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THE METHOD OF APPLICATION OF SEA IN STRUCTURAL DESIGN PROCESS FOR REDUCING STRUCTURE-BORNE NOISE IN MACHINERY PART 1: GENERAL PROCEDURE

SUMMARY

The paper focuses on the finding which parts of machinery should be modified in order to reduce the noise. The countermeasure is decided by being based on the theoretical formulation the statistical energy analysis (SEA). The proposed process is applied to determine the countermeasures to structure-borne noise in a commercial product (A4 laser printer). This study is the first report of a series to develop a process based on the SEA.

***Keywords**: statistical energy analysis, method of vibration analysis, vibration of continuous system, modeling, vibration control, noise control, identification*

*METODY REDUKCJI HAŁASU MASZYN
Z WYKORZYSTANIEM STATYSTYCZNYCH METOD ANALIZY PRZEPŁYWU ENERGII.
CZĘŚĆ 1. PROPOZYCJA METODY*

Artykuł poświęcono metodzie wyszukiwania podukładów maszyn, które powinny być zmodyfikowane w celu obniżenia poziomu hałasu. Metoda bazuje na statystycznej analizie przepływu energii (SEA). Zaproponowaną metodę zastosowano do obniżenia hałasu wytwarzanego przez drukarkę laserową. Artykuł jest pierwszą częścią raportu poświęconego omawianej metodzie.

***Słowa kluczowe**: statystyczna analiza energii, metoda analizy drgań, drgania układów ciągłych, modelownie, sterowanie drganiami, sterowanie dźwiękiem, identyfikacja*

1. INTRODUCTION

In order to reduce structure-borne noise radiated from machinery, the following procedure is generally considered. An experimental or a numerical model of the structure of the machinery is first constructed by using conventional methods such as the experimental modal analysis (EMA), the finite element method (FEM) or the boundary element method (BEM). Next, the input forces into the structure during real-world operation of the machinery are identified. The structural model is then adjusted to match the vibration and the noise behaviour of the machinery and to determine the necessary countermeasures which need to be applied to the structural optimization algorithms in order to reduce structure-borne noise.

It is, however, not easy to systematically implement of the procedure described above. Modelling structures using conventional methods is often cumbersome and takes a long time. For structures with a large numbers of elements engineers might have difficulty identifying the forces acting on the system during operation. If force identification is possible, it takes a long time and considerable human effort to perform the experiments. Moreover, finding the countermeasures and the adoption of the optimization algorithms is very cumbersome since the analysis of structure-borne sound requires wider and higher frequency calculations while considering a lot of resonances. Of course, conventional methods which are focused on the natural modes of structures are very useful tools for analyzing and designing partial structures of machinery; however, they are not suitable for implementing the process of vibration and noise reduction to complete systems.

On the other hand, statistical energy analysis (SEA) (Lyon and DeJong 1998; Norton 1989) has been used for many years to predict the response of complex engineering systems in the case of high modal density. At present, there is a rough classification of the methods based on SEA into three kinds, where one is the classical analytical SEA based on theoretical parameter formulation, another is experimental SEA whose parameters are determined by performing experiments on the existing system, and the third one is SEA combined with FEM, which corresponds to numerical experimental SEA. The advantages of SEA over the conventional approaches lie in the fact that relatively few degrees of freedom are involved, it is easy to determine how the external power input to the system, and the SEA parameters have physical meaning, which is very useful for considering the countermeasures.

Therefore, the implementation of the structure-borne noise reduction process described above can be achieved by using the SEA methodology. This paper is the first part of a series to develop a process based on the SEA methodology. This paper focuses on the finding which real-world product parts should be modified in order to reduce the noise produced by the product. The second paper describes the optimization design for the subsystem which should be

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modified to make noise reduction by using the combination of FEM and the optimization procedure. However in this first paper, the countermeasure is decided by being based on the theoretical formulation of the SEA parameters. This proposed process takes a few days to determine the countermeasures to structure-borne noise in a commercial product such as an office printer. The application of the results from this study to an A4 laser printer is also presented to demonstrate that the process works well.

