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MODELLING OF TWO PHASE FILTRATION IN FRACTURES OF HYDRAULIC FRACTURING

1. INTRODUCTION

Technology of hydraulic fracturing of layers have received wide spreading in world practice of oil production for an intensification of fluids inflow in wells opening low permeable productive layers [1–10]. The system of extensive drainage network is formed by natural micro cracks disclosing and creation of vertical big extent crack in well bottom zone. It allows including productive seam opened by well to development, increasing discharge, reducing depression on layer and raising of deposit's oil recovery factor.

Discharges of extracting wells and intake capacity of injection well are increased as a result of hydraulic break of layers caused by reducing of hydraulic resistance in well bottom zone and increasing of filtration surface of well. In articles [5–10] are made analyses through optimization and advance of breakdown as one of the effective methods of intensification of liquid influx to borehole cavity. In spite of wide application of this method not all operations of breakdown that have been made in borehole are effective which of explaining by insufficiently founded choice of technology for specific boreholes and ill-posed characterization of its behavior. Therefore the search of growth efficiency of breakdown in oil-wells is a problem of today.

The highest efficiency of breakdown is provided with comprehensive approach for designing based on such factors as conductivity of rock, system of well spacing, cleft mechanics, characteristics of breakdown agent and propping agent, technological and economical limitations. For implementation of this approach besides crack formation models it is necessary to create filtration models with well system which were crossed by cracks of

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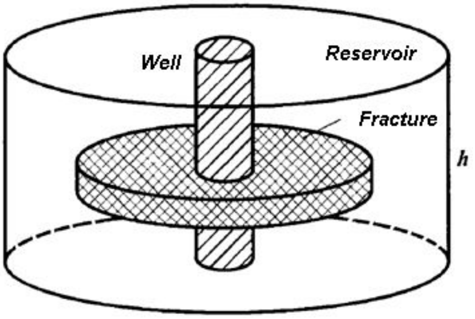
hydraulic fracturing, study features of fluid flow in vicinity of crack including heterogeneous and watered stratum.

2. MATHEMATICAL MODEL

In oil-saturated stratum is made a horizontal crack by the hydraulic fracturing method [7]. The crack has a form like cylindrical disc with a width h_f and diameter D_f (Fig. 1a). Permeability of the crack is much more than permeability of the surrounding oil-saturated stratum. The oil layer of the stratum with power h and diameter D_c props up bottom water. In pursuance of a symmetry problem will be solved in a cylindrical coordinate (Fig. 1b).

a)



b)

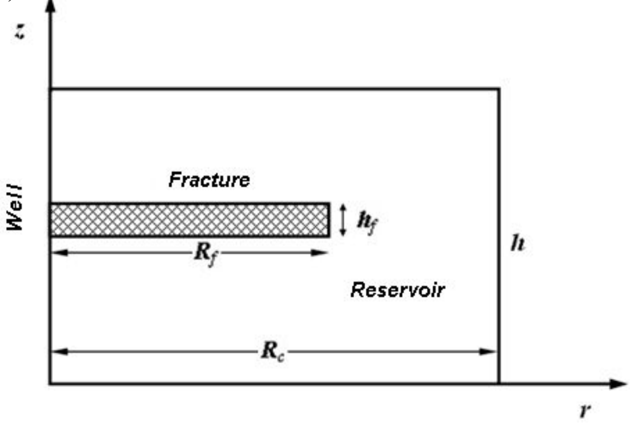


Fig. 1. Scheme of oil reservoir with discal fracturing crack

Navier-Stocks equations for small Reynolds numbers are used by Golf-Raht T.D. [11] for derivation of equation of flow in fractures. He had modeled fluid flow in single crack like flow between plates. He defined dependence of averaged on plate thickness flow velocity on potential difference multiplied on squared distance. Darcy law could be received by definition factor in front of potential gradient which is proportional to squared distance of crack opening displacement as fracture's permeability following to Pirverdyan [12].

