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OPTIMAL HAND-ARM SYSTEM OF VIBRATION ISOLATION INCLUDING HUMAN SENSITIVITY TO VIBRATION**

SUMMARY

In the paper the optimal vibration isolation systems (OVIS) have been analytically obtained for a new hand-arm models (HAS) subjected to stochastic excitations. The models are described by its impedance, weighted by standard frequency domain curve $W_h(s)$ depicting hand-arm discomfort level. The analytical functions depicting OVIS have been carried out in the general form for random stationary acceleration excitations, general forms of impedance Z(s) of hand-arm system and selected criterion of isolation.

***Keywords**: hibrid hand-arm models, optimal vibration isolation*

*OPTYMALNA WIBROIZOLACJA UKŁADU RĘKA-RAMIĘ
PRZY UWZGLĘDNIENIU WRAŻLIWOŚCI CZŁOWIEKA NA WIBRACJE*

W artykule przedstawiono analityczne wyrażenia opisujące optymalne układy wibroizolacji (OVIS) dla nowych, hybrydowych modeli układu ręka-ramię (HAS) poddanych wymuszeniom przypadkowym. Modele hybrydowe zostały zbudowane na podstawie znajomości impedancji układu ręka-ramię Z(s) oraz podanej w normach funkcji wagowej $W_h(s)$ opisującej wrażliwość człowieka na wibracje przekazywane przez ręce. Optymalizację przeprowadzono dla przyjętego a priori kryterium. Wyrażenia analityczne opisujące OVIS zilustrowano przykładami numerycznymi.

***Słowa kluczowe**: hybrydowe modele układu ręka-ramię, optymalne układy wibroizolacji*

1. INTRODUCTION

Vibrating tools in manufacturing processes, mining, and other branches of industry where hands and finger grasp or push vibrating objects are the principal causes of hand injuries due to severe vibration and shocks. A wide range of vibrating objects can bring about such magnitudes of acceleration, which jointly with the exposure time produce observable injurious effects to the hand-arm system (HAS) of the human operator body. There are many biomechanical models of this system gathered and compared in (Książek *et al*. 1995; Rakheja *et al*. 2002), all of which resulted from investigation of its dynamic characteristics in particular situations. The problem of severe vibration on humans has been described in detail in (Griffin 1990). In some papers concerning the hand-arm system the usual way of theoretical approaches to vibration isolation based on simple, passive biomechanical models and selected, a priori known, passive structures of isolators. Some works concerning active human body models (AHBM) and their optimal vibration isolation were already published in (Książek 1997, 1999; Książek and Basista 1998). In the present article a general procedure for analytical construction of optimal vibration isolation systems for hand-arm models including weighting function depicting level of discomfort, is proposed, based on Wiener – Hopf filtration theory (Gupta and Hasdorf 1981).

2. FUNDAMENTAL COMPONENTS OF VIS – HAS SYSTEM

There are some principal factors that influence the problem of vibration isolation of whole – body and hand-arm systems. These factors are presented in block form in Figure 1 where E represents excitation, VIS represents the vibration isolation system, HAS represents the hand-arm system, HS represents the human sensitivity controller, and *J* represents the criterion of isolation. The block represented by *J*, in spite of its virtual character, has been introduced among the physical components to underline the capital influence of theperformance specifications on the VIS structure. Components presented in Figure 1 have different physical character. The HAS and HS components depend on physical structure and way of functioning of the structure of hand-arm-human body. The J component depends on subjective decision of the VIS designer. The VIS, as a final result depends on the procedure of composition of the previous components and applied mathematical tools. The excitations transmitted to the hands of hand-held tool operators can have a very diverse character.

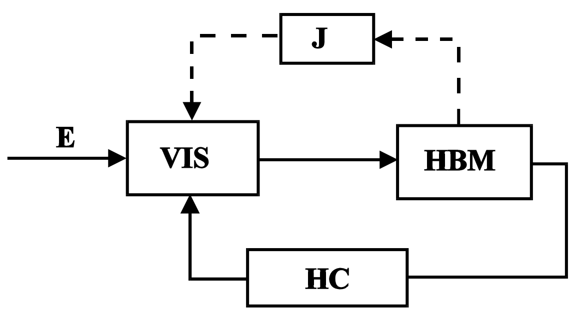

Fig. 1. Factors describing the problem of the vibration isolation of hand-arm system

