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CONTROL SYSTEM OF THE HYDRAULIC CYLINDERS MOTION SYNCHRONIZATION WITH THE CONTROLLERS DESIGNED ON THE BASIS OF THE DIRECT LAPUNOV'S METHOD

SUMMARY

The following article deals with the subject of control of the hydraulic cylinders motion synchronization. The subject of the paper is the analysis of throttling-controlled motion synchronization system in the range of potential control algorithms. The main aim was to elaboration and research of the control system of n (where n ≥ 2) hydraulic valves with a view to ensuring the highest possible efficiency of cylinders motion synchronization. The elaborated control system enables effective motion synchronization of n hydraulic cylinders with a view to ensuring a system stability. In the suggested control system controllers designed on the basis of the direct Lapunov's method were used. The number of control systems is equal to the number of the synchronized cylinders.

***Keywords**: control system, nonlinear control, motion synchronization, hydraulic drive*

UKŁAD STEROWANIA SYNCHRONIZACJĄ RUCHU SIŁOWNIKÓW HYDRAULICZNYCH Z REGULATORAMI ZAPROJEKTOWANYMI Z WYKORZYSTANIEM BEZPOŚREDNIEJ METODY LAPUNOWA

W artykule podjęto problematykę sterowania synchronizacją ruchu siłowników hydraulicznych. Przedmiotem artykułu jest analiza sterowanego dławieniowo układu synchronizacji ruchu w zakresie możliwości zastosowania potencjalnych algorytmów sterowania. Głównym celem było opracowanie i zbadanie układu sterowania n (dla n ≥ 2) hydraulicznymi zaworami, zapewniającego możliwie wysoką efektywność synchronizacji ruchu siłowników. Przedstawiony układ sterowania umożliwia skuteczną synchronizację ruchu n siłowników hydraulicznych wraz z zapewnieniem stabilności układu. W proponowanym układzie sterowania zastosowano nieliniowe regulatory zaprojektowane z wykorzystaniem bezpośredniej metody Lapunowa. Liczba układów regulacji równa jest liczbie synchronizowanych siłowników.

***Słowa kluczowe**: układ sterowania, sterowanie nieliniowe, synchronizacja ruchu, napęd hydrauliczny*

1. INTRODUCTION

The systems composed of more than two hydraulic cylinders are used to moving of constructions, elements of assembly lines or as components of moving machinery. The frequent problem is unequal motion velocity of the cylinders in such systems. In spite of the fact that scientific research into hydraulic control systems has been conducted in many scientific institutes for years, the subjects still raises a lot of interest and constitutes the mainstream of advanced control techniques implementation to the control of hydrostatic systems. A lot of theoretical and practical aspects still require solutions. The lack of a satisfactory concept for control of a system consisting of more than two cylinders was the main reason for undertaking the following subject.

In the throttling control the rule of the working liquid throttling is used and it is expressed in such a formula:

$$Q = C_d \sqrt{\frac{2}{\rho}} A_d \sqrt{\Delta p} \tag{1}$$

The algebraic equation (1) which describes the intensity of the working liquid flow Q in the dependence from a decrease of the pressure Δp in the controlling valve and from a surface of the throttling gap A_d is strongly nonlinear (Tomasiak 2001). The throttling of the working liquid is a basic structural nonlinearity in the hydraulic and electro-hydraulic control systems (Pizoń 1995). It is one of the circumstances which indicates that the throttling-steered cylinder is a nonlinear object. Such a cylinder characterizes features of the nonlinear object, mainly lack of a superposition principle so the response y on the input u which is a linear combination of the input: $u_1, u_2, \ldots, u_m$ $u = \sum_{i=1}^{m} a_i u_i$ is not equal to the linear combination of the responses: $y_1, y_2, \ldots, y_m$ $\left(y = \sum_{i=1}^{m} a_i y_i \right)$ making an assumption, that y_i is the response of the system on the input u_i (Kaczorek *et al.* 2005). Apart from this feature, nonlinear systems are characterized by (Jędrzykiewicz 2002):

1) the amplitude of control signal or/and the amplitude disturbances influence on the response of the nonlinear system,
2) the response of the nonlinear system in the steady state, apart from the frequency coming from the input or/and disturbances, can additionally contain other harmonics,
3) the stability of the nonlinear system depends on the initial conditions values,

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On the basis of the presented features it is possible to conclude that the correct work of the control systems with the controllers which are linear state function (for example PID controllers) is limited outside the chosen working point, for which controllers parameters were selected. The working point is a point where the linearization of the equations of the object condition was made. Application of the linear algorithms PID for the objects which have a strong structural nonlinearity enables to achieve stable work of the system only in the narrow surrounding of the chosen working point (Bania 2000). That is why the analysis of the object stability described by the linear state equations is correct only in the equilibrium point which is the working point and its closest surrounding – the local stability. In the result of the analysis conducted in this way, information about the system behaviour beyond the closest working point surrounding cannot be obtained. That is why the linear control should be designed in such a way as the system retained in its "linear range" which comprises serious limitation to the linear control application.

