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MODELLING OF MATRIX DEGRADATION IN PIEZOCOMPOSITE ACTUATORS USING GRADED MATERIAL APPROXIMATION

SUMMARY

The work deals with modelling of material degradation in piezoelectric fiber composite (PFC) layers, which operate as actuators in active laminated plates. The considered degradation process concerns reduction of the matrix material stiffness and is described based on the functionally graded material approach. It is assumed that distribution of the matrix damage, which develops at the outer surface of the actuator layer and progresses toward its inner part, can be approximated according to a power function. The dynamic analysis and calculations are focused on changes in the natural frequencies and amplitude-frequency characteristics of the active laminated plate depending on the degradation level.

***Keywords**: laminated plate, piezoelectric control, piezocomposite actuator, material degradation, graded material*

MODELOWANIE DEGRADACJI OSNOWY W PIEZOKOMPOZYTOWYCH AKTUATORACH Z ZASTOSOWANIEM KONCEPCJI MATERIAŁU GRADIENTOWEGO

Praca dotyczy modelowania degradacji materiałowej w warstwach piezokompozytowych stosowanych jako aktuatory w aktywnych płytach laminowanych. Rozważany proces degradacji ograniczono do redukcji sztywności osnowy i opisano zgodnie z koncepcją materiału gradientowego. Założono, że rozkład uszkodzenia osnowy, które rozwija się na powierzchni zewnętrznej warstwy aktuatora i rozszerza do jej wnętrza, można w przybliżeniu opisać funkcją potęgową. Analizę dynamiczną i obliczenia ukierunkowano na określenie zmian częstości drgań własnych i charakterystyk amplitudowo-częstotliwościowych badanej aktywnej płyty w zależności od zakresu degradacji osnowy.

***Słowa kluczowe**: płyta laminowana, sterowanie piezoelektryczne, piezokompozytowy aktuator, degradacja materiałowa, materiał gradientowy*

1. INTRODUCTION

In recent years advanced composites with integrated piezoelectric sensor/actuator layers have been considered as attractive materials to develop thin walled structures and improve their dynamic behaviour. In many areas of technical applications, such as aerospace, automotive and mechanical engineering, active plate-like elements are used as structural components, which have ability to reduce unwanted vibration. In order to achieve a satisfactory control effectiveness relatively large deformations of piezoelectric actuator layers have to be generated due to the control process. Severe, alternative in time interfacial shear stresses and also environmental conditions create geometrical and material degradation, which may progress significantly during the service life and finally lead to a failure of the system. The geometrical degradation introduced as delamination reduces the stiffness, strength and fatigue properties of laminated composite and also affects the structure dynamic response and may reduce operational control effectiveness. Material degradation is primarily a matrix material process created by stresses due to mechanical vibration, and environmental conditions including temperature, chemical corruption and solar radiation to which polymeric materials are exposed (Sevostianov *et al.* 2003). The matrix damage is mainly characterized by a change of Young's modulus and a development of micro-cracks, which finally can lead to local delamination for layered composites as well as fiber-interface damage for fiber-based piezocomposites.

With the development of composite materials and their various applications the damage detection of the structure has become an important subject. Well established are techniques based on vibration responses, such as the modal frequency approach, transmittance functions approach, resonance approach, mechanical impedance approach, etc. cf (Diaz Valdes and Soutis 1999; Sampaio *et al.*1999). The literature review about various detection strategies one can find in (Zou *et al.* 2000).

The present work deals with modelling of material degradation in piezoelectric fiber composite (PFC) actuator layers. The degradation process concerns matrix material and is described based on a functionally graded material concept. The actuator layer is treated as a stack of sublayers reinforced with PZT (lead-zirconate-titanate) fibers, whose thickness relates to dimensions of the fiber cross-section. It is assumed that the matrix material stiffness diminishes through the thickness of the actuator according to a power function and reaches the minimal value near the outer surface. The dynamic analysis is based on the classical

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laminated plate theory. The calculations are focused on changes in the natural frequencies and vibration characteristics of the laminated plate depending on the degradation level.

