AGH Journal of Mining and Geoengineering • vol. 36 • No. 3 • 2012

*Beata Trzaskuś-Żak**

AN ASSIGNATION AND COMPARISON OF THE PROGNOSIS RESULTS OF PAID-IN-TERM RECEIVABLES IN OPENCAST MINE "X" VIA AUTOREGRESSIVE MODEL AND PERIODIC TRENDS METHOD**

1. Introduction

Forecasting aids taking management decisions — in the case of this article of the decisions related to receivables. Many statistical methods have been developed for the purpose of forecasting (prognosis, prediction) and they are put into practice by means of development of a larger number of forecasting models, as a result of which the final forecast is most usually calculated as the arithmetic mean or weighted average of partial forecasts (the so-called combined method). The article takes into account the development of the analysed phenomenon in time, i.e. development of paid-in-term receivables in the analysed period of 72 months (six years). When looking for a formula of a forecasting model, the last six values were excluded from the analysis since they were used for the purpose of verification of the model's (forecast's) correctness.

The models used in this article for the purpose of preparation of forecasts are time series models with seasonal fluctuations, that is an autoregressive model and a periodic trends method.

2. Definition of receivables

Receivables means the right to receive a monetary payment in a specific amount and in a specific time [4].

* AGH University of Science and Technology, Krakow

** The article was written in 2012 under the statutory research registered in the AGH University of Science and Technology in Krakow at no. 11.11.100.481.

Receivables can be considered in various legal contexts [3]:

— of a balance sheet law pursuant to which receivables of a company are classified, regardless of their source and legal grounds, according to the criterion of maturity into:
  - short-term receivables maturing 12 months from the balance-sheet date,
  - long-term receivables maturing more than 12 months from the balance-sheet date,
— civil law, pursuant to which entrepreneurs conclude civil law contracts on the basis of which the seller acquires a right to request payment from the buyer for a product issued to the buyer, and the said right is then called a receivable,
— tax law, i.e. in the case when the VAT calculated is in excess of the VAT due, or the issue of including receivables in revenue in the context of income taxes,
— bankruptcy law which regulates a completely different problem, that is the method of recovery of debts,
— commercial law (the Polish Commercial Companies Code) which specifies the scope of liability towards creditors for obligations in commercial companies.

2.1. Receivables from the point of view of the balance sheet law

The basic criterion of classification of receivables is their repayment period (twelve-month payment period) on the basis of which receivables are classified into short-term and long-term ones. The receivables analysed in this article, that is those arising from deliveries and services are an exception from this rule since they are treated as short-term receivables regardless of their maturity period.

The short-term receivables balance sheet item includes all the receivables arising from deliveries and services regardless of their contractual payment date (maturity) as well as all other receivables maturing within 12 months from the balance sheet date.

Pursuant to the Polish Accounting Act receivables belong to the assets of a business entity and they occur in the current assets items:

— item ‘B.II. Short term receivables’, divided into ‘receivables from related entities’ and ‘receivables from other entities’.

Additionally, these two groups are divided into:

— receivables maturing within 12 months,
— receivables maturing in the period exceeding 12 months from the balance sheet date.

Apart from the already-mentioned balance sheet items which refer to short-term receivables there are also ‘receivables from tax, subsidy, customs, social security, health and other benefits’. They include amounts arising from taxes (e.g. VAT) and due from the national budget and local government budgets, overpaid local taxes and other amounts not related to taxes, i.e. contributions for the Social Insurance Company, Labour Fund, Guaranteed Employment Benefit Fund.