2. PROCESS FOR STRUCTURE-BORNE SOUND REDUCTION BASED ON SEA

2.1. Power balance equation for SEA

In SEA, the system is regarded as an assembly of subsystems. The energy of the i-th subsystem is written as E_i. The system is considered to be resonant, so that the total energy E_i, which is the sum of the kinetic and the potential energy, is equal to twice the kinetic energy. The external power input to this subsystem is denoted by P_i. Considering the power balance leads to a set of equations, and the SEA equation is written in the matrix form

$$\begin{Bmatrix} P_1 \\ \vdots \\ \vdots \\ P_N \end{Bmatrix} = \begin{bmatrix} \eta_1 + \sum_{i \neq 1}^{N} \eta_{1i} & -\eta_{21} & \cdots & -\eta_{N1} \\ -\eta_{12} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ -\eta_{1N} & \cdots & \cdots & \eta_N + \sum_{i \neq r}^{N} \eta_{Ni} \end{bmatrix} \begin{Bmatrix} E_1 \\ \vdots \\ \vdots \\ E_N \end{Bmatrix}$$

$$\text{or } \mathbf{P} = \omega \mathbf{L} \mathbf{E} \qquad (1), (2)$$

Here, $\mathbf{P}$ is the external power input vector, $\mathbf{E}$ is the vector of the subsystem energies, ω is the angular center frequency of the band, and $\mathbf{L}$ is the loss factor matrix, which is composed of the damping loss factors (DLF) η_i of the i-th subsystem and the coupling loss factors (CLF) η_{ij} from the i-th to the j-th subsystem. The coupling loss factors are estimated theoretically in the case of analytical SEA (Lyon and DeJong 1998; Norton 1989), experimentally in experimental SEA (Yamazaki *et al.* 2000), and numerically in the SEA combined with FEM (Simmons 1991; Mace and Shorter 2000; Yamazaki and Kamata 2001). The estimation of the damping and the coupling loss factors is called the construction of the SEA model.

2.2. Process for structure-borne sound reduction based on SEA

Consider the case of mass products such as automotive vehicles, office machines and home appliances. In those cases, there exist base systems for finding the countermeasures by using experiments, and the experimental SEA can be applied. The SEA model of the system includes either both structural and acoustical subsystems or only structural subsystems. If the acoustical subsystems are not considered in the process of noise reduction, the relation between the noise levels and the responses of the structural subsystems should be determined, and scaling factors relating the structural subsystem energies to the sound pressure levels at the noise assessment points need to be introduced.

In summary, the process proposed here is as follows:

Step 1. Constructing an experimental SEA model for the target product system
Step 2. Evaluating the scaling factors from the energies of the structural subsystems to the sound pressure levels at the assessment locations
Step 3. Identifying external power input to the subsystems during real-world operation
Step 4. Specifying loss factors which should be changed to reduce the sound pressure levels
Step 5. Designing the structure necessary for changing the desired loss factors

The details of each step in the process are described below.

Step 1: Constructing an experimental SEA model

Firstly, the system is subdivided into subsystems (Kamata *et al.* 2002). The measured responses of the subsystems are then used to construct the experimental SEA model, which is used for estimating the damping and the coupling loss factors. Various experimental methods can be used to determine the loss factors.

Step 2: Evaluating the scaling factors for noise

If the SEA model already includes the acoustical subsystems necessary for reducing the sound pressure levels, this step can be omitted. However, reducing noise by considering only the structural subsystems requires the following step to evaluate the scaling factors relating the structural subsystem energies to the sound pressure levels at the assessment locations.

The relation between the individual structural subsystem energy and the sound pressure level at a given assessment location is expressed as the scaling factor β relating the i-th subsystem energy E_i to the squared sound pressure V^2 at that location by using the experimental SEA model for the structures constructed in the first step described in Step 1.