2. FORMULATION OF THE PROBLEM

The considered system is a rectangular symmetrically laminated plate composed of conventional orthotropic layers (e.g. graphite-epoxy, glass-epoxy) integrated with PVDF (polyvinylidene fluoride) sensors and PFC actuators operating in closed loop with a constant gain velocity feedback. Uniformly aligned PZT fibers are embedded within matrix material. Due to longitudinal polarization they are electrically supplied by means of interdigitated electrodes designed with finger like sections of alternating polarity, which are mounted on the top and bottom surfaces of the actuator layer. To produce the bending mode action the midplane symmetric actuators are with either opposite polarization or opposite applied electric field. For example, the laminate cross-section is presented in Figure 1. Here the thickness of the component layers and the global z and the local z_l co-ordinates are shown.

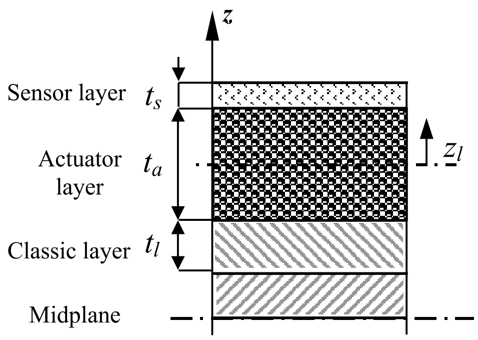

Fig. 1. The laminate cross-section

Assuming Kirchhoff's simplifications and the plate orthotropic behaviour, the transverse vibration $w(x, y, t)$ of the considered active laminated plate subjected to the external distributed load $q(x, y, t)$ is described by the equation

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + \\ + \rho t_c w_{,tt} = q(x,y,t) - p(x,y,t) \tag{1}$$

where D_{ij} (i, j = 1, 2, 6) are the elements of the bending stiffness matrix, which are complex for a viscoelastic material, t_c and ρ are the total thickness and equivalent mass density of the plate, respectively, $p(x, y, t)$ is the control system loading.

The control loading $p(x, y, t)$ is produced by the actuator layers driven by the voltage induced in the sensor layers and in the considered case transformed according to the velocity feedback. By solving the governing equation (1) with simply supported boundary conditions the transverse displacements of the active plate are obtained and for the steady-state case presented in terms of frequency response functions.

3. ACTUATOR AND SENSOR RELATIONS

The considered material degradation concerns the matrix material (resin) surrounding PZT fibers. Due to the bending stresses the imperfections develop at the outer surfaces of the actuator and progress toward the inner part of the piezocomposite forming a damaged part of the layer, whose thickness increases with time during service. It is assumed that the matrix damage by micro-cracks causes reduction of the elastic moduli mainly. But, in the case of the fiber-based composite actuator it may also reduce effective electromechanical coupling due to partial destruction of the piezoceramic fiber-interface decreasing the system control capability. Considering the stiffness reduction only, the damaged region is treated as functionally graded material. Modelling of functionally graded piezocomposites is discussed in (Pietrzakowski 2006, 2008; Tylikowski 2004), among others. Thus, its effective electromechanical properties are assumed to be averaged in-plane, while the elastic properties change in the thickness direction depending on the micro-crack number. For the critical state the damaged material becomes predominant within the actuator layer.

In the paper the actuator is considered as stacked of the sub-layers with PZT fibers aligned according to the rectangular packing pattern. Schematic visualization of the applied concept of the actuator damage modelling is shown in Figure 2.