TABLE 1

The value of individual balance sheet items related to the receivables in the analysed 'X' mine, PLN

Item/Year	'X' hard rock strip mine					
	1	2	3	4	5	6
Long-term receivables	0.00	0.00	0.00	0.00	0.00	0.00
a) from related entities	0.00	0.00	0.00	0.00	0.00	0.00
b) from other entities	0.00	0.00	0.00	0.00	0.00	0.00
Short-term receivables	6,170,853.52	8,570,720.10	7,192,547.31	9,910,704.66	7,223,913.12	10,761,308.92
a) receivables from related entities	1,676,497.18	2,189,471.10	2,154,263.98	2,467,623.63	2,608,308.44	3,663,915.19
– for deliveries and services with maturity period up to 12 months	1,676,497.18	2,189,471.10	2,154,263.98	2,467,623.63	2,608,308.44	3,663,915.19
– for deliveries and services with maturity period over 12 months	0.00	0.00	0.00	0.00	0.00	0.00
– other	0.00	0.00	0.00	0.00	0.00	0.00
b) receivables from other entities	4,494,356.34	6,381,249.00	5,038,283.00	7,443,081.03	4,615,604.68	7,097,393.73
– for deliveries and services with maturity period up to 12 months	3,987,765.82	5,675,055.38	4,465,431.98	5,020,756.48	3,664,976.17	6,173,438.54
– for deliveries and services with maturity period over 12 months	0.00	0.00	0.00	0.00	0.00	0.00
– from tax, subsidy, customs, social security, health and other benefits	307,579.08	303,791.17	154,246.73	1,786,119.18	365,869.47	461,159.96
– other	199,011.44	402,402.45	418,604.62	636,205.37	584,759.04	462,795.23
– claimed in court	0.00	0.00	0.00	0.00	0.00	0.00

Source: Own analysis on the basis of the 'X' mine's balance sheets.

The next balance sheet item — 'other receivables' includes all the receivables which have not been included elsewhere, e.g. the receivables from companies to which tangible assets were sold, from the employees to whom loans for housing purposes were granted, from property and personal insurance agencies for damages, etc. The item 'receivables claimed in court' includes the receivables pursued in court in order to obtain a right to recover them as well as receivables questioned by the debtor [6].

According to the Table 1, no receivables claimed in court were revealed within the analysed 6-year period. What is interesting here is the increase in successive years of receivables due from related entities for deliveries and services, as well as a high level of receivables from other entities, in relation to the drop of production (Fig. 1) and sale in the year 4 and 5 of the analysis.

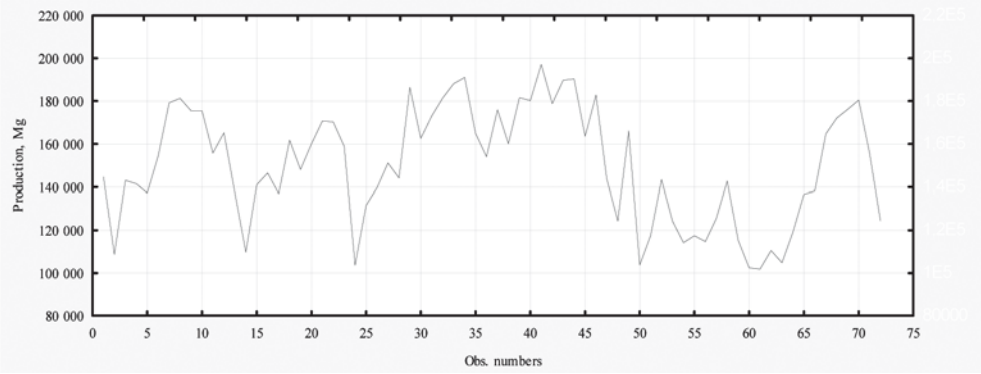

Fig. 1. Production volume of the "X" mine in the analysed period. Source: Own analysis

Due to the specific situation of the analysed mine "X" in the months 46–66 caused by modernisation of the production line which resulted in reduction of production (Fig. 1) and sales, an intervention in the form of a zero-one variable was introduced into the regressive model. The intervention which turned out to be statistically significant in the series of paid-in-term receivables of the "X" mine is the intervention in the months 52–66.