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The power spectral densities of hand-tool excitations have irregular shape and are difficult to approximate by rational functions of the complex variable s. The HAS itself is a very complex, active biological system and there have been many attempts to measure its dynamical characteristics (Bystroem *et al*. 1978; Daikoku and Ishikawa 1990; Reynolds and Keith 1977), Miwa in (Rakheja *et al*. 2002; Basista 2002, 2006; Basista and Książek 2004). It is very difficult to build a general model of such a system. The existing models are mostly finite degree of freedom mechanical systems. They are of functional character and their structure is obtained from measurements of the mechanical impedance, apparent mass or transmissibility functions. These models were considered as lumped parameter, passive, and minimum-phase systems. In the majority of cases the biomechanical models of HAS were designed on the basis of the driving-point impedance (2.1) measurements.

$$Z(s)=\frac{F(s)}{\dot{x}(s)} \tag{2.1}$$

where $F(s)$, $\dot{x}(s)$, $\ddot{x}(s)$ are correspondingly Laplace transforms of force and velocity applied and measured at the chosen point of the hand. In hand-held tools or other vibrating objects the vibration isolation systems are applied as internal (Basista 2006) or external (Reynolds and Jetzer 1999) parts of the principal construction. External isolators, whether passive or active, are built between the tool or other vibrating object and the handle. The external devices (usually rubber mountings between the handle assembly and the power unit) may also include provision for heating the handle by either exhaust gases or electricity. Internal VIS are the results of the global conception and design of a tool that can have material, passive, or state control character. The present state of VIS is the result of increased interest by tool manufactures, primarily a result of pressure from tool users and the general increase in knowledge of the injuries character of vibrating equipment.

3. SENSITIVITY OF HAND-ARM SYSTEM TO VIBRATION

In the paper the frequency weighting curve W_h, defined in BS 6842, 1987b, was chosen for further calculations as a measure of hand-arm system sensitivity to vibration and plotted in Figure 2. This curve is compatible with ISO 5349 and is considered as the most representative characteristics for hand transmitted vibration. It is expressed by product of two rational functions as following

$$W_h(s)=H_{fb}(s)\,H_h(s) \tag{3.1}$$

with filter transmissibility

$$H_{fb}(s)=\frac{s^2 4\pi^2 f_2^2}{\left(s^2+\frac{2\pi f_1}{Q_1}s+4\pi^2 f_1^2\right)\left(s^2+\frac{2\pi f_2}{Q_1}s+4\pi^2 f_2^2\right)} \tag{3.2}$$

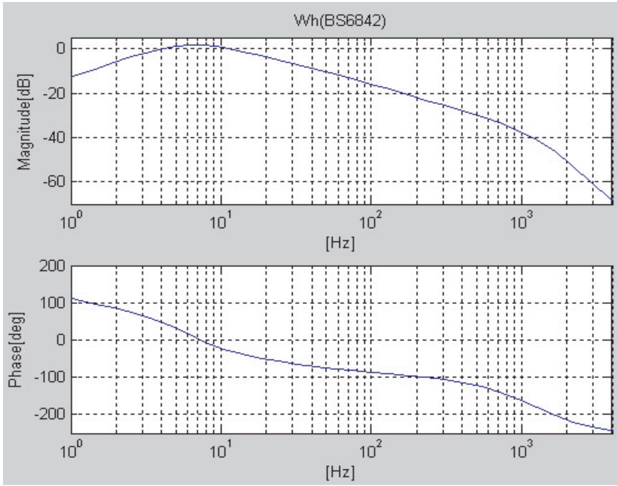

Fig. 2. The frequency weighting curve W_h

and frequency weighting function

$$H_h(s)=\frac{(s+2\pi f_3)}{\left(s^2+\frac{2\pi f_4}{Q_2}s+4\pi^2 f_4^2\right)}\,\frac{2\pi K f_4^2}{f_3} \tag{3.3}$$

The numerical values of the parameters in formulae (3.2), (3.3) are given in Table 1.

Table 1

Characteristics of band-limiting filters and frequency weightings defined in BS 6842

f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	Q_1 [–]	Q_2 [–]	K [–]
6.3	1250	16	16	0.71	0.64	1.0

4. FORMULATION OF HAS VIBRATION ISOLATION PROBLEM

The target of the proposed approach is to find the optimal VIS for the hand-arm system subjected to stationary random excitations described by their power spectral of acceleration. The biomechanical model composed of the two masses (m_1, m_2), damper (α) and spring (k) proposed by Miwa in (Rakheja *et al*. 2002), shown on the left side of Figure 4, is considered in the presented paper as the passive part of hand-arm system (HAS). The total model of HAS was assumed in the paper as a hybrid model described by the product of the impedance (or apparent mass) of the passive model and weighting curve W_h described in British Standards and ISO for the hand-arm system.