The analysis of the global stability of the nonlinear object is possible due to the use of the direct Lapunov's method. This method is a generalization notion of the energy connected with the mechanical system. Movement of such a system is stable if its total mechanical energy decreases constantly. Therefore, for the nonlinear system the energy scalar should be found and it is called the Lapunov's function V. Taking into notion the Lapunov's function, Lapunov theorem can be described: "the nonlinear system described by the equation $dx/dt = f(x)$ is stable asymptotically in the field Ω, containing the beginning of the co-ordinate system (the equilibrium point), if such positively described in a field Ω Lapunov's $V(x)$ function, whose time derivative along the state trajectory is the negatively described function in this field Ω, is possible to select," (Kaczorek *et al*. 2005). For the selected nonlinear system it is possible to build a number of different Lapunov's functions which enable to conclude that if one function does not enable to describe whether given nonlinear system is stable it does not indicate that other function is impossible to describe such stability (Gibson 1963).

2. DIRECT LAPUNOV'S METHOD IN THE CONTROLLERS DESIGN

Apart from the examination of the global stability the direct Lapunowa's method can be used in the design of the nonlinear controllers (Slotine and Li 1991). In such method two ways can be enumerated: the backstepping method and Lapunov's redesign. The use of backstepping method for the design of the force nonlinear control in the electro-hydraulic systems was presented in (Alleyne 1996; Sohl and Bobrow 1999; Ursu *et al.* 2005; Kaddissi *et al.* 2006), and the elaboration of the robust control with the state variables feedback in the electro-hydraulic systems in (Yu *et al.* 2003). The use of the Lapunov's redesign for the robust control elaboration in the hydraulic systems was presented in (Kim 2004). Particularly interesting seems to be the use of the backstepping method. On the basis of the solution analysis in (Alleyne 1996); Yu *et al.* 2003; Ursu *et al.* 2005; Kaddissi *et al.* 2006) three stages of the design of the controller using backstepping can be named:

1. The elaboration of the nonlinear mathematical model in the states space:

$$\frac{dx(t)}{dt} = f(x(t), u(t), t) \tag{2}$$

2. The definition of the errors understood as differences between the planned values of the state variables and the values in the real object:

$$e_i(t) = x_{zi}(t) - x_i(t) \tag{3}$$

3. Finding positively defined the Lapunov's function being the error function:

$$V(t) = f(e_i(t)) \tag{4}$$

4. The determination of the derivative of Lapunov's function.
5. The choice of the control law in such a way that the derivative Lapunov's function will be negatively defined.

3. DESIGN OF THE NONLINEAR CONTROLLER FOR CYLINDER

The assembly consisting of the cylinder and hydraulic control valve can be described by the nonlinear equations system:

$$\begin{cases} \dfrac{dy_i(t)}{dt} = v_i(t) \\ \dfrac{dv_i(t)}{dt} = \dfrac{1}{m_{rs}}\left[A_t p_{Ai}(t) - F_t v_i(t) - F_{0i}(t)\right] \\ \dfrac{dp_{Ai}(t)}{dt} = \dfrac{E_C}{y_i(t)A_t + V_0}\left[C_d\sqrt{\dfrac{2}{\rho}}tg\dfrac{\alpha}{2}y_{zi}^2(t)\sqrt{p_1(t) - p_{Ai}(t)} - v_i(t)A_t\right] \\ \dfrac{dp_1(t)}{dt} = \dfrac{E_C}{V_{pw}}\left[Q_P - C_d\sqrt{\dfrac{2}{\rho}}tg\dfrac{\alpha}{2}y_{zi}^2(t)\sqrt{p_1(t) - p_{Ai}(t)}\right] \end{cases} \tag{5}$$

Where:

- y_i(t) – displacement of the cylinder piston rod,
- $v_i(t)$ – speed of the cylinder piston rod,
- m_{rs} – mass of the movable elements of cylinder,
- A_t – surface of the piston,
- $p_{Ai}(t)$ – pressure in A chamber of the cylinder,
- F_t – friction coefficient,
- $F_{0i}(t)$ – external load of the cylinder,
- E_c – elasticity modulus of the working liquid,
- V_0 – initial volume in A chamber of the cylinder,
- C_d – flow resistance coefficient,
- ρ – density of the working liquid,
- α – the angle which depends on the construction of throttle valve,

$y_{zi}(t)$ – displacement of the slide in the throttling valve,
$p_1(t)$ – pressure in the hydraulic conduit between valve and the supplying pump,
V_{pw} – volume of the hydraulic conduit between valve and the supplying pump,
Q_p – capacity of the supplying pump.