2.2. Time series elements

Economic time series usually consist of the following elements: trend, seasonal fluctuation, interventions and random fluctuations. These elements can be seen on a chart of the analysed phenomenon and therefore a chart of the analysed series has been prepared and presented in Figure 2. On the basis of this chart it can be said that in the analysis of the development of paid-in-term receivables in the analysed period there are all the components listed above, that is: trend (however, it is necessary to find out the level of the trend), interventions as well as seasonal and random fluctuations.

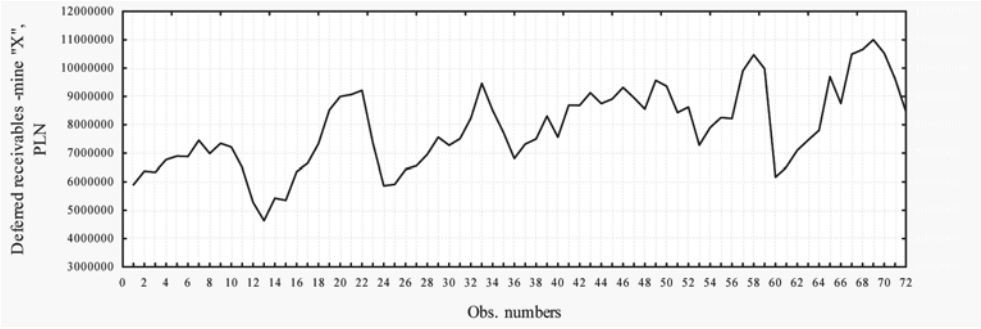

Fig. 2. Development of paid-in-term receivables of the „X" mine in the analysed period.
Source: Own analysis

3. Construction of the forecasting model of „X" mine's paid-in-term receivables with the use of an autoregressive model

The autoregressive model assumes that implementation of the phenomenon depends on the values assumed by the phenomenon in previous periods. Forecasting often uses delayed variables, i.e. the value of a variable is forecast on the basis of previous observations of such a variable. In order to identify periodical fluctuations from a series, in the first place it is necessary to exclude trend from it and then, on the basis of a series without a trend an auto-correlation function can be determined. In the course of the procedure it is necessary to define delayed variables by providing such values as 'one month ago', 'two months ago', 'one year ago', etc. The purpose of these variables is to help you capture the effect of seasonal fluctuations. In the next step delayed variables are introduced into the model and successive variables insignificant for the model are eliminated by means of a multiple regression method [7, 8].

The steps taken for the purpose of identification of seasonal fluctuations in the course of development of a forecasting model include removal of trend and taking into consideration the intervention in the months 52–66.

The trend was determined by means of a downward multiple regression method. On the basis of the results presented in Table 2 a trend formula was established, taking into account the intervention, and then it was deducted from the original series.

$$Trend + Interventi\,on = 6071098{,}80 + 54982{,}99 \cdot t - 1043289{,}84 \cdot I \tag{1}$$

where:

t — subsequent month of the time series,
I — intervention in the months 52–66.

TABLE 2

The final summary of the results of the multiple regression of the calculation of the trend equation, taking into account the intervention

N = 66	**Summary of the variable dependable regression: Paid-in-term receivables, n = 66 R = 0.6133, R^2 = 0.3762, Corrected R^2 = 0.3563 F(2, 63) = 18.99 p < 0,00000, Std. estimation error: 1040825.42**					
Absolute term	b^*	Std. Error from b^*	***b***	Std. Error from *b*	*t*(63)	*p*
t			**6 071 098.80**	293 048.2	20.717	0.000000
Intervention	0.813593	0.144691	**54 982.99**	9 778.3	5.623	0.000000
Std. error	–0.339600	0.144691	**–1 043 289.84**	444 508.0	–2.347	0.022078

Source: Own analysis.

On the basis of the series without trend and including the intervention, an autocorrelation function, presented on Figure 3, was determined.