The right side of Figure 3 presents schematically handle (m) and the dynamical structure of VIS described by the function $\varphi(s)$. The source of excitation is described by stationary random acceleration signal $\ddot{x}_0(t)$.

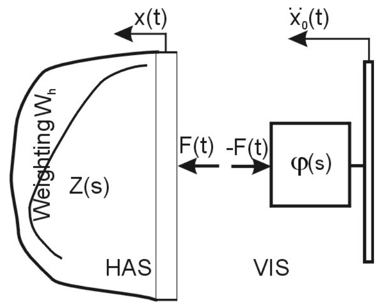

Fig. 3. Schematic drawing of the HAS, handle and VIS

5. ANALYTICAL DESIGN OF VIS OF HAS

5.1. Analytical form of criterion of vibration isolation

As the criterion of synthesis of the optimal vibration isolation of hand-arm system, the following expression was assumed

$$J = \sigma^2_{x-x_0} + \lambda\sigma^2_{\ddot{x}} = \min \tag{5.1}$$

where:

- $\sigma^2_{x-x_0}$ – variance of the relative displacement between tool and handle,
- $\sigma^2_{\ddot{x}}$ – variance of acceleration of the handle,
- λ – Lagrange multiplier.

The mean square values σ^2_y can be calculated as follows:

$$\sigma^2_y = \frac{1}{i}\int_{-i\infty}^{+i\infty} \left|H_y(s)\right|^2 S_{\ddot{x}_0}(s)ds \tag{5.2}$$

In (5.2) the index y denotes, respectively, $x - x_0$ and $\ddot{x}$. $H_y(s)$ denotes respectively transfer functions between the variables $x - x_0$, $\ddot{x}$ and $\ddot{x}_0$. Criterion (5.1) is a particular case of more general criterion proposed in (Książek 1999). The variances (5.2) are calculated under an assumption concerning the source of vibration. It was assumed that the excitation is depicted by the power spectral density (PSD) of acceleration denoted as $S_{\ddot{x}_0}(s)$ in the formula (5.3):

$$S_{\ddot{x}_0}(s) = S_{0r}s^4\psi_r(s)\psi_r(-s), \quad r = 1,2 \tag{5.3}$$

where:

- $r = 1$ – white noise acceleration excitation – WN,
- $r = 2$ – narrow band noise acceleration excitation – NBN.

And $S_{\ddot{x}_0}(s)$ can be expressed by its factorized functions $\psi_r(s)\psi_r(-s)$, $r = 1.2$ of complex variable s. These two power spectral densities can be considered as two extreme cases of excitation between harmonic (single frequency) and pure random signals. The very first approximation of most power spectral densities of real vibration sources can be obtained by combining and shaping of the densities given by formulae (5.4) and (5.5).

$$S_{\ddot{x}_0}(s) = \frac{\sigma^2}{2\pi} \tag{5.4}$$

$$S_{\ddot{x}_0}(s) = \frac{\gamma\sigma^2}{\pi}\frac{\Omega^2 - s^2}{\left(\Omega^2 + s^2\right)^2 - 4\gamma^2 s^2} \tag{5.5}$$

Due to the great variety of existing power spectral densities, this approach allows estimation of the extreme references for the optimum hand-arm VIS.

5.2. General procedure of design of optimum vibration isolation system (OVIS)

As shown in Figure 3, the force $F_T(t)$ is a force of contact between a hand-arm structure and VIS. To obtain the optimal VIS, the Wiener-Hopf theory has been applied. Let

$$\varphi(s) = F(s)/\ddot{x}_0(s) \tag{5.6}$$

where $\ddot{x}_0(s)$ and $F(s)$ are the Laplace transforms of random time functions $\ddot{x}_0(t)$ and $F(t)$ respectively. $\varphi(s)$ is the function describing the optimal VIS that does not have any poles in the right-hand side pf the s plane. The criterion of vibration isolation (5.1) can be written as

$$J = \sigma^2_{x-x_0}(L(s), W_h(s), \varphi(s), \psi(s), \lambda) + \\ +\lambda\sigma^2_{\ddot{x}}(L(s), W_h(s), \psi(s), \varphi(s), \lambda) \tag{5.7}$$

where:

- $W_h(s)$ – the frequency weighting function representing comfort preferences of hand-arm system,
- $L(s)$ – function depicting physical properties of the chosen model of hand.