The space states model of the mentioned above system can be described by the equations:

$$\begin{cases} \dfrac{dx_{1i}(t)}{dt} = x_{2i}(t) \\ \dfrac{dx_{2i}(t)}{dt} = \dfrac{1}{m_{rs}}\left[A_t x_{3i} - F_t x_{2i}(t) - u_{2i}(t)\right] \\ \dfrac{dx_{3i}(t)}{dt} = \dfrac{E_C}{x_{1i}(t)A_t}\left[C_d\sqrt{\dfrac{2}{\rho}} tg\dfrac{\alpha}{2} u_{1i}^2(t)\sqrt{x_4(t)-x_{3i}(t)} - x_{2i}(t)A_t)\right] \\ \dfrac{dx_4(t)}{dt} = \dfrac{E_C}{V_{pw}}\left[u_3 - C_d\sqrt{\dfrac{2}{\rho}} tg\dfrac{\alpha}{2} u_{1i}^2(t)t\sqrt{x_4(t)-x_{3i}(t)}\right] \end{cases} \tag{6}$$

Where:

$x_{1i}(t) = y_i(t)$, $u_{1i}(t) = y_{zi}(t)$,

$x_{2i}(t) = v_i(t)$, $u_{2i}(t) = F_{0i}(t)$,

$x_{3i}(t) = p_{Ai}(t)$, $u_3(t) = Q_p$.

$x_4(t) = p_1(t)$,

To make this model easier such additional denotations were adapted:

$$f_3(t) = \frac{E_C}{x_{1i}(t)A_t} C_d K_Q \sqrt{x_4(t) - x_{3i}(t)}$$
$$f_2(t) = \frac{E_C}{x_{1i}(t)} \tag{7}$$
$$K_Q = \sqrt{\frac{2}{\rho}}\text{tg}\frac{\alpha}{2}$$

After replacement (7) to (6) such a result was obtained in the state space:

$$\begin{cases} \dfrac{dx_{1i}(t)}{dt} = x_{2i}(t) \\ \dfrac{dx_{2i}(t)}{dt} = \dfrac{1}{m_{RS}}\left[A_t x_{3i} - F_t x_{2i}(t) - u_{2i}(t)\right] \\ \dfrac{dx_{3i}(t)}{dt} = f_3(t)u_{1i}^2(t) - f_2(t)x_{2i}(t) \\ \dfrac{dx_4(t)}{dt} = \dfrac{E_C}{V_{pw}}\left[u_3 - C_d K_Q u_{1i}^2(t)\sqrt{x_4(t)-x_{3i}(t)}\right] \end{cases} \tag{8}$$

For a system described in this way, it was accepted that the input value of the cylinder displacement $y_z(t)$ is a desired value of the state variable $x_{1i}(t)$, that is why control error can be described:

$$e_{1i}(t) = y_z(t) - y_i(t) = x_{1z}(t) - x_{1i}(t) \tag{9}$$

Control error square was applied as the Lapunov's function (Alleyne 1996; Yu *et al.* 2003; Ursu *et al.* 2005; Kaddissi *et al.* 2006):

$$V_{1i}(t) = \rho_1 e_{1i}^2(t) \tag{10}$$

On the assumption that $\rho_{1i} > 0$ the Lapunov's function is positively described for every $x_{1i}(t)$.

After differentiation of the Lapunov's function it was obtained:

$$\frac{dV_{1i}(t)}{dt} = 2\rho_1 e_{1i}(t)\frac{de_{1i}(t)}{dt} \tag{11}$$

Where the derivative of control error is described by the relationship:

$$\frac{de_{1i}(t)}{dt} = \frac{y_z(t)}{dt} - \frac{x_{1i}(t)}{dt} = \frac{y_z(t)}{dt} - x_{2i}(t) \tag{12}$$

Due to the use of the third equation from the model (8) relationship describing $x_{2i}(t)$ was established:

$$x_{2i}(t) = \frac{1}{f_2(t)}\left[f_3(t)u_{1i}^2(t) - \frac{dx_{3i}(t)}{dt}\right] \tag{13}$$

After the replacement (12) and (13) to (11) the derivative Lapunov's function formula was found:

$$\frac{dV_{1i}(t)}{dt} = 2\rho_1 e_{1i}(t)\left[\frac{y_z(t)}{dt} + \frac{1}{f_2(t)}\frac{dx_{3i}(t)}{dt} - \frac{f_3(t)}{f_2(t)}u_{1i}^2(t)\right] \tag{14}$$

According to the Lapunov's theory the derivative of the function V must be negatively described to make the control system look stable. Therefore, such a pattern is desirable:

$$\frac{dV_{1i}(t)}{dt} = -2\rho_1 k_1 e_{1i}^2(t) \tag{15}$$

On the assumption that $k > 0$.