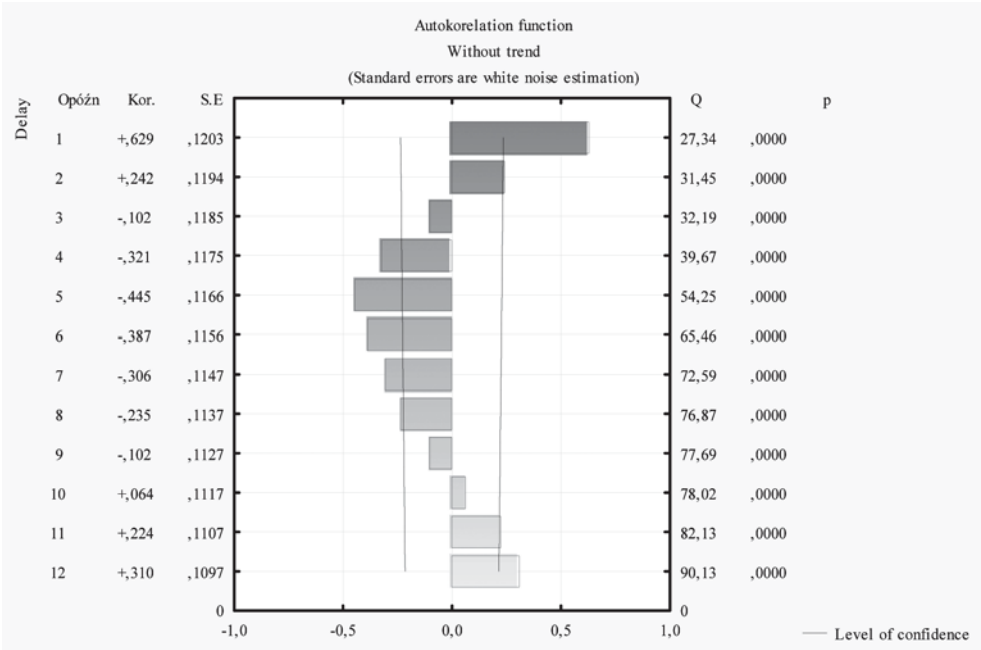

Fig. 3. The autocorrelation function of the analysed phenomenon after excluding the trend and taking into account the intervention. Source: Own analysis

The values of the autocorrelation function indicate that it can be suspected that in the time series there are periodical components with the following periods: 1, 2, 4, 5, 6, 7, 8, 11 and 12 months, however, the autoregressive component of the first row is the most distinct. In the next step, by using the method of downward multiple regression a model was developed which contained provisionally identified periodical components in the form of properly delayed autoregressive variables. After the provisional model was estimated a final model was developed (on the basis of the results presented in Tables 2 and 3), illustrated with the formula No. 2.

Equation of the autoregressive model I:

$$MODEL(I) = 6071098,8 + 54982,99 \cdot t - 1043289,84 \cdot I + \\ + 0,6266 \cdot O(-1) - 0,2967 \cdot O(-2) - 0,3165 \cdot O(-5) - 0,2527 \cdot O(-8) \quad (2)$$

TABLE 3

The final results of multiple regression after introduction of delayed variables into the model

***N* = 58**	**Summary of the variable dependable regression: Without trend taking into account intervention R = 0.7280, R^2 = 0.5299, Corrected R^2 = 0.4945 F(4,53) = 14.938 p< 0,00000, Std. estimation error: 765222.32**					
	b^*	Std. Error from b^*	***b***	Std. Error from *b*	*t*(63)	*p*
Absolute term			–8 970.5657	100860.5	–0.08894	0.929465
O(–1)	0.627761	0.132117	**0.6266**	0.1	4.75155	0.000016
O(–2)	–0.296797	0.129787	**–0.2967**	0.1	–2.28680	0.026230
O(–5)	–0.310284	0.104316	**–0.3165**	0.1	–2.97446	0.004411
O(–8)	–0.224859	0.110200	**–0.2527**	0.1	–2.04047	0.046299

Source: Own analysis.

where:

t — subsequent month of the time series,

I — intervention in the months 52–66.