Our aim is to find $\varphi(s)$ that minimizes J and that has no poles in the right-hand side of the s plane (for stability and realizability reasons). The method used to this end is a technique of variational calculus of variations (Gupta and Hasdorf 1981; Książek 1999). Assume that $\varphi(s)$ is subjected to a small variation, so that

$$\hat{\varphi}(s) = \varphi(s) + \varepsilon\eta(s) \tag{5.8}$$

Then J becomes $\hat{J}$ where

$$\hat{J}(\varepsilon) = \sigma^2_{x-x_0}(L(s), W_h(s), \hat{\varphi}(s), \psi(s), \lambda) + \\ +\lambda\sigma^2_{\ddot{x}}(L(s), W_h(s), \hat{\varphi}(s), \psi(s), \lambda) \tag{5.9}$$

To minimize J we calculate the following difference

$$\delta J = \hat{J} - J \tag{5.10}$$

and we minimize $\hat{J}(\varepsilon)$ as $\varepsilon \to 0$. This will be the case if

$$\left.\frac{d(\delta J)}{d\varepsilon}\right|_{\varepsilon=0} = 0 \tag{5.11}$$

As the final result of the above operations we obtain

$$\varphi(s) = f(s, L(s), W_h(s), \psi(s), \lambda) \tag{5.12}$$

5.3. Application of the procedure for vibration isolation of the hybrid biomechanical model of the hand-arm

Let us assume that hand-arm system can be represent by a hybrid biomechanical model described by the product of the mechanical impedance and weighting frequency function W_h. Taking this into account the force $F(s)$ can be written as

$$F(s) = \frac{Z(s)}{s} W_h(s)\ddot{x}(s) \tag{5.13}$$

Applying (5.13) and (5.6) the mean square values of (5.2) can be expressed as follows

$$\sigma^2_{x-x_0} = \frac{1}{j}\int_{-j\infty}^{+j\infty}\left|\frac{\varphi(s)}{sZ(s)W(s)} - \frac{1}{s^2}\right|^2 S_{\ddot{x}_0}(s)ds \tag{5.14}$$

$$\sigma^2_{\ddot{x}} = \frac{1}{j}\int_{-j\infty}^{+j\infty}\left|\frac{s\varphi(s)}{Z(s)W(s)}\right|^2 S_{\ddot{x}_0}(s)ds \tag{5.15}$$

Introducing

$$G(s) = \frac{s}{Z(s)W(s)} \tag{5.16}$$

and supposing that the spectral density can be factorized in the way shown in (5.7) criterion reduces to

$$J(\varphi(s),\lambda) = \frac{1}{i}\int_{-i\infty}^{+i\infty}[\left|\varphi(s)G(s) - 1\right|^2 + \\ +\lambda s^4 \left|\varphi(s)G(s)\right|^2]S_0\psi(s)\psi(-s)ds \tag{5.17}$$

Executing all steps of the procedure written in paragraph 5.2. The function (5.12) can be written as follows

$$\varphi(s) = \frac{1}{D^+(s)\psi^+(s)}\left[\frac{G(-s)\psi(s)}{D^-(s)}\right]_+ \tag{5.18}$$

where

$$D^+(s) = \left[G(s)G(-s)(1+\lambda s^4)\right]_+ \tag{5.19}$$

$$D^-(s) = \left[G(s)G(-s)(1+\lambda s^4)\right]_- \tag{5.20}$$

We can show that

$$1 + \lambda s^4 = R(s)R(-s) \tag{5.21}$$

where

$$A = \lambda^{0.5},\ B = 2^{0.5}\lambda^{0.25},\ C = 1 \tag{5.22}$$

$$R(s) = As^2 + Bs + C \\ R(-s) = As^2 - Bs + C \tag{5.23}$$

so

$$D^+(s) = \left[\frac{s}{Z(s)W(s)}\frac{-s}{Z(-s)W(-s)}(1+\lambda s^4)\right]_+ = \\ = \frac{sR(s)}{Z(s)W(s)} \tag{5.24}$$

$$D^-(s) = \left[\frac{s}{Z(s)W(s)}\frac{-s}{Z(-s)W(-s)}(1+\lambda s^4)\right]_- = \\ = \frac{-sR(-s)}{Z(-s)W(-s)} \tag{5.25}$$

finally

$$\varphi(s) = \frac{F(s)}{\ddot{x}_0(s)} = \frac{Z(s)W(s)}{sR(s)\psi(s)}\left[\frac{\psi(s)}{R(-s)}\right]_+ \tag{5.26}$$

Final formula for $\varphi(s)$ depends on the forms of driving-point impedance of the isolated object, frequency weighting function $W(s)$ and acceleration excitation $\psi(s)$.