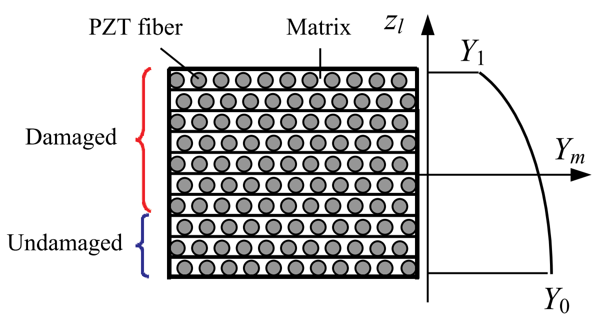

Fig. 2. The actuator cross-section divided into sub-layers and the distribution of the matrix material Young's modulus

It is supposed that the matrix stiffness is constant for each sub-layer and graded through the total thickness t_a of the actuator according to the power function

$$Y_m(z_l) = Y_0 - (Y_0 - Y_1)\left(\frac{1}{2} + \frac{z_l}{(m-1)t_p}\right)^p \tag{2}$$

where: z_l is the local co-ordinate measured from the middle surface of the actuator, t_p is the thickness of each sub-layer,

Y_0 and Y_1 denotes the maximal (health state) and minimal Young's modulus, respectively, p is the power function exponent and m is the total number of sub-layers in which the actuator layer is divided.

The actuator performance is described by the constitutive equation reduced to the 3–1 plane of material axes and transformed to the x, y plate reference axes as follows

$$\bar{\boldsymbol{\sigma}} = \bar{\mathbf{c}}^{ef}\, \bar{\boldsymbol{\varepsilon}} - E_3\, \mathbf{R}\, \mathbf{e}^{ef} \tag{3}$$

where $\bar{\boldsymbol{\sigma}} = [\sigma_x, \sigma_y, \tau_{xy}]^T$ and $\bar{\boldsymbol{\varepsilon}} = [\varepsilon_x, \varepsilon_y, \gamma_{xy}]^T$ is the in-plane stress and strain representation, respectively, $\bar{\mathbf{c}}^{ef}$ is the effective stiffness matrix, $\mathbf{e}^{ef} = \left[e_{33}^{ef}, e_{31}^{ef}, 0\right]^T$ is the matrix of piezoelectric stress coefficients, E_3 is the electric field along the fibers in the 3-axis direction, $\mathbf{R}$ is the transformation matrix related to the skew angle between the piezocomposite material axes 3 and 1 and the plate reference axes x and y.

The uniform field method based on the rule of mixtures is applied to obtain the effective properties of the two-phase composite material of each sub-layer. Details of the piezocomposite model and the way of formulation of the effective constitutive relations can be found in (Bent and Hagood 1997; Pietrzakowski 2003).

The effective piezoelectric constants strongly depend on the electrode surroundings, i.e. the electric parameters of the matrix material, and also the distance between the electrode and the PZT fiber. The electric path length differs for the particular sub-layer and increases when the sub-layer is located further from the electrode. Assuming that the actuator sub-layers are of the same thickness t_p, the averaged path length within ith sub-layer can be approximated as

$$l_p^{(i)} = s + (2i-1)\, v_{2m}\, t_p \tag{4}$$

where s is the spacing of the electrode sections, v_{2m} is the linear fraction of matrix material measured in the 2-axis direction perpendicular to the plate (constant for a distinct sub-layer), i denotes the sub-layer number in the sequence from the upper (or lower) electrode to the opposite face, $i = 1, 2, \ldots, m$.

The analysis concerns two-dimensional actuation effect transferred from the actuators to the host structure. Assuming that the actuator layers are perfectly integrated with the laminate the interaction may be described by the control moment distributed along the edges of the activated area. The moment resultant $\mathbf{M}^E$ for the particular actuator is obtained taking into account the electric part of the constitutive equation (3). Considering the actuator consisting of m sub-layers the moment produced can be expressed as

$$\mathbf{M}^E = \left[M_x^E, M_y^E, M_{xy}^E\right]^T = \sum_{i=1}^{m} \mathbf{R}\left(\mathbf{e}^{ef}\right)^{(i)} t_p z_0^{(i)} E_3^{(i)} \tag{5}$$

where $z_0^{(i)}$ is the distance of the ith sub-layer from the mid-plane of the plate, $\left(\mathbf{e}^{ef}\right)^{(i)}$ is the effective piezoelectric constant matrix and $E_3^{(i)}$ is the electric field supplying the ith sub-layer.