O (–1) — data (paid–in -term receivables of the „X" mine) delayed by one month,

O (–2) — data (paid-in-term receivables of the „X" mine) delayed by two months,

O (–5) — data (paid-in-term receivables of the „X" mine) delayed by five months,

O (–8) — data (paid-in-term receivables of the „X" mine) delayed by eight months.

Further verification of the model consists in assessment of the properties of residuals. Normal distribution of the residuals can be verified by means of the Shapiro-Wilk test. According to Figure 4 the value of the Shapiro-Wilk's statistics is 0.9798 whilst the value of test probability is 0.3751. Therefore, there are no grounds for rejecting a hypothesis about normal distribution of residuals on the significance level of $\alpha = 0.05$.

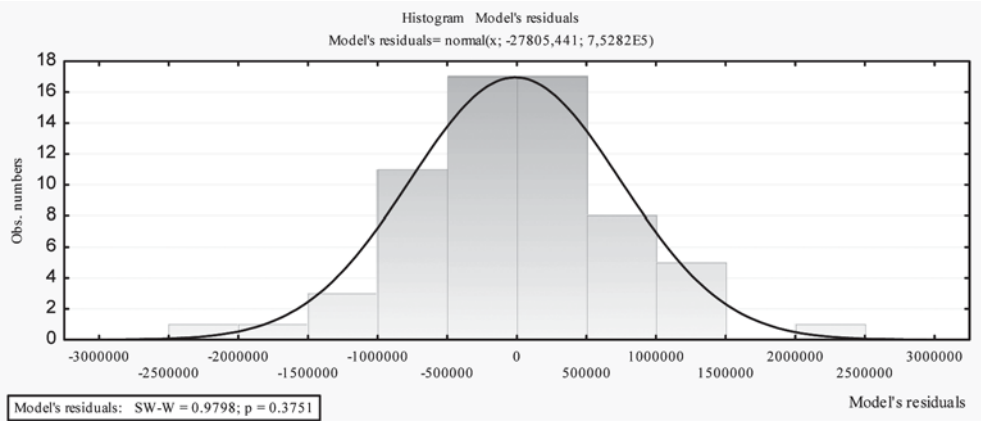

Fig. 4. Graph of the normality of legal opinions s of the autoregressive model I.
Source: Own analysis

4. Development of a forecasting model of „X" mine's paid-in-term receivables with the use of the method of periodic trends method

This method consists in estimating appropriate trend models based on the formula No. 3, separately for empirical data of the variable forecast on the basis of particular seasons (phases of the cycle), in this case months, whereas in practice, for every phase of the cycle linear functions are most frequently used. The forecast for a given season is prepared each time on the basis of an appropriate model for individual phases of the cycle [5].

$$y_t = f(t) + \xi \tag{3}$$

where:

y_t — variable which illustrates the level of the analysed phenomenon over time,
$f(t)$ — model of the trend,
ξ — random component.

By using the periodic trends method appropriate trend models were developed separately for the data from individual months in the years 1–6. The last six monthly values of the series were used for the purpose of verification of the forecast of model II (excluding intervention) and model III (including intervention). Results of the method are presented in Table 4.

TABLE 4

Estimated trend formulas for the given real values of paid-in-term receivables

Months	The periodic trends model without taking intervention into account	The periodic trends model without taking intervention into account
January	*44 711 093.08+550 816.28 t*	*4 363 329.8+669 857.68 t*
February	*5 381 299.55+471 459.28 t*	*5 033 536.27+620 500.68 t*
March	*5 413 721.54+473 041.65 t*	*5 065 958.26+622 083.06 t*
April	*6 092 687.76+357 707.9 t*	*5 605 819.16+596 174.14 t*
May	*6 484 146.95+487 659.12 t*	*5 597 278.36+726 125.37 t*
June	*6 703 964.37+260 404.03 t*	*6 356 201.09+409 445.44 t*
July	*7 508 806.86+220 250.38 t*	*7 091 490.92+428 908.34 t*
August	*7 571 219.23+220 816.29 t*	*7 153 903.29+429 474.26 t*
September	*7 449,982.29+494 603.48 t*	*732 666.96+703 261.45 t*
October	*6 959,285.5+660 418.02 t*	*6 541 969.56+869 075.99 t*
November	*5 532,330.43+854 696.08 t*	*5 115 014.5+1 063 354.04 t*
December	*5 193,171.72+446 777.89 t*	*4 775 855.78+655 435.86 t*