To obtain control law $u_{1i}(t)$ should be described as:

$$u_{1i}^2(t) = \frac{f_2(t)}{f_3(t)}\left[\frac{1}{f_2(t)}\frac{dx_{3i}(t)}{dt} + \frac{y_z(t)}{dt} + k_1 e_{1i}(t)\right] \tag{16}$$

After the replacement (7) to (16):

$$u_{1i}^2(t) = \frac{A_t}{K_Q\sqrt{x_4(t)-x_{3i}(t)}} \cdot \left[\frac{x_{1i}(t)}{E_C}\frac{dx_{3i}(t)}{dt} + \frac{y_z(t)}{dt} + k_1 e_{1i}(t)\right] \tag{17}$$

Finally, after taking into consideration (9), the nonlinear control takes the form:

$$u_{1i}(t) = \sqrt{\frac{A_t}{K_Q\sqrt{x_4(t)-x_{3i}(t)}}\left[\frac{x_{1i}(t)}{E_C}\frac{dx_{3i}(t)}{dt} + \frac{x_{1z}(t)}{dt} + k_1\left(x_{1z}(t)-x_{1i}(t)\right)\right]} \tag{18}$$

To verify the correctness of the assigned function $u_{1i}(t)$ the expression (18) was inserted to (14) and it was agreed that the derivative of the Lapunov's function is negatively described while $k_1 > 0$.

The value of the k_1 factor is not constant for the whole range of the synchronization system's work. Therefore, it should be selected in an adaptable way. The adaptive methods used in control were described in (Kowal 1996). Among the systems presented, the most interesting one seems to be the adaptive system with the indirect control. The correction of the controller's setting takes place on the basis of the selected parameters values of the controller's object identified on current basis. The adaptive control implemented in hydraulic systems on the basis of this method was described in (Cendrowicz 2001), where the adaptive control system was presented for the excavator's manipulator with the hydraulic drive. The adaptation of the controller's settings took place through the programme change of the values of this settings depending on the selected values measured during the work of this system.

On the basis of the results, it was stated that the best quality of the control is obtained if the prevalent variable is obtained on the basis which the value of the factor k_1 will be the biggest difference between the pressures in chambers of the particular cylinders. Hence the value k_1 will be marked:

$$k_1(t) = a_1 \max\left(\left|p_{Ai}(t) - p_{A1,\ldots An}(t)\right|\right) \tag{19}$$

In order to perform the required condition according to which the value k_1 must always have the positive value to enumerate k_1 the biggest absolute difference between the pressures will be taken. The value of the a_1 factor will be determined in the simulation examinations.

The block diagram which presents control algorithm with the nonlinear controllers is presented in Figure 1.