When voltage $V_a(t)$ is applied to the actuator interdigitated electrodes, which cover the both actuator faces, the portion of electric field within ith sub-layer can be approximated due to the electric field-voltage formula

$$E_3^{(i)} = V_a(t)\left(\frac{1}{l_p^{(i)}} + \frac{1}{l_p^{(m+1-i)}}\right) \tag{6}$$

The voltage V_a relates to the voltage V_s generated by the deformed PVDF sensor layer due to the direct piezoelectric effect and then transformed according to the velocity feedback low. The constitutive equation referred to the PVDF layer polarized transversally along the 3-axis and with the in-plane material axes 1 and 2 parallel, respectively, to the plate x, y axes has the form

$$D_3 = \mathbf{e}^T \boldsymbol{\varepsilon} + \epsilon_{33}\, E_3 \tag{7}$$

where D_3 and E_3 are the electric displacement and the external electric field in the 3-axis direction, respectively, ε is the in-plane strain representation of sensor layer, $\mathbf{e}$ denotes the piezoelectric coefficient matrix of the form $\mathbf{e} = [e_{31}, e_{32}, 0]^T$, ϵ_{33} is the material permittivity constant.

The voltage V_s is obtained by integration the charge stored on the sensor electrodes. Neglecting the external electric field, after using the standard relation for capacitance and the geometric relation between strain and transverse displacement the following formula is derived

$$V_s^k = -\frac{t_s\, z_0^k}{\epsilon_{33}\, A_s}\mathbf{e}^T \int_0^a\!\!\int_0^b \left[w_{,xx}, w_{,yy}, 2w_{,xy}\right]^T dxdy \tag{8}$$

Finally, the control loading $p(x, y, t)$ produced by a pair of actuator layers located symmetrically about the middle of the plate is as follows

$$p(x,y,t) = 2\left(M_{x,xx}^E + M_{y,yy}^E + 2M_{xy,xy}^E\right) \tag{9}$$

The twisting moment component M_{xy}^E vanishes when the plate axes and the piezocomposite material axes coincide, respectively.

4. RESULTS OF CALCULATIONS

Calculations have been carried out for a simply supported cross-ply laminated plate of dimensions 400×400×2 mm. The plate consists of classic graphite-epoxy layers of thickness t_l = 0.2 mm, PVDF sensors of thickness t_s = 0.1 mm, and PFC actuator layers of thickness t_a = 0.5 mm.

Table 1

Properties of PFC components [1]

Parameter	ρ_a kgm^{-3}	c_{11} GPa	c_{12} GPa	c_{13} GPa	c_{33} GPa	G GPa	e_{31} Cm^{-2}	e_{33} Cm^{-2}	$\in_{33}/\in_0$
PZT-5H	7650	127	80.2	84.7	117	36.3	–4.42	15.5	1392
Matrix	1200	8.15	4.01	4.01	8.15	2.33	0	0	11.2

The layers are stacked according to the symmetric order $[S/A/0^o/90^o]_s$, where the symbols "S" and "A" indicate the sensor and actuator, respectively. The PFC actuators are a two-phases composite with the PZT volume fraction equal to 0.8 and the PZT fibers spatial configuration which enables the division into ten distinct sub-layers of the same thickness t_p = 0.05 mm and having constant effective properties throughout their thickness. The stiffness parameters of the graphite-epoxy composite are assumed as: Y_{11} = 150 GPa, Y_{22} = 9 GPa, G_{12} = 7.1 GPa, and the equivalent mass density is equal ρ = 1600 kg/m^3. The properties of PVDF material are following: Y_s = 2 GPa, ρ_s = 1780 kg/m^3 and piezoelectric strain constants $d_{31} = 2.3 \cdot 10^{-11}$ mV^{-1}, $d_{32} = 3 \cdot 10^{-12}$ mV^{-1}. The equivalent damping of the plate composite material is modelled due to the Voigt-Kelvin approach with the following retardation time values: $\mu_1 = 10^{-6}$ s, $\mu_2 = \mu_{12} = 4 \cdot 10^{-6}$ s for orthotropic graphiteepoxy layers, $\mu_s = 2 \cdot 10^{-6}$ s for PVDF layers and $\mu_m = 8 \cdot 10^{-6}$ s for matrix material in PFC actuators. The electromechanical properties of piezocomposite components are listed in Table 1.