In the next step the models were verified by assessing the properties of residuals. Normal distribution of residuals was verified by means of the Shapiro-Wilk test. According to Figure 5 the value of the Shapiro-Wilk's statistics for model II is 0.9781 whilst the value of test probability is 0.2416. Therefore, there are no grounds for rejecting a hypothesis about normal distribution of residuals on the significance level of $\alpha = 0.05$. For model III, on the other hand, the value of the Shapiro-Wilk's statistics (according to Fig. 6) is 0.9872 whilst the value of test probability is 0.6815. Therefore, there are no grounds for rejecting a hypothesis about normal distribution of residuals on the significance level of $\alpha = 0.05$.

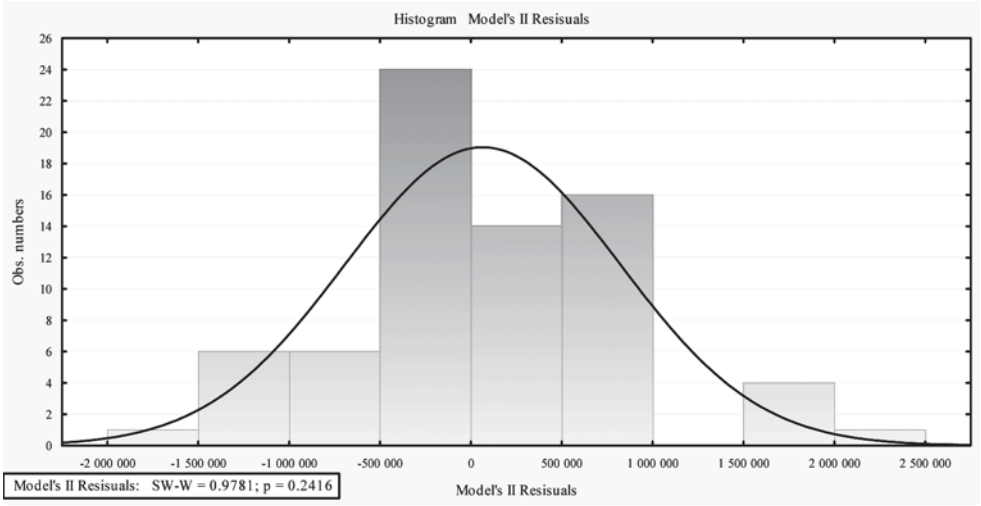

Fig. 5. Graph of the normality of residuals of model II. Source: Own analysis

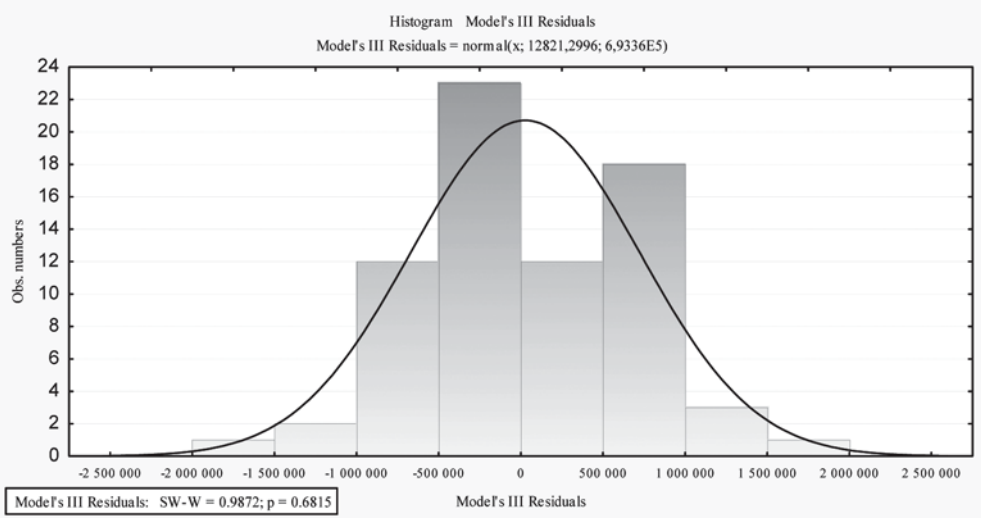

Fig. 6. Graph of the normality of residuals of model III. Source: Own analysis

5. Summary and conclusions

Taking into account calculated values of three constructed in this article models, their movements are presented on Figure 7. The values of prognosis models are compare to real analysed time series data.

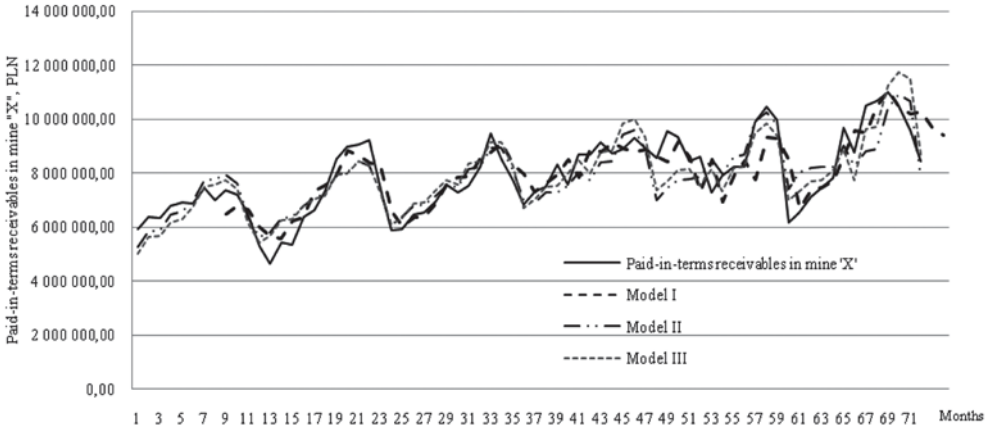

Fig. 7. Developing of the values of models I, II and III, taking into account real values of „X" mine's paid-in-term receivables. Source: Own analysis