5.4. Forms of acceleration excitation

Case of white noise (WN) acceleration excitation

$$S_{\ddot{x}_0}(s) = S_{01} = \frac{\sigma_0^2}{2\pi} \tag{5.27}$$

$$\psi_1(s) = \frac{1}{s^2},\quad \psi_1(-s) = \frac{1}{(-s)^2} \tag{5.28}$$

$$\left[\frac{\psi_1(s)}{R^-(s)}\right]_+ = \left[\frac{1}{s^2(As^2 - Bs + C)}\right]_+ = \frac{Ks + L}{s^2} \tag{5.29}$$

where the parameters K and L must be calculated from (5.23) and system of algebraic equations

$$KA + U = 0 \\ -KB + W + LA = 0 \\ KC - LB = 0 \\ LC = 1 \tag{5.30}$$

Finally, the expression for the optimum function $\varphi(s)$ for WN excitation takes the form

$$\varphi(s)=\frac{Z(s)W(s)}{sR^{+}(s)\psi(s)}\left[\frac{\psi(s)}{R^{-}(-s)}\right]_{+}=$$
$$=\frac{Z(s)W(s)}{s(As^2+Bs+C)\frac{1}{s^2}}\frac{Ks+L}{s^2}= \tag{5.31}$$
$$=\frac{Z(s)W(s)(Ks+L)}{s(As^2+Bs+C)}$$

Case of narrow band noise (NBN) acceleration excitation

$$S_{\ddot{x}_0}(s)=S_{02}\frac{\Omega^2-s^2}{(\Omega^2+s^2)-4\gamma^2s^2},\quad S_{02}=\frac{\gamma\sigma_0^2}{\pi} \tag{5.32}$$

$$\psi_2(s)=\frac{s+\Omega}{s^2(s^2+2\gamma s+\Omega^2)}$$
$$\psi_2(-s)=\frac{-s+\Omega}{(-s)^2(s^2-2\gamma s+\Omega^2)} \tag{5.33}$$

$$\left[\frac{\psi_1(s)}{R^{-}(s)}\right]_{+}=\left[\frac{s+\Omega}{s^2(s^2+2\gamma s+\Omega^2)}\frac{1}{(As^2-Bs+C)}\right]_{+}=$$
$$=\frac{Ms^3+Ns^2+Ps+Q}{s^2(s^2+2\gamma s+\Omega^2)} \tag{5.34}$$

The parameters M, N, P, Q are to be calculated from the following system of algebraic equations:

$$\begin{aligned}&MA+V=0\\&-MB+NA+2\gamma V+T=0\\&MC-NB+PA+V\Omega^2+2\gamma T=0\\&NC-PB+QA+T\Omega^2=0\\&PC-QB=1\\&QC=\Omega\end{aligned} \tag{5.35}$$

Finally

$$\varphi(s)=\frac{F_T(s)}{\ddot{x}_0(s)}=\frac{Z(s)W(s)}{sR^{+}(s)\psi(s)}\left[\frac{\psi(s)}{R^{-}(-s)}\right]_{+}=$$
$$=\frac{Z(s)W(s)(Ms^3+Ns^2+Ps+Q)}{s(As^2+Bs+C)(\Omega+s)} \tag{5.36}$$

6. OPTIMAL VIS FOR SELECTED HAND-ARM MODELS

6.1. VIS – $\varphi(s)$ for HAS model of 1-DOF-pure mass

$$Z(s)=ms,\quad W(s)=W_h(s) \tag{6.1}$$

Case 1: WN acceleration excitation

$$\varphi(s)=\frac{Z(s)W(s)(Ks+L)}{s(As^2+Bs+C)}=$$
$$=\frac{mH_{fb}(s)\,H_h(s)(Ks+L)}{(As^2+Bs+C)} \tag{6.2}$$