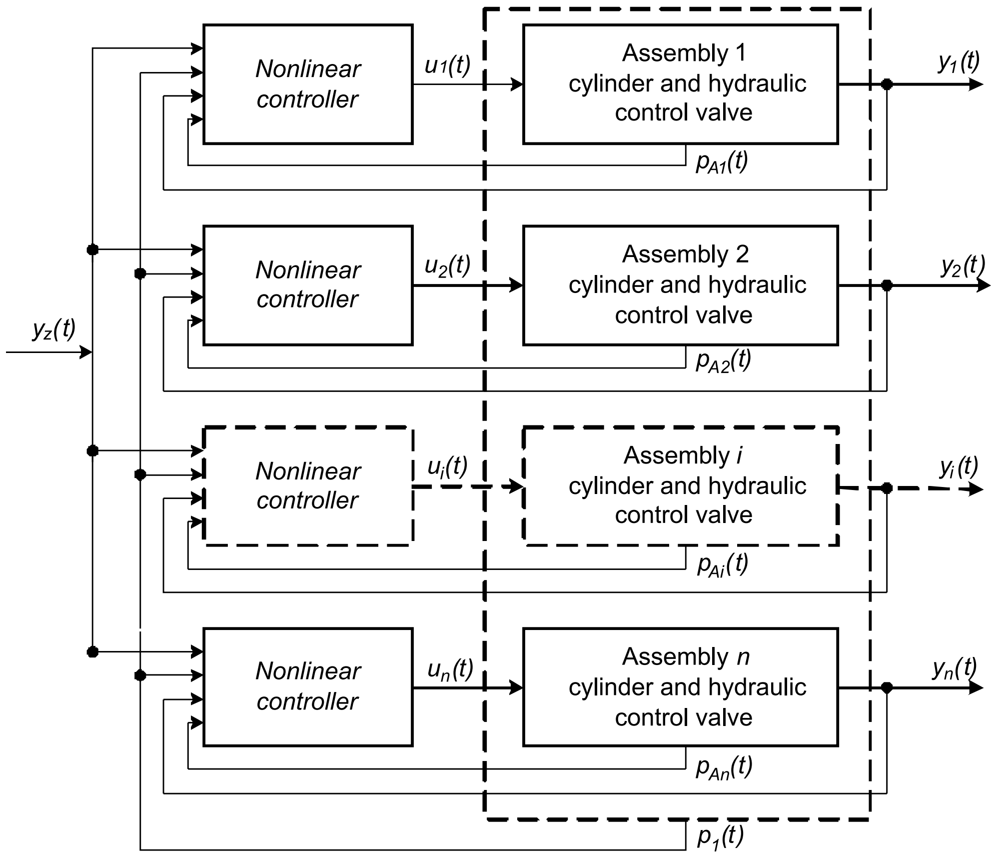

Fig. 1. The control system with the nonlinear controllers

4. SIMULATION RESEARCH

The characteristics of the piston rods' cylinder motion $y_i(t)$ and the slides in the hydraulic control valve $y_{zi}(t)$ were determined with the step value change of the external loads $F_{0i}(t)$ were treated as disturbances. The simulated step change of the values were assigned according to Figure 2.

The characteristics of the quantities mentioned above were marked for the disturbances value in the control system no 1 according to Figure 3.

During the preliminary simulation researches the most appropriate value of the factor a_1 was assigned (a_1 = 5000). The characteristics of the simulated synchronization errors were presented in Figures 4a, 5a, 6a and the slide's movement in the hydraulic control valves in Figures 4b, 5b, 6b. In comparison to algorithm with PID controllers, the increase of the maximal synchronization error value $\Delta y_{i\max}$ was observed by 5% with the step change of the disturbance value in the control system of the one cylinder. At the same time, overshoot $\kappa_i(t)$ was reduced by 50%. The value of $\Delta y_{i\max}$ was significantly reduced with the disturbances value linearly growing in comparison to the value obtained with the use of control algorithm with PID controllers about 44%. The working liquid pressure $p_1(t)$ (Fig. 7) is slightly lower. The static deviation is not eliminated (which is visible in Fig. 5a), but its value is contained in the set tolerance and its total is 0.01 [mm].

The main aim of the use of the controllers designed on the basis of the direct Lapunov's method was to provide stable work of the synchronization system in the whole range of the variation of the disturbances values which was achieved. In order to present the achieved effect the simulated responses were generated according to Figure 8.

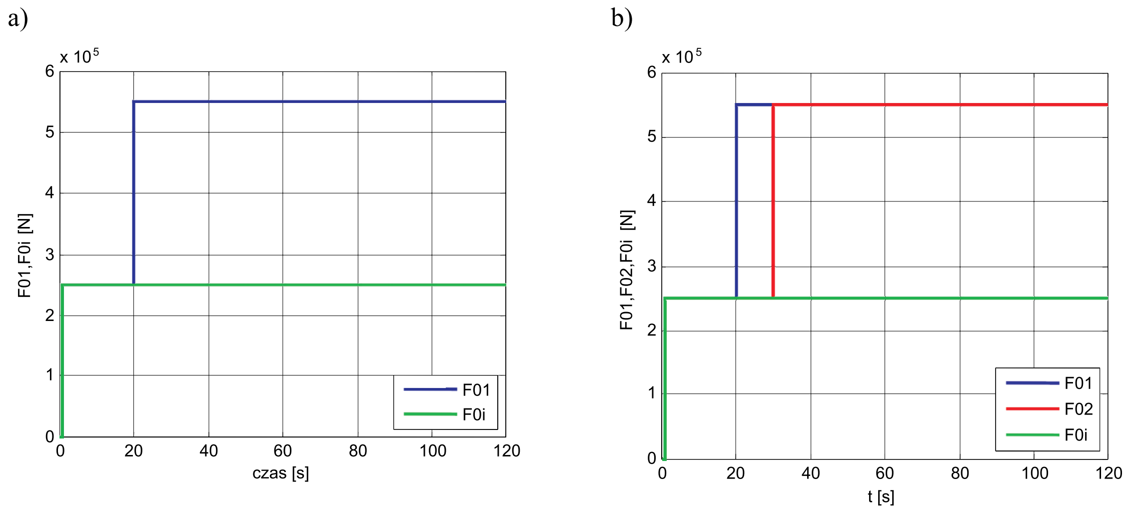

Fig. 2. Step change of the disturbances values: a) in control system of the assembly no 1; b) in control systems of the assembly no 1 and 2