On the basis of the results of three obtained forecasting models a weighted average forecast was determined. In the next step the ex post-MSE (mean squared error) was calculated and then its root, that is RMSE (root mean square error), as well as the ex post-MAPE (mean absolute percentage error) for the forecast for the next 72 months since, as it has already been said before, the time series which consisted of 72 values of monthly paid-in-term receivables of the analysed „X" mine was reduced by six last monthly values which were then used for the purpose of verification of the developed models.

Taking into account the calculated values of the ex post RMSE — root mean square error — it can be said that the model with the lowest fit error (apart from the weighted average forecast) is model I (autoregressive) whilst the one with the highest fit error — model II. On the basis of analysis of the values of ex post-MAPE (mean absolute percentage error) we can conclude that the percentage error value of model I is 6.66% when forecasting the values of the „X" mine's paid-in-term receivables. This model has the lowest value of this error, whereas the highest MAPE value, similarly as in the case of the RMSE forecast error, occurs in model II which means that the percentage error value of this model is 9.86% when forecasting the value of paid-in-term receivables. Due to the complex character of the analysed time series which includes all possible elements, i.e. trend, seasonal fluctuations, interventions and random fluctuations, it can be said that model I, despite the fact that its mean absolute percentage error ex post-MAPE is 6.66%, can be applied.

The results of these calculations find their confirmation in Table 5 below, as well as on Figure 8.