The i-th subsystem is individually excited, for example, by an impulse hammer. The vibration response of the excited subsystem is measured to evaluate the subsystem energy normalized by the power input E_{ii}. The other subsystem energies E_{ji} ($j \neq i$) under the i-th subsystem are predicted by the previously constructed experimental SEA model. Simultaneously, the squared sound pressure V_i^2 is also measured. Iterating this process over the entire set of subsystems yields

$$\begin{bmatrix}\beta_1\\ \beta_2\\ \vdots\end{bmatrix}=\begin{bmatrix}E_{11} & E_{21} & \cdots\\ E_{12} & E_{22} & \cdots\\ \vdots & \vdots & \ddots\end{bmatrix}^{-1}\begin{bmatrix}V_1^2\\ V_2^2\\ \vdots\end{bmatrix} \tag{3}$$

Here, β_i is the scaling factor relating the i-th subsystem E_i to the squared sound pressure produced by the i-th subsystem. Thus, the total squared sound pressure is determined from

$$V^2=\sum_{i=1}^{N}\beta_i E_i \tag{4}$$

where V^2 is the total sound pressure which needs to be reduced. It should be noted that there is no information about either the phase of the squared sound pressure or whether the acoustic SEA subsystems are considered.

Step 3: Identifying external power inputs

It is very important to identify the external power inputs to the subsystems during real-world operation since the resulting countermeasures strongly depend on them. The SEA methodology allows this to be done easily with equations (1) or (2) by only measuring the energies of the subsystems.

Moreover, the SEA methodology yields valuable information about the energy transfer P_{ij} from the i-th to the j-th subsystem in the following form

$$P_{ij}=\omega\eta_{ij}E_i-\omega\eta_{ji}E_j \tag{5}$$

Step 4: Specifying loss factors

In SEA, it is easy to understand which SEA parameters affect the responses on the subsystems since the degrees of freedom in SEA are fewer than those in the conventional methods. The SEA model and the external power inputs yield the sensitivities of the subsystem energies to each loss factor by using techniques such a sensitivity analysis and the perturbation method.

This paper introduces a new formula relating the sensitivities of the subsystem energies to each loss factor based on the perturbation method (Nakagiri 1992). In addition, the sensitivity of the squared sound pressure is also formulated.

The loss factor η_n can be expressed by using the small parameter α_n in the form

$$\eta_n=\overline{\eta_n}\left(1+\alpha_n\right) \tag{6}$$

Here, $\overline{\eta_n}$ is the deterministic (expected) value of the damping or the coupling loss factor η_n without perturbations.

If the external power input $\mathbf{P}$ in equation (2) is assumed not to change when the structure is modified, the perturbations of the loss factor indicate that the loss factor matrix $\mathbf{L}$ and the subsystem energy matrix $\mathbf{E}$ in equation (2) can be approximated by using the first order of Taylor's expansions as

$$\mathbf{L}=\overline{\mathbf{L}}+\sum_{n=1}^{N_L}\frac{\partial\mathbf{L}}{\partial\alpha_n}\alpha_n \quad\text{and}\quad \mathbf{E}=\overline{\mathbf{E}}+\sum_{n=1}^{N_L}\frac{\partial\mathbf{E}}{\partial\alpha_n}\alpha_n \tag{7), (8}$$

Here, N_L is the total number of damping and coupling loss factors, and $\overline{\mathbf{L}}$ and $\overline{\mathbf{E}}$ are the deterministic (expected) values of the loss factor η_n and the subsystem energy E_i without perturbations.

The respective substitution of equations (7) and (8) into equation (2) yields

$$\mathbf{P}=\overline{\mathbf{L}}\,\overline{\mathbf{E}} \quad\text{and}\quad \overline{\mathbf{L}}\frac{\partial\mathbf{E}}{\partial\alpha_n}=-\frac{\partial\mathbf{L}}{\partial\alpha_n}\overline{\mathbf{E}} \tag{9), (10}$$

From equation (10), the sensitivity of the subsystem energy matrix to small changes in the loss factor as shown in equation (6) can be derived as

$$\frac{\partial\mathbf{E}}{\partial\alpha_n}=-\overline{\mathbf{L}}^{-1}\frac{\partial\mathbf{L}}{\partial\alpha_n}\overline{\mathbf{E}} \tag{11}$$

Furthermore, the variance of the subsystem energy matrix can be written as

$$\Delta\mathbf{E}=\frac{\partial\mathbf{E}}{\partial\alpha_n}\alpha_n \tag{12}$$

Moreover, the sensitivity and the variance of the squared sound pressure averaged over the assessment locations can be written as

$$\frac{\partial V^2}{\partial\alpha_n}=\sum_{i=1}^{N_L}\frac{\partial E_i}{\partial\alpha_n} \quad\text{and}\quad \Delta V^2=\sum_{i=1}^{N_L}\beta_i\Delta E_i \tag{13), (14}$$

The sensitivities of the subsystem energies and the sound pressure to each loss factor determine which loss factors exert the greatest influence on the system and should therefore be modified in order to obtain the most efficient design structure featuring low vibration and noise.

