \cdot \Delta \mathbf{r} + \mathbf{B} \cdot \Delta \mathbf{k} \tag{7}$$

where ΔX is a vector of perturbed unknowns $[\delta X_i] = [\delta x, \delta y, \delta z]$, $\Delta \mathbf{r}$ is a perturbation of coordinates of particular detectors $[\delta r_i] = [\delta x, \delta y, \delta z]$ $i = 1, ..., 3$ and $\Delta \mathbf{k}$ is a perturbation of direction of seismic rays $[\delta k_i] = [\delta \phi, \delta \varphi]$ $i = 1, ..., 3$. Using formula (7) the vector of hypocenter coordinates error can be evaluated as:

$$\Delta X = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot (\mathbf{I} \cdot \Delta \mathbf{r} + \mathbf{B} \cdot \Delta \mathbf{k}) \tag{8}$$

Of course, the location error calculated with first and second method depends on the number of sensors, their space configuration and individual measurement errors of parameters (e.a. errors of P-wave onsets or errors of directional angles of seismic ray). In mining practice the space configuration of seismic detectors is periodically optimized to guarantee the best possible source location in vicinity of the current exploration areas. Next, errors of the onsets of determination depend on the sampling rate of seismic equipment and the noise level of recorded seismograms. Similarly, the errors of angle evaluation depend on the quality of linear phase determination which is also determined by the noise level and sensor quality. It is necessary to mention that for the approximately planar measurement net which consists of 1C detectors the error of source depth location is significantly higher than error of epicenter determination.

If the net consists of both 1C and 3C sensors or in the case when 3C sensors register the time of the P-wave onsets, the errors ΔX can be determined independently by the two methods discussed above and the total error can be evaluated as the average of the individual errors.

4. Practical example

An interesting example of the evaluation of the source of the location error which combines both location methods can be presented based on the “Rudna” mine. The location of the seismic sources in exploitation area is performed by a seismic network consisting of 1C, vertical seismometers which are supplemented by a single 3C detector (accelerometer). All detectors are mounted on the exploration level. Using the algorithms described in the previous paragraph an effective method of hypocenter location (together with error determination) of induced seismic events can be proposed. The idea of the method is presented below:

— Source epicenter is determined using the 1C seismometers and the Times of the P-wave onset. Of course the onset recorded by the 3C detector can also be used.
— The hypocenter is sought on the vertical line that crosses the epicenter.
— The position of the hypocenter is located in that point of the line which is placed closed to the ray trajectory determined by 3C sensor.

The simple idea described above allows for the increase in accuracy of the source location by mixed 1C/3C net of detectors located on the level of exploration. That is the case of the “Rudna” mine where single 3C detectors were fixed close to some exploitation fields.

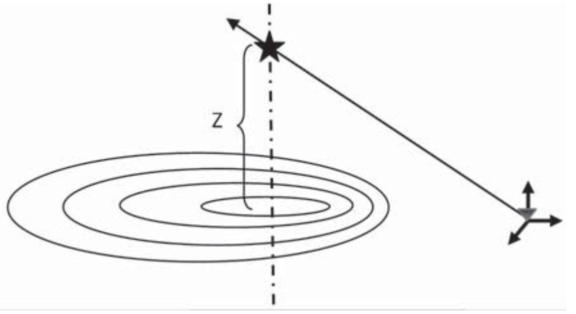

Fig. 2. Idea of hypocenter determination by seismic network that consists of 1C, vertical seismometers and single 3C detector

The limitation of the method presented can be formulated and in the cases described below the method cannot be used:

— The epicenter is located close to the 3C detector. The seismic ray to 3C detector is almost vertical in which case and the error of the "Z" component of the hypocenter is evaluated by directional method and significantly large. If that error is larger than the error determined by the 1C seismometers and the P-wave onsets, the method described above cannot be used.

— When the error of the azimuth and inclination is large. This happens when the energy of the longitudinal P-wave is small and noise level is high. In this case a non-standardized detector of linearity has little value (usually less than one) and determination of the direction of seismic ray is unreliable.