By assuming that the fluid motion is subjected to Darcy law as in low permeable stratum and in cracks on bases of interpenetrative continuum mechanics [13, 14] we get mathematical model two-phase filtration. System of motion equation of immiscible fluids in cylindrical coordinates (r, z) is represented in the form [15, 16]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(s)\frac{\partial p}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda(s)\frac{\partial p}{\partial z}\right)=0 \tag{1}$$

$$u_T=-\lambda(s)\frac{\partial p}{\partial r},\quad v_T=-\lambda(s)\frac{\partial p}{\partial z} \tag{2}$$

$$\phi\frac{\partial s}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(rf_w u_T\right)+\frac{\partial}{\partial z}\left(f_w v_T\right)=0 \tag{3}$$

where:

p – pressure,
$\lambda(s)$ – total mobility,
r – radial coordinate,
z – vertical coordinate,
s – water saturation,
f_w – water fraction in flux,
u_T, v_T – components of total velocity.

Functions in equations (1)–(3) are defined as:

$$\lambda(s)=\lambda_w(s)+\lambda_o(s)=K\left(\frac{k_w}{\mu_w}+\frac{k_o}{\mu_o}\right),\quad f_w=\frac{\lambda_w(s)}{\lambda_w(s)+\lambda_o(s)} \tag{4}$$

where:

λ_w, λ_o – water and oil mobility,
K – permeability tensor,
k_w, k_o – relative permeability,
μ_w, μ_o – water and oil viscosity.

Boundary conditions for pressure [15, 16]:

at well: 1) constancy of fluid rate, 2) pressure constancy;
on upper boundary and bottom – impermeability;
on external boundary of reservoir – pressure constancy;

Boundary conditions for water saturation:

at well – Neumann boundary condition;
on upper boundary and bottom – water saturation constancy.

The layer and fracture interface is satisfied to conjugation condition, i.e. normal component of velocity and pressure are continuous.

3. NUMERICAL CALCULATION SCHEME

Systems of equations (1)–(3) are solved in a rectangular area (Fig. 1b). The region under consideration is broken to calculation mesh with sides Δz_i, Δr_j, entire calculation knots are in the center of calculation mesh (pressure, saturation), and velocity components are in the edge of the calculation mesh. For realizing of cracks with height 50 mm an uneven computational grid is used.

The algorithm of solution of equations (1)–(4): from equation (1) the pressure is calculated by iteration method at each time step then from the ratio (2) velocity components are defined, after this from the equitation (3) water saturation is calculated. The saturation equation is approximated by explicit scheme, where convective parts with a second kind upstream difference [15–19]. For ensuring the scheme stability time step is chosen from stability condition of Courant.

4. DISCUSSION OF CALCULATION DATA

There are used physical-geological conditions in these calculations: depression (and/or flow rate) in the well – 1 MPa (40 m^3/day), well penetration ratio – 50%, viscosity of oil and water – 7 mPa*s and 0,49 mPa*s respectively, aquifer thikness – 5 m, initial reservoir pressure – 10,4 MPa, saturation pressure – 8,3 MPa. Radius of external boundary of reservoir – 75 m, reservoir thickness – 25 m, radius of penny-shaped crack – 70 m, height – 50 mm.

In the Figure 2 there were showed the field distribution of water saturation in the different moment of time at constant selection of wells (flow rate q = 20 m^3/day). In all pictures there are drawn cracks of hydraulic fracturing with white lines.

It is clear from the Figure 2 that water first get in the initial part of the crack then the rest of the crack is filled up with water. It is account for by significant pressure falls occur in the initial part of the crack where were set up the condition of water breeding. The breakthrough of bottom water happens in 863 days after bringing well into production.

To value a work efficiency of the well with a discal crack, results of calculation are compared with data without cracks.

In the Figure 3 there were shown a change of levels of well watering with cracks (dash line) and without it (solid line). Dashed lines corresponds with different magnitude of ratio of crack permeability to the permeability of lowpassing stratum in the range from (k_{ch} = 50) to (k_{ch} = 150). Other parameters in calculations were same. Wells without cracks water earlier than with cracks (Fig. 3). The crack makes heightened depression and excites oil inflow from oil-saturated stratum with low filtrational-capacity options.