Step 5: Designing the structure

After the loss factors have been determined in Step 4, the countermeasures for the structural modification with the desired loss factor change must be considered. The increment of the damping loss factors is determined by the increment of the damping of the corresponding subsystem, for example, by the attachment of a damping sheet. The modification of the coupling loss factors is implemented by the alternation of the subsystem configuration or the coupling conditions.

This paper focuses on the modification of the coupling loss factors based on the theoretical expression of the line coupling between two plates in the analytical SEA method, for example, in the following form (Norton 1989).

$$\eta_{ij}=\frac{2L_{ij}\tau_{ij}}{\pi S_i}\sqrt{\frac{h_i}{\omega}}\sqrt[4]{\frac{E_i}{12\rho_i\left(1-\nu_i^2\right)}} \tag{15}$$

Here, h_i, E_i, ν_i, ρ_i and S_i are the thickness, the Young's modulus, the material density, and the surface area of the i-th subsystem, respectively. L_{ij} and τ_{ij} are the coupling length and the transmission efficiency between the *i*-th and the *j*-th subsystems, respectively.

Since the countermeasures determined on the basis of the above way are overly general, the finite element method and the structural optimization algorithms can be useful for finding the detailed design of the configuration of the subsystem and the coupling (Kuroda *et al*. 2009).

3. EXAMPLE APPLICATION TO A LASER PRINTER

The results from applying the above framework to a laser printer indicate that the process proposed in this paper works well for reducing structure-borne noise.

3.1. Target printer

The printer used in the experiment is shown in Figure 1. This machine is intended for A4 monochrome printing at a speed of 14 pages per minute. The printer was tested without its outer casing as shown in Figure 1. The sound pressure level was measured at four points, which correspond to the position of a standing person (1.5 m height and 1m from front, back, left and right), as specified in the ISO-7779 standard for free-field conditions. The sound pressure level was an average of the values at the four positions; the conditions for the real-world operation were set to the continuous printing of 14 pages. The sound pressure was regarded as the sound spectrum averaged over the time needed to print pages 2 through 13.

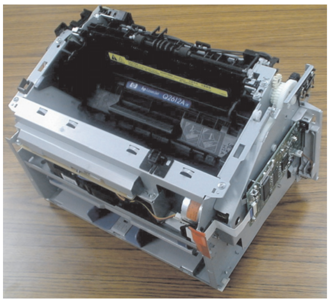

Fig. 1. Test laser printer

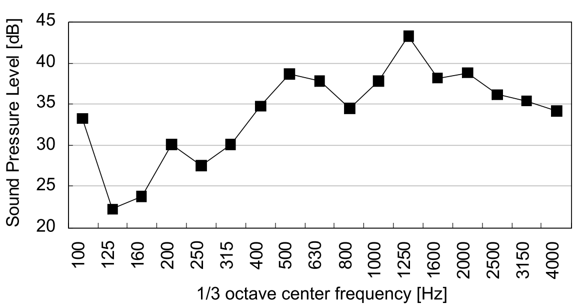

Fig. 2. Sound pressure levels of the test printer

The sound pressure level measured in 1/3 octave bands is shown in Figure 2. The main noise sources were the rotating motor, which had a fundamental frequency of 504 Hz, and the sound produced by the photoconductive drum, which was around 1.2 kHz during electrification and around 1.8 kHz during the development. Figure 2 presents the noise level peak at the 500 Hz, 1.25 kHz and 2 kHz 1/3 octave bands. The noise at 1.25 kHz is especially high. However, this noise can not be reduced by means of structural modification since it is an air-borne sound. Therefore, the noise at 500 Hz 1/3 octave band was targeted for reduction based on the process proposed here, as it is a structure-borne sound.

3.2. Building an SEA model

The experimental SEA model was constructed in Step 1. The subdivision is shown in Figure 3, and the information about each subsystem is shown in Table 1.