— The onset of longitudinal P-wave is not the first one registered. Another wave (e.g. a refractive wave) comes to the receiver first. The direction of the ray of the refractive wave differs significantly from the direction of direct P-wave and do not point the source. In this case the directional method of source location cannot be used.

If the method of hypocenter location described above is possible to apply, we can achieve an increase of location accuracy. As an example, the analysis of error distribution in the exploitation field G-26 of "Rudna" copper mine is presented. Such analysis is usually done before the 3C detectors are mounted and mining work starts. The reason is to appropriately configure the seismic net in the vicinity of the mining area to reach the highest quality of observation. In the discussed case the observation subnet consists of six geophones 1, 14, 29, 30, 31, 40 from among the 30 sensor is three component (Fig. 3).

As explained earlier the depth of the source („Z" component of hypocenter) is evaluated using epicenter location determined with P-wave onsets time method. The distribution of the epicenter location errors for the mining field G-26 of the "Rudna" mine at a level of –900 m below ground is presented in figure 4. For the evaluation of location error as a value of perturbation the mean error (squared root of variance) of the appropriate parameter was used, e.a. the error of the sensor coordinate was set to 1m, the error of P wave velocity was equal to 300 m/s and error of P wave onset determination to 2 ms.

Using the distribution of epicenter location errors the distribution of errors of hypocenter location was evaluated (Fig. 5). The method described above combines epicenter location

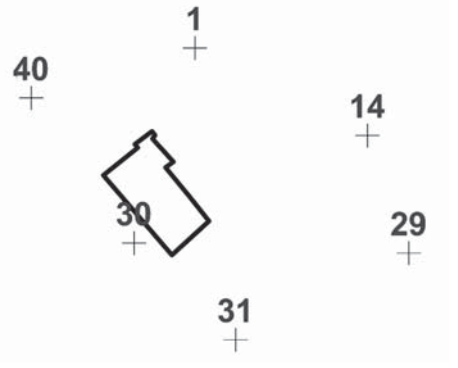

Fig. 3. Mining field G-26 (XX/1) of “Rudna” copper mine with seismometers marked as crosses. Only detector 30 is three component (3C), the other are one component (1C)

with P-wave onsets time method and depth location (“Z” component) with the directional method was used. Errors of angle determination were set to 5° in that case.

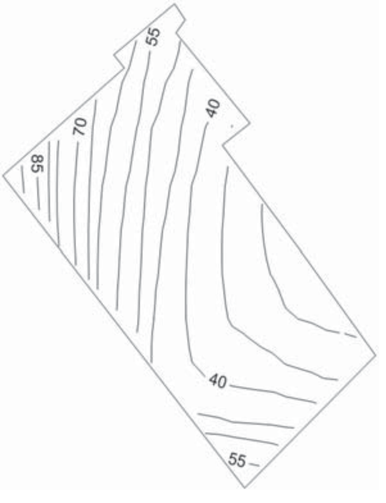

Fig. 4. Isolines of epicenter location errors (in meters) evaluated in P-wave onsets time method

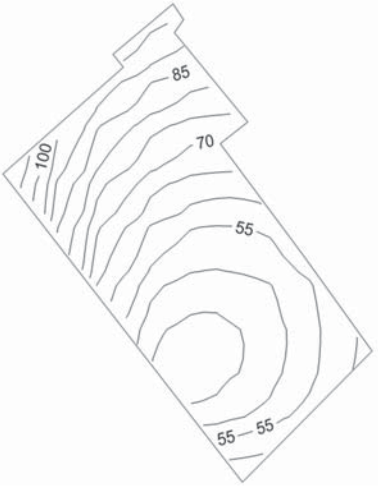

Fig. 5. Isolines of hypocenter location errors (in meters) evaluated as a combination of epicenter location with P-wave onsets time method and depth location (“Z” component) with the directional method

If the location used with the P-wave onsets time method is the only one that was used the average error of hypocenter location for the area of the mining field G-26 at a level –900 m below ground surface is equal to 97.7 m. The error of epicenter location is 46.5 m in this case and depth location is 85.2 m. The incorporation of the 3C sensor to location results decreases the average error of depth location (“Z” component) to 41,4 m and error of hypocenter location to 63.8 m.