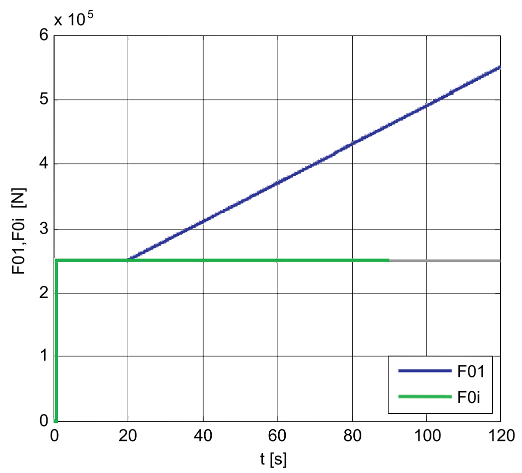

Fig. 3. Simulated linear growing of the disturbances values

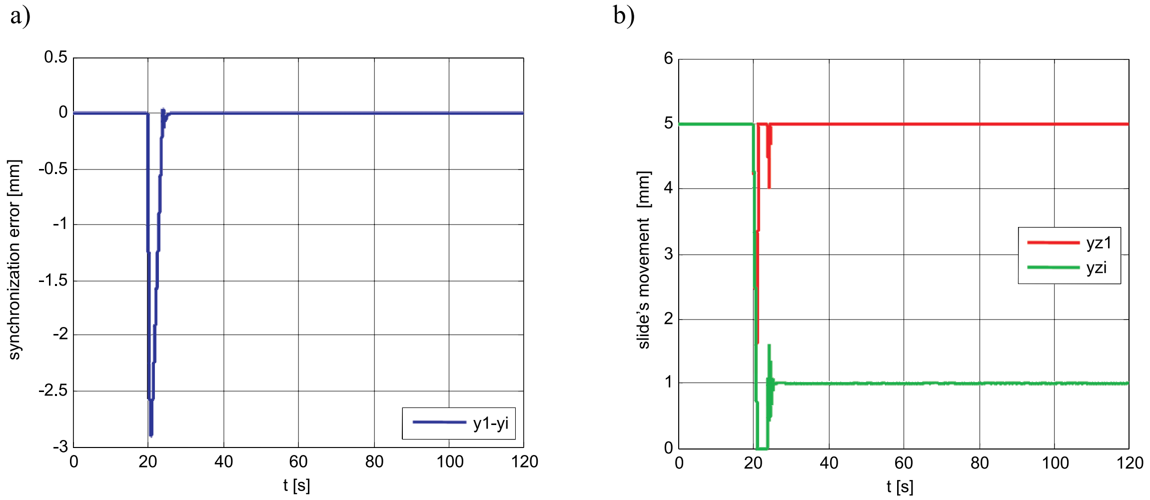

Fig. 4. Response of the system to the step change of the disturbances value in the no 1 assembly: a) synchronization error; b) slide's movement in the hydraulic control valves

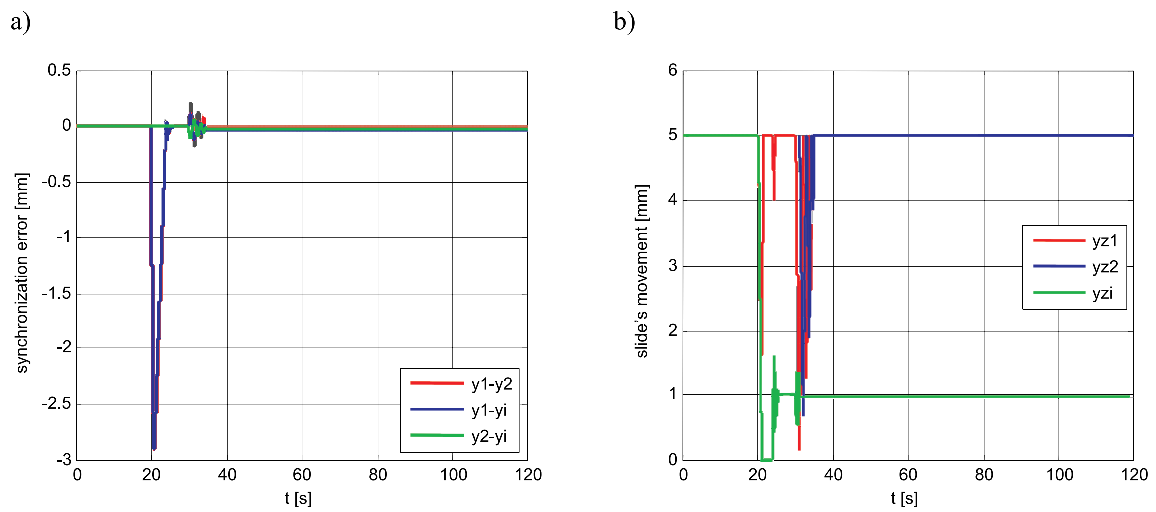

Fig. 5. Response of the system to the step change of the disturbances value in the no 1 and 2 assemblies: a) synchronization errors; b) slide's movement in the hydraulic control valves