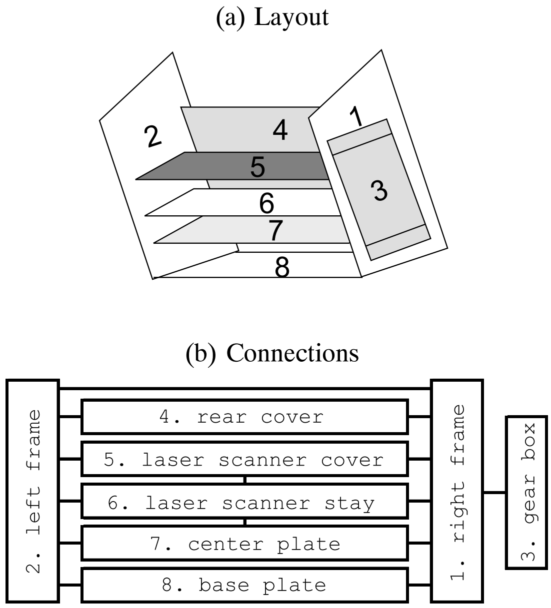

Fig. 3. Subdivision of the test printer

Table 1

Subsystem information

Subsystem No & Name		Thickness [mm]	Area [m^2]	Weight [kg]
1	Right frame	0.8	0.038	0.24
2	Left frame	0.8	0.038	0.23
3	Gear box	0.8	0.024	0.20
4	Rear cover	0.8	0.030	0.13
5	Laser scanner cover	0.6	0.033	0.16
6	Laser scanner stay	1	0.025	0.19
7	Center plate	0.6	0.022	0.10
8	Base Plate	0.8	0.053	0.33

The damping and the coupling loss factors were evaluated by measuring of the subsystem energies and the power inputs from (Yamazaki *et al.* 2000).

$$\eta_{ij} = \frac{E_{ji}/P_i}{\omega E_{ii}/P_i \cdot E_{jj}/P_j} \quad \text{and}$$

$$\eta_i = \frac{1-\omega\sum\limits_{j\neq i}^{n}\left(\eta_{ij}E_{ii}/P_i - \eta_{ji}E_{ji}/P_i\right)}{\omega E_{ii}} \qquad (16),\ (17)$$

Here, E_{ij} is the i-th subsystem energy as measured by the 6 accelerometers at two points per subsystem individually for power P_j inputted to the j-th subsystem with the impulse hammer.

In order to verify the precision of this experimental SEA model thus constructed, the subsystem energies corresponding to the excitation of the subsystem 1 were calculated by using this model. As an example, the result for the energy of the 6th subsystem was founded to be in fairly good agreement with the measured results under impact excitation above the 500 Hz 1/3 octave band, as shown in Figure 4. Even for frequencies below the 400 Hz band, there are small differences between the measured and the predicted results. It is also confirmed that the proposed process can be applied to reduce the noise at 500 Hz.

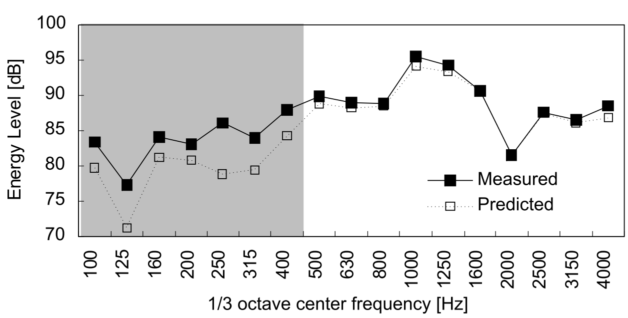

Fig. 4. Comparison between the measured and the predicted values of the 6th subsystem energy level for the experimental SEA model constructed under the excitation of the 1st subsystem

3.3. Identifying external power inputs during printing

The power inputs to the subsystems during printing were identified by measuring the subsystem energies, as shown in Figure 5. The grey hatching area represents the frequency bands at which the verification of the SEA model constructed was not satisfactory. The power inputs to the 1st and the 2nd subsystems were observed mainly in the 500 Hz band. These results were consistent with the presence of a driving motor in subsystem 1 and a rotor driven by this motor in subsystem 2. Finally, some negative power inputs are observed in Figure 5, which are dependent on the precision errors of the SEA model.