The gradation of the matrix material stiffness across the actuator is determined by the power function (Eq. (2)) with Young' modulus Y_m decreasing toward the outer actuator surface from the health state value Y_0 to the minimal value Y_1.

To compare dynamic behaviour of the active plate depending on the matrix degradation the amplitude-frequency characteristics near the 1–3 resonance region are presented in Figures 3 and 4. The characteristics are calculated at the plate point co-ordinates $x = y$ = 100 mm assuming that the plate is subjected to the uniformly distributed harmonic load of the amplitude q_0 = 1 Nm^{-2}. Two parameters are used to describe the degradation level. The relative inhomogeneity parameter r denotes the range of changes in the matrix Young's modulus across the actuator thickness and is defined as the ratio $r = (Y_0 - Y_1)/Y_0$. The degradation depth relates to the exponent p of the applied power distribution. The depth of damaged region is minimal for greater exponent values and considered as maximal for a linear distribution (p = 1).

Figure 3 shows the influence of the depth of degradation (exponent p) on the 1–3 resonance characteristics assuming the constant relative inhomogeneity, r = 0.5. The effect of variations in the matrix inhomogeneity (parameter r) caused by the degradation process is presented in Figure 4. In this case, calculations are accomplished for the distribution function with the exponent p = 3. It can be noticed that with increasing both the degradation depth and the range of matrix inhomogeneity the resonance peaks related to the frequency ω_{13} shifts to lower frequency values comparing with the undamaged actuator (dotted line). The resonant amplitude values are almost insensitive to the actuator degradation considered.

The characteristics presented in Figures 5 and 6 relate to the considered plate excited by the external harmonic load of the amplitude intensity q_0 = 1 Nm^{-2} which is uniformly distributed over a square surface limited by the co-ordinates $x_1 = y_1$ = 250 and $x_2 = y_2$ = 350 mm. Calculations are performed to show the effect of the matrix stiffness inhomogeneity on the dynamic response within wide frequency range assuming the Young's modulus gradation with the expo-

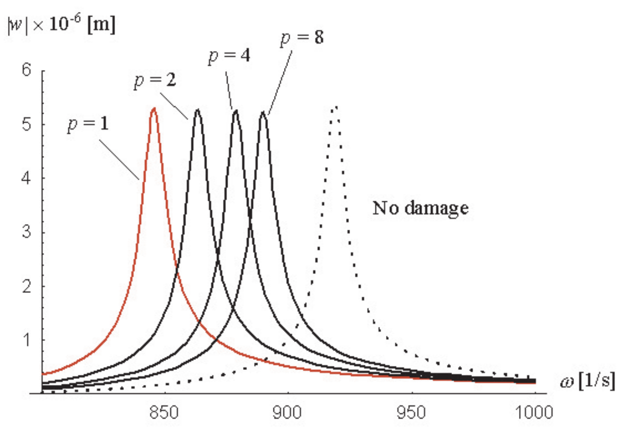

Fig. 3. Effect of variations in the exponent p near the 1–3 resonance region (r = 0.5). Dotted line – undamaged actuator