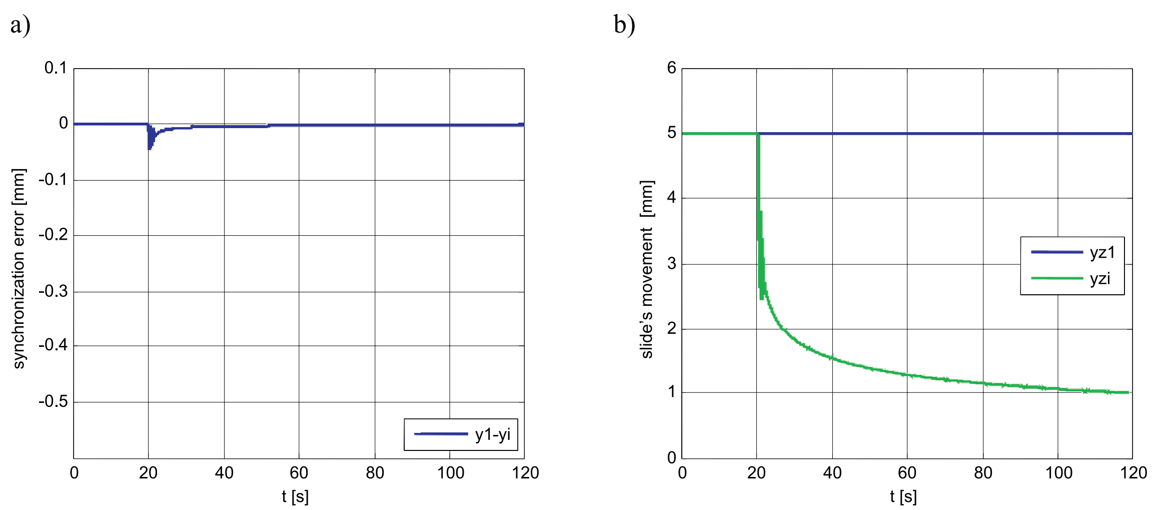

Fig. 6. Response of the system to the linear growing change of the disturbance value in the no 1 assembly: a) synchronization error; b) slide's movement in the hydraulic control valves

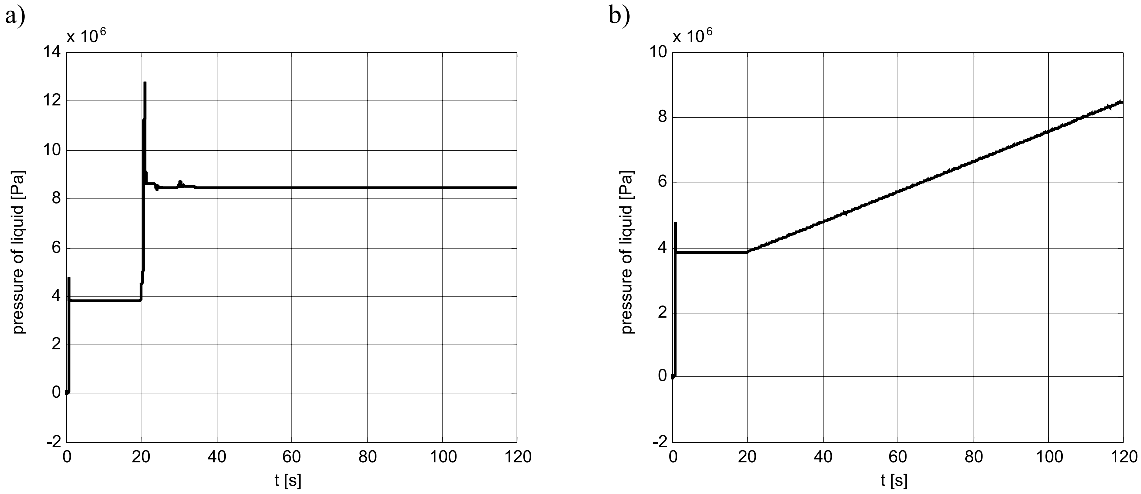

Fig. 7. Working liquid pressure: a) for the step change of the disturbances values in the no 1 and 2 assemblies; b) for the step change of the disturbances values in the no 1 assembly