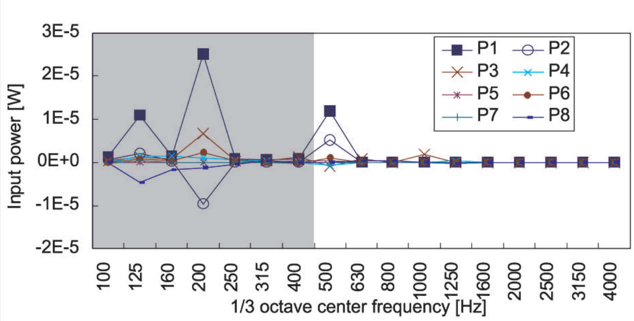

Fig. 5. Identified power input during printing

The energy transfer between the subsystems can be estimated using equation (5), as shown in Figure 6, which describes how the external power inputs to the 1st and the 2nd subsystems are transmitted through the subsystems.

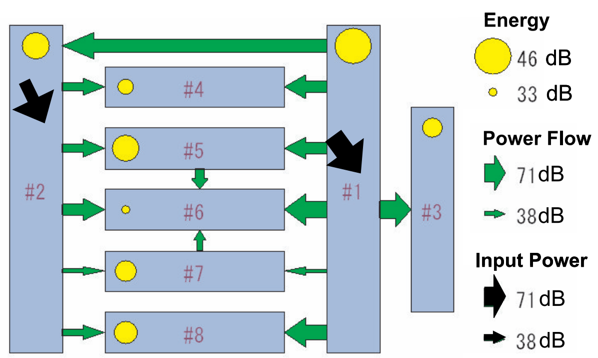

Fig. 6. Energy transfer path at the 500 Hz 1/3 octave band frequency during printing

3.4. Specifying the loss factors which should be changed in order to reduce noise

In order to reduce the noise produced by the printer during printing, the sensitivity of the squared sound pressure was first calculated from equation (13), with the parameter α_n = 0.1, as shown in Figure 7. For example, from Figure 7a, the damping loss factor with the highest sensitivity is η_6, which relates to subsystem 6. Considering the coupling loss factors in Figure 7b, the coupling loss factors η_{13} and η_{16} have higher sensitivity, and it is clear that η_6 and η_{13} should be larger, while η_{16} should be smaller. Furthermore, the ease of applying the countermeasures was considered, and consequently focus was turned to making the coupling loss factor η_{16} smaller for the purpose of noise reduction.

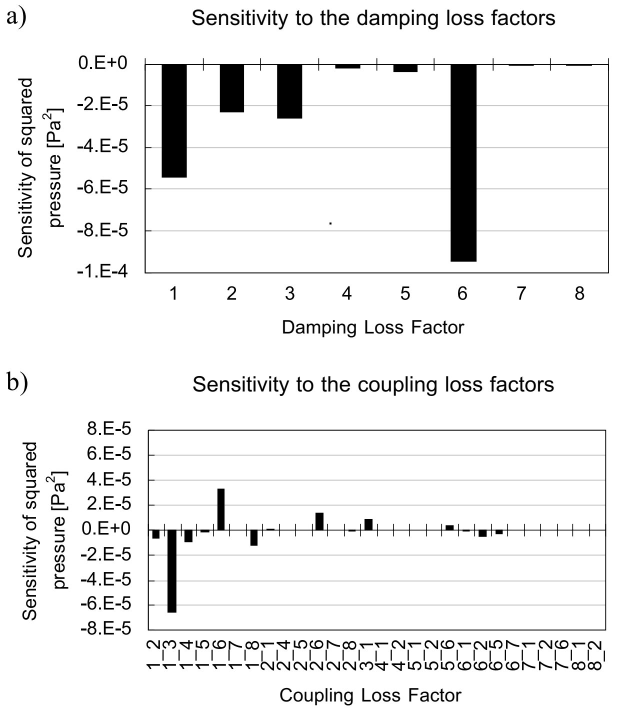

Fig. 7. Sensitivity of the sound pressure levels with respect to fluctuations in the loss factors at 500 Hz at the 1/3 octave band during printing

3.5. Designing the structure and coupling in accordance with specified loss factors

It was found that in order to decrease the coupling loss factor η_{16}, the structures of the 1st and the 6th subsystem and the coupling between them must be redesigned. In this paper, equation (15) is used for the necessary modifications.