The aim of the application of the 3C sensor was to increase the accuracy of depth determination of the epicenter. It would be interesting to compare the distributions of the errors for the both cases — but when only the P-wave onsets location method was used and for the case when the direction of the P-wave ray was also used. The distribution for such cases is presented respectively in figure 6a and 6b. It is easy to notice that for the analyzed field the depth determination the error is twice as small at almost all points when the directional method is used.

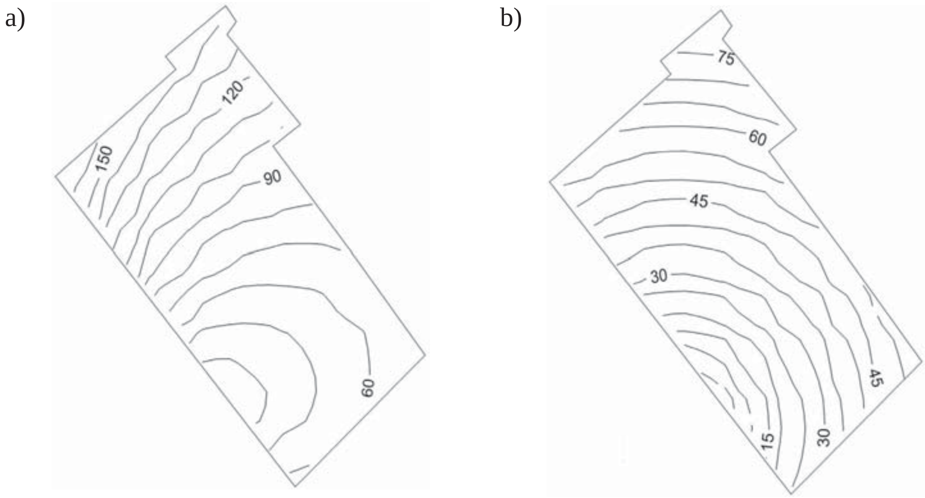

Fig. 6. Isolines of depth („Z" component): a) determination errors (in meters) evaluated with P-wave onsets time method, b) as the combination of P-wave onsets time method and the directional method directional method

5. Summary

Using the results presented in the previous paragraphs it is possible to increase the accuracy of the depth of the hypocenter of seismic source determination. The described method can be a valuable option for the classical location for algorithms implemented in mining and geophysical stations. It can play especially important role for the wide, flat seismic nets which are common in Polish copper mines. The distances between the exploration fields in these mines many times exceed several kilometers and can be effectively monitored by a limited number of sensors (usually les than 10). Nowadays the number of 3C detectors installed is small mainly because of the high requirements as for the transmission lines and 3C sensor costs. Consequently 3C detectors are mounted in mutually large distances in places where enlarged induced seismicity is expected. In such a case effective monitoring can be performed by single 3C sensor supported by a few 1C seismometers. Only in a limited number of cases of strong seismic events the two or more 3C sensors and over a dozen 1C sensors will be used for event location.

As mentioned above, the location which uses the direction of the seismic ray has to take into account ray bending which is a result of the heterogeneity of velocities of the geological medium. The success of directional location essentially depends on the knowledge of geological structure and exactness of polarization analysis which can result in the précised determination of inclination and azimuth angles. Quite often these conditions are not fulfilled. Only simplified geological models are known and only approximated ray tracing can be performed. Similarly, high noise level decreases the precision of direction angle evaluation. In these cases the accuracy of the hypocenter location is similar for both methods. The only realistic possibility to decrease the errors of location is to increase the number of working sensors.

The papers was prepared partially in the framework of statutory research 11.11.140.032 and partially from industry grant 5.5.140.086 financed by KGHM S.A.

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