Numerical data of water breakthrough time to well and well watering rate in 3600 days are shown in table.

For example with ratio of permeability (k_{ch} = 50) time of water breakthrough equals to 315 days, then for wells without cracks (k_{ch} = 1) – 150 days (Tab. 1) i.e. more than 2 times. Therefore at constant selection (flow rate) in the well with crack break time of bottom water can be increased and be raised productivity of the well.

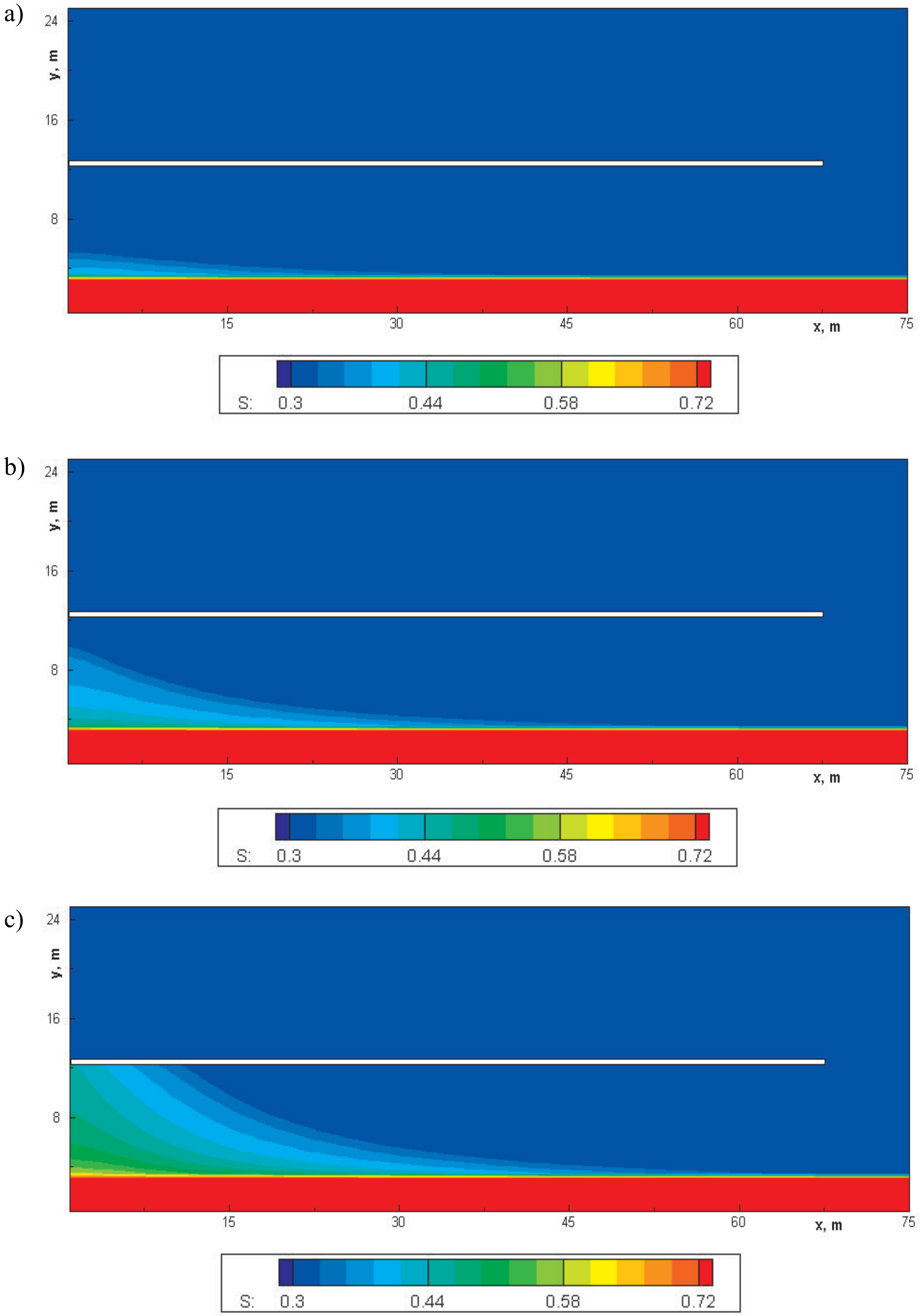


Fig. 2. Water saturation distribution: a) 210 days; b) 647 days; c) 3237 days

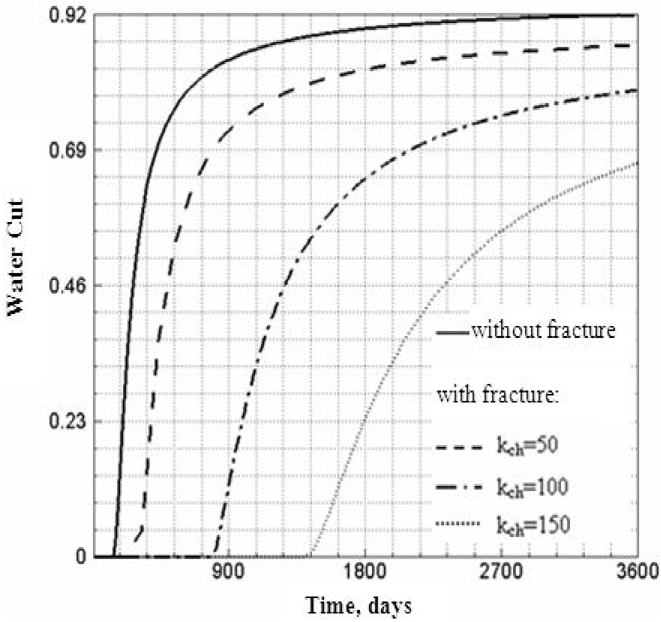


Fig. 3. Evolution of well watering rate with (dashed lines) and without (solid line) crack

Table 1

Numerical data of water breakthrough time and well watering rate

No	Ratio of crack's and seam's permeability	Breakthrough time, day	Well watering rate in 3600 days, %
1	$k_{ch} = 1$	150	92%
2	$k_{ch} = 50$	315	86%
3	$k_{ch} = 100$	809	79%
4	$k_{ch} = 150$	1456	66%

For the well without cracks level of watering reaches 92% in 3600 days. Then as well as for the well with cracks level of watering is 66–86%, thereby it is conducive to huge oil accumulation. This implies significant practical result that the amount of oil accumulation raises for the well with cracks.

In the Figure 4 were presented calculation results when the well exploits with constant hole pressure. For comparing the well production with cracks and without was brought for assessment of efficiency of hydraulic fracturing method. Calculations were made with ratio of crack permeability to permeability low permeable stratum k_{ch} = 150, other secure parameters were same as previous one.

Calculation data in Figure 4 showed that there is a relaxation between flow rate and initial pressure drop in bottom hole formation zone [20, 21]. During relaxation time initial flow rate of the well with cracks descent from 210 m^3/days to 50 m^3/days and get stabilization at this level which is made by crack depression and amount of filtration flow. The crack with high permeability makes depression and oil inflow from stratum with low filtrational-capacitive options which provides flow rate 50 m^3/days prior to time of water break to well (800 days).

In the well without cracks initial flow rate of well descends from 70 m^3/days till 12,5 m^3/days in relaxation time and accommodates at this level until time of water break (100 days).