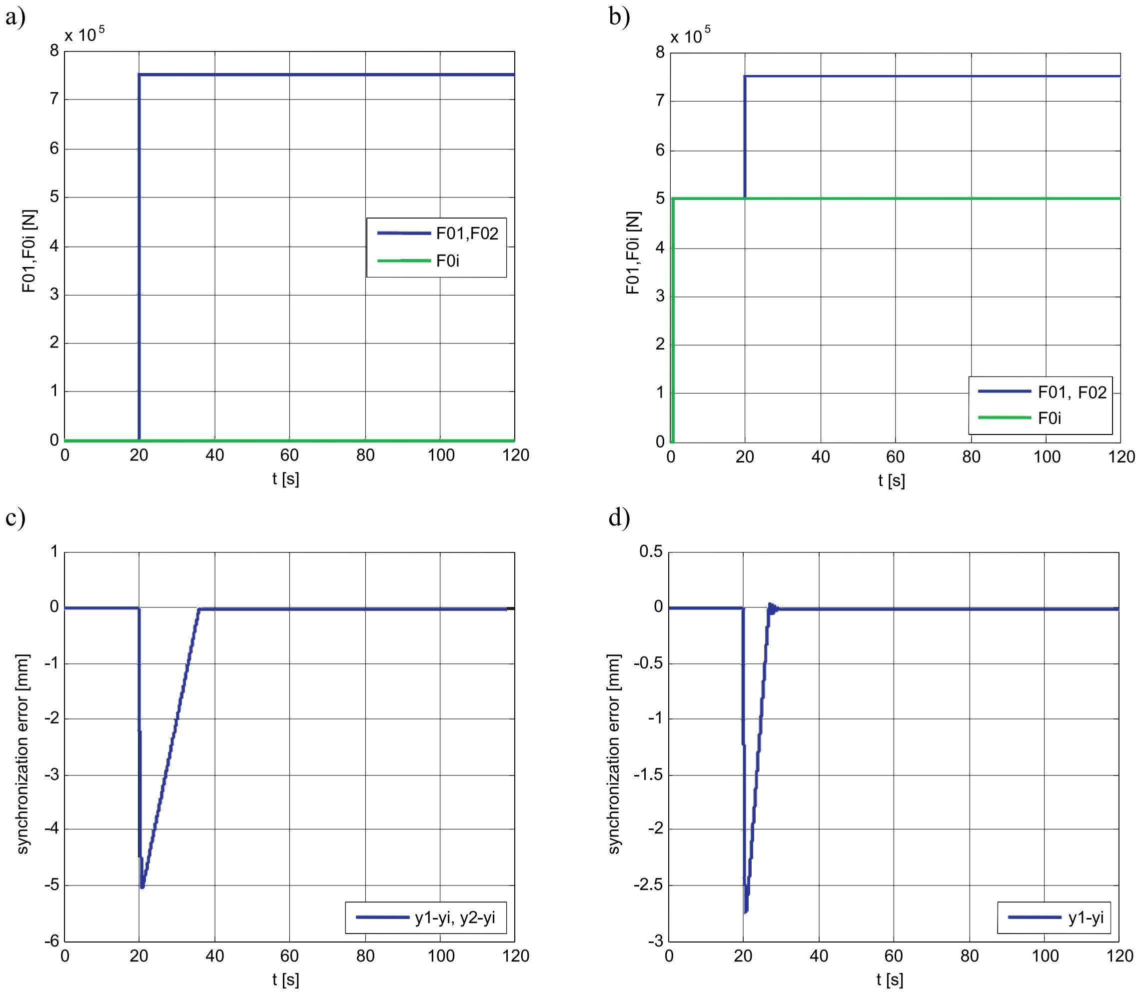

Fig. 8. The response of the control system for different value of the disturbances: a) step change (750 kN) of the disturbances values in the no 1 and 2 assemblies; b) step change (250 kN) of the disturbances values in the no 1 and 2 assemblies; c) synchronization error; d) synchronization error

The constant value k_1 = 300 was accepted. It is a value selected in order to achieve a desirable control quality with step changes of the disturbances values about 750 kN (Fig. 8a). For the similar parameter k_1 value the response was generated with the step change of the disturbances values equal 250 kN (Fig. 8b). The control algorithm with the linear controllers PID which settings were selected for the step change of the disturbances values equal 750 kN did not enable to preserve the stability with the change of disturbances equal 250 kN. The control algorithm designed with the nonlinear controllers enabled stable work of the system regardless of the value occurred perturbation changes which was showed in the Figures 8c and 8d. The use of the nonlinear controllers led to the statistic deviation being generated and its value was located in the set range ±0.01 mm.

5. CONCLUSION

1. Stable work of the synchronization system can be ensured by the application of the controllers based on the direct Lapunov's method. These controllers ensure stable work in the whole range of the value differences changes among the loads of the cylinder.
2. Apart from the control system stability, the application of the controllers designed on the basis of the direct Lapunov's method enables to obtain the control quality improvement in the range of the reduction of overshoot $\kappa_i(t)$ as well as the value of the maximal synchronization error $\Delta y_{i\max}(t)$ in comparison to algorithm with PID controllers. In the simulation experiments overshoot $\kappa_i(t)$ was reduced by 50% for the step change of the disturbances value in no 1 and 2 assemblies. The value of the maximal synchronization error $\Delta y_{i\max}(t)$ was reduced by 44% for the linear growing change of the disturbance value in no 1 assembly.
3. The elaborated by author control system with the controllers designed on the basis of the direct Lapunov's method doesn't enable the elimination of the static deviation, which describes differences among cylinder displacements after the characteristics have been stabilized. Hence, in the future research the ways of the correction in this field should be searched.
4. The value of the parameter k_1 occurring in transfer function of the designed controllers should be changed according to the load differences among synchronized cylinder.

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