TABLE 5

Summary of deviations of forecasts from real values including ex post error values (RMSE, MAPE)

Months of the forecast	Deviations from real values	Deviations from real values	Deviations from real values	Deviations from real values
	Model I	Model II	Model III	Weighted average forecast
67	1 002 298.72	1 658 805.08	824 173.26	1 072 189.96
68	169 306.16	1 759 913.66	925 281.78	722 668.62
69	–75 368.53	583 452.57	–251 179.36	–4 279.27
70	50 288.42	–400 421.64	–1 235 053.52	–452 892.45
71	–579 090.42	–1 047 567.52	–1 882 199.31	–1 091 467.12
72	–1 826 952.10	574 646.86	–259 985.02	–853 837.41
RMSE	**514 195.97**	**1 277 339.09**	**767 083.09**	**248 058.88**
MAPE	**6.66%**	**9.86%**	**8.87%**	**7.13%**

Source: Own analysis.

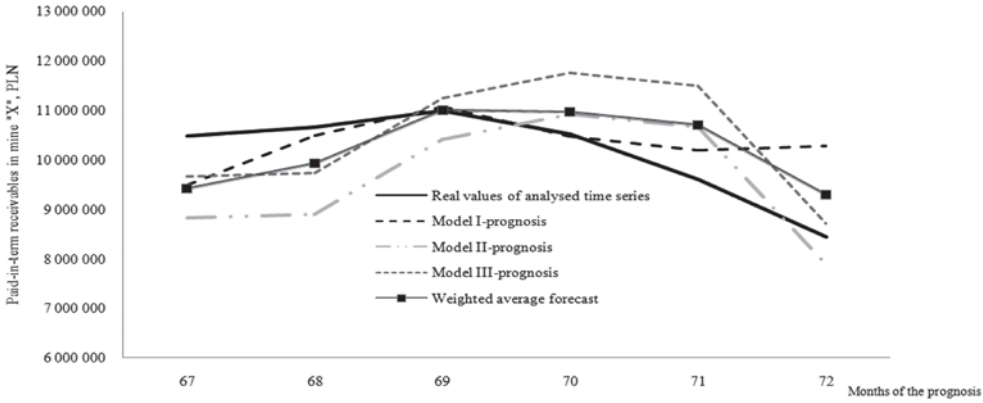

Fig. 8. Movements of forecasts in particular models. Source: Own analysis

REFERENCES

[1] *Czekaj J., Dresler Z.*: Zarządzanie finansami przedsiębiorstw. Podstawy teorii, Wydawnictwo Naukowe PWN, Warsaw 1999.

[2] *Dittman P., Szabela-Pasierbińska E., Dittman I., Szpulak A.*: Prognozowanie w zarządzaniu przedsiębiorstwem, Oficyna Wolters Kluwer Polska Sp. z o.o., Kraków 2009.

[3] *Kłak B.*: Skutki ekonomiczno-podatkowe wykorzystania rabatu w procedurach windykacji należności przez przedsiębiorstwa górnicze. PhD thesis, Akademia Górniczo-Hutnicza im. St. Staszica w Krakowie, Wydział Górnictwa i Geoinżynierii, Kraków 2006.

[4] *Kreczmańska-Gigol K., Pajewska-Kwaśny R.*: Faktoring. Przewodnik dla przedsiębiorcy. Infor Biznes Sp. z o.o., Warsaw 2010.

[5] *Klóska R., Hundert M., Czyżyński R.*: Wybrane zagadnienia z prognozowania, ECONOMICUS, Warsaw 2007.

[6] *Sierpińska M., Jachna T.*: Ocena przedsiębiorstwa według standardów światowych, Wydawnictwo Naukowe PWN, Warsaw 2004.

[7] *Sokołowski A.*: Prognozowanie i analiza szeregów czasowych. Course materials. Statsoft, Kraków 2010

[8] *Sokołowski A.*: Prognozowanie sprzedaży-studium przypadku, www.statsoft.pl, Kraków 2011.