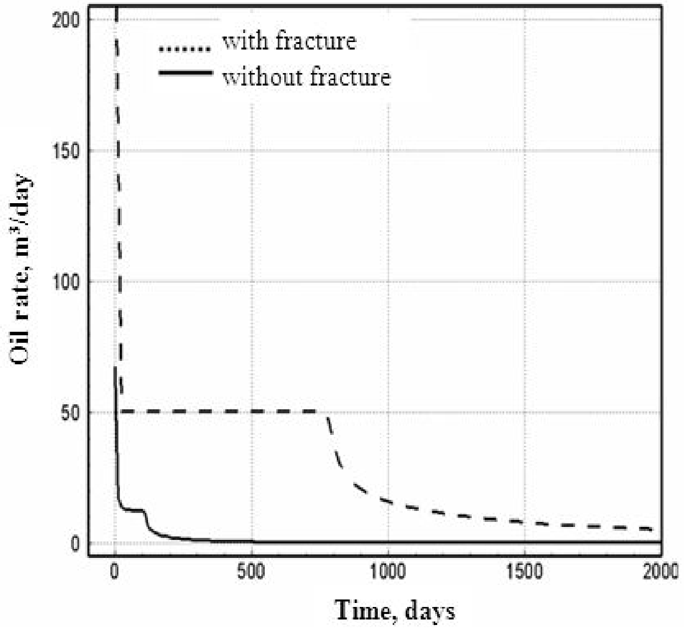


Fig. 4. Evolution of oil rate without (solid line) and with (dash line) crack

The break of bottom water to the well brings to descent of oil flow rate. But productivity of the well with cracks all the time stays higher than level of the flow rate of general well (Fig. 4).

Dynamics of development of accumulated oil of well defines efficiency of influence method to stratum (Fig. 5).

Dependence of the curve of accumulated oil volume in case of well with cracks rises nearly with rectilinear way till water break in the well, then steadily goes to stabilization (Fig. 5). Then as in the well without cracks volume of the accumulated oil will be nearly 20 times lower.

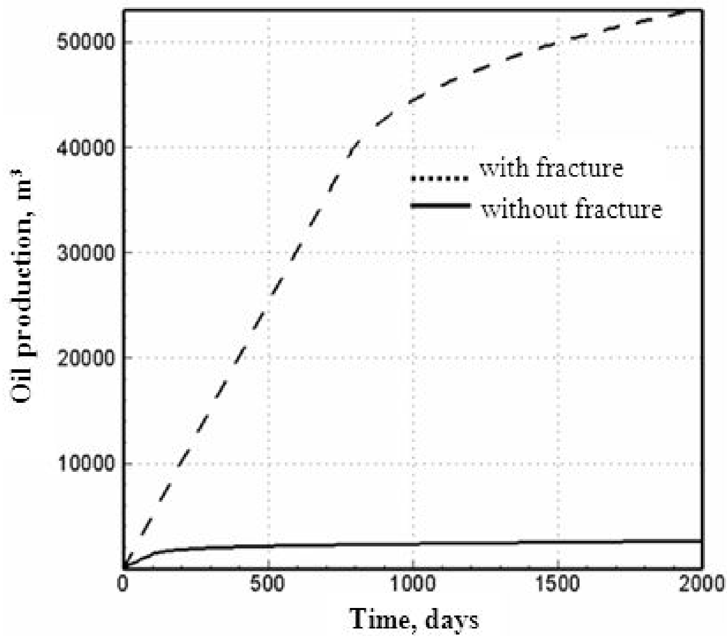


Fig. 5. Cumulative oil production's evolution

Therefore the method of hydraulic fracturing which makes cracks in oil-saturated stratum raises flow rate of the well. Amount of accumulated oil and coefficient of oil extraction are increased. Calculation data are satisfies agreement with characteristics of hydraulic fracturing which is got in industrial tests of petroleum deposit [6, 7, 10].

5. CONCLUSIONS

Following conclusion could be made by received results:

1. Generalize mathematical model allows to make efficient method of two phase filtration in fracturing seam. It is shown that fracture in seam increases well rate and cumulative oil production at the same seam's geophysical condition. Reducing of well watering rate is achieved by fracture using.
2. Comparative calculations are filled at following conditions on well: a) well rate constancy; b) pressure constancy. Relaxation time between initial pressure difference and well rate is taken place at calculations with constant well pressure. Flow filtration caused by pressure drawdown of fracture is steadied at relaxation time. Reducing of water cut and increasing of well productivity are shown in both cases.
3. Method of hydraulic fracturing is increase well rate. Numerical data are in good agreement with industrial testing [6, 7, 10].

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