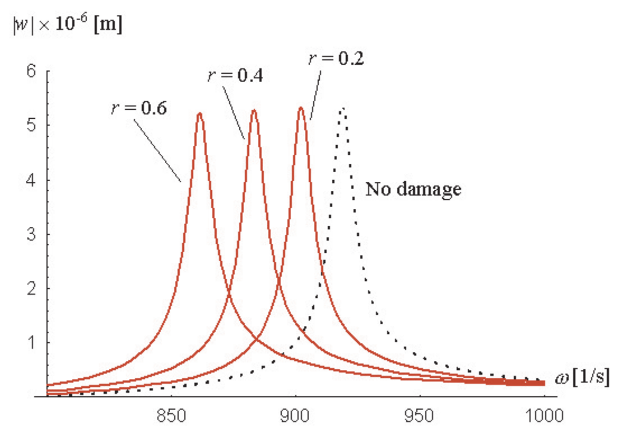

Fig. 4. Effect of variations in the inhomogeneity parameter r near the 1–3 resonance region (p = 3). Dotted line – undamaged actuator

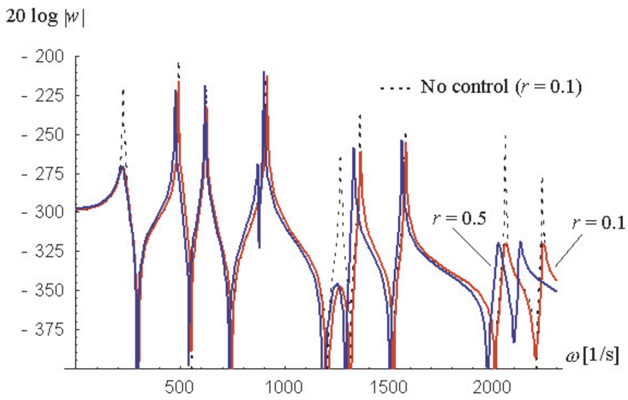

Fig. 5. Near field dynamic responses within wide frequency range depending on the matrix inhomegeneity parameter. Dotted line – uncontrolled plate

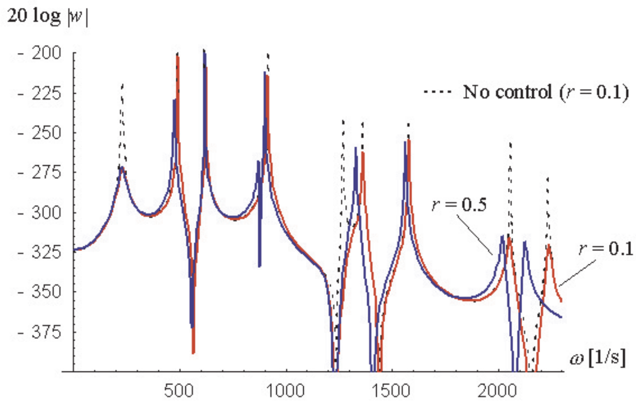

Fig. 6. Far field dynamic responses within wide frequency range depending on the matrix inhomegeneity parameter. Dotted line – uncontrolled plate

nent $p = 3$. Two measure points of the plate are taken into account – the near field at $x = y = 300$ mm (Fig. 5) and far field at $x = y = 100$ mm (Fig. 6). To demonstrate the active damping effect the dynamic characteristics of the uncontrolled plate are also shown (dotted line). Within the frequency range tested the resonance peaks observed depend on the external loading arrangement and the measure point localization.

The characteristics referring to the matrix damage described by the greater stiffness inhomogeneity ($r = 5$) confirm decreasing of the plate natural frequencies, which is a result of the global stiffness reduction. The natural frequency shift is more significant for higher frequencies. The comparison of the actual amplitude-frequency characteristic with that of the healthy system can be used to recognize the damage level.

5. CONCLUSIONS

The matrix degradation modelling based on the functionally graded material concept is implemented to describe the piezocomposite actuator damage. Considering the matrix Young's modulus gradient determined by a power function, the influence of the exponent value, which relates to the depth of degradation, and the inhomogeneity parameter, which relates to the difference between extreme Young's modulus values, on the active plate dynamic response is numerically examined. The observed changes in the natural frequencies depend on the level of matrix material degradation. Incorporating the material degradation model, the range of the natural frequency shift can be used for detection of the actuator damage and also recognizing a critical state of the structure.

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