In order to decrease the coupling loss factor η_{16}, the coupling strength should be weakened. Equation (15) indicates that the thickness and the Young's modulus E_1 of the subsystem 1 need to be decreased in order to achieve the desired effect. Consequently, the easiest countermeasure was to decrease the coupling length L_{16}.

The coupling length L_{16} was controlled by widening the slit between the 1st and the 6th subsystems from 8 mm to 31.5 mm, as shown in Figure 8. Figure 9 shows the comparison of the coupling loss factor η_{16} as measured before and after implementing the countermeasure. It is confirmed that widening the slit yields a smaller coupling loss factor at 500 Hz, which was the desired effect.

(a) Original (b) Countermeasure

Fig. 8. Structural modification for weakening the coupling strength between the 1st and the 6th subsystem

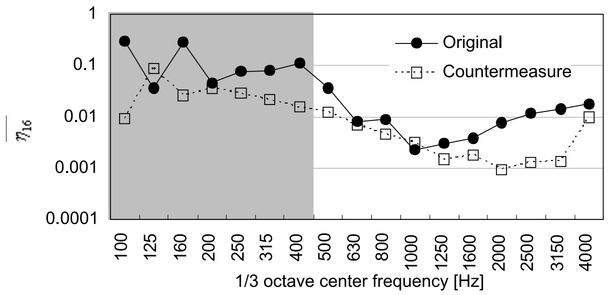

Fig. 9. Comparison between the original and the countermeasure values of the coupling loss factor from the 1st to the 6th subsystem

3.6. Verifying the noise reduction

The squared sound pressure levels were measured before and after the implementation of the countermeasure in order to confirm the noise reduction, and the results are shown in Figure 10. As visible from the comparison, noise reduction of 4.1 dB was achieved at the 500 Hz 1/3 octave band. Figure 9 indicates that the implementation of the countermeasure reduced the coupling loss factor η_{16} and the external power inputs to the 1st and the 2nd subsystems to about 35% and 65% of their original values, respectively, although in this paper it is assumed that the countermeasure implemented in accordance with the proposed process does not change the external power inputs. It is very difficult to predict changes in the external power inputs, and even more difficult to discuss the relative contribution weight of the changes of the coupling loss factor and the power inputs before and after the countermeasure. Therefore, the relative contributions have been estimated while maintaining the power inputs unchanged, and it was found that the decrease in the values of the coupling loss factor and the power inputs entailed noise reduction of 1.7 dB and 2.4 dB, respectively. Therefore, the contribution analysis indicated that the process of reducing structure-borne sound by using SEA as proposed in this paper worked well for reducing the noise produced by the printer.

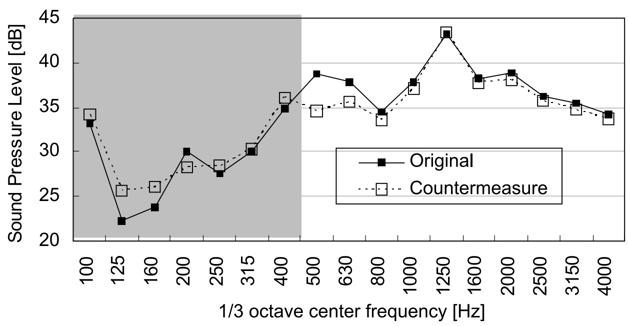

Fig. 10. Comparison of the sound pressure levels with and without the countermeasure

4. CONCLUSIONS

The process of reducing structure-borne sound in machinery using experimental SEA was proposed, since such process is difficult to implement using conventional methods such as EMA, FEM, and BEM.

Application of the proposed method to a laser printer for the purpose of reducing the structure-borne noise produced during printing demonstrated the validity of the process. The work on this problem will proceed with designing the geometry of the structural subsystems and the coupling by using FEM and structural optimization algorithms (Kuroda *et al.* 2009).

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