Case 2: NBN acceleration excitation

In this case the form of expression is the same as in case 1. The final form of function $\varphi(s)$ is

$$\varphi(s)=\frac{Z(s)W(s)(Ms^3+Ns^2+Ps+Q)}{s(As^2+Bs+C)(\Omega+s)}=$$
$$=\frac{mH_{fb}(s)\,H_h(s)(Ms^3+Ns^2+Ps+Q)}{(As^2+Bs+C)(\Omega+s)} \tag{6.3}$$

6.2. VIS – $\varphi(s)$ for HAS model of 2 DOF

In this case HAS is described by hybrid 2 DOF model shown in Figure 4. The dynamical reaction of operator's hands on the handle vibration is depicted by the mechanical impedance $Z(s)$ given by the formula (6.4) and presented in Figure 5.

$$Z(s)=\frac{s\left[m_1m_2s^2+\alpha(m_1+m_2)s+k(m_1+m_2)\right]}{m_2s^2+\alpha s+k} \tag{6.4}$$
$$W(s)=W_h(s)$$

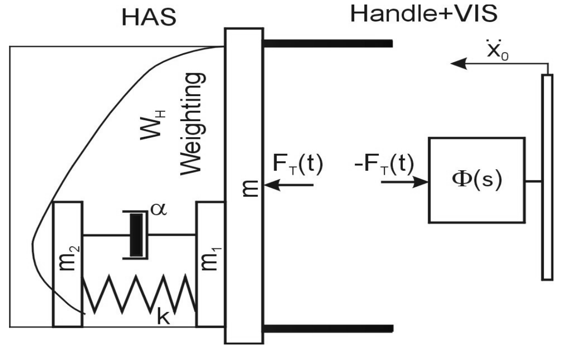

Fig. 4. The exemplary hybrid HAS model with handle and VIS

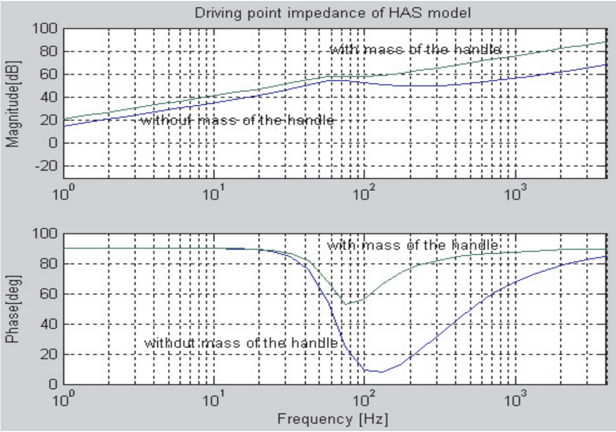

Fig. 5. The driving point impedance of Miwa's HAS

The data concerning the HAS model shown together with handle and VIS in Figure 4 are as follows: $m_1 = 0.1$ [kg], $m_2 = 0.8$ [kg], $\alpha = 250$ [Ns/m], $k = 130{\cdot}10^3$ [N/m]. Mass of handle was taken $m = 1$[kg].

Case 1: WN acceleration excitation

$$\varphi(s) = \frac{[m_1 m_2 s^2 + \alpha(m_1 + m_2)s + k(m_1 + m_2)]}{m_2 s^2 + \alpha s + k} \times \frac{H_{fb}(s)\, H_h(s)(Ks + L)}{(As^2 + Bs + C)} \tag{6.5}$$

Case 2: NBN acceleration excitation

$$\varphi(s) = \frac{[m_1 m_2 s^2 + \alpha(m_1 + m_2)s + k(m_1 + m_2)]}{m_2 s^2 + \alpha s + k} \times \frac{H_{fb}(s)\, H_h(s)(Ms^3 + Ns^2 + Ps + Q)}{(As^2 + Bs + C)(\Omega + s)} \tag{6.6}$$

Final formula for $\varphi(s)$ depends on the forms of impedance of the isolated object and weighting function $W(s)$.

7. NUMERICAL EXAMPLE

As an numerical example the optimum vibration isolation system has been calculated for hybrid double mass, weighted hand-arm model. In this example the mass of handle was neglected.

The following parameters of the power spectral density for the narrow-band excitation were assumed: $\sigma^2 = 1$ [m^2s^{-4}], $\gamma = 0.1$[s^{-1}], $\Omega = 6$ [s^{-1}] – (resonance of magnitude of $W_h(s)$) and $\Omega = 60$ [s^{-1}] – (resonance of magnitude of $Z(s)$). The numerical values of the parameters of (6.6) have been calculated on the basis of these data. In Figure 6 the magnitudes and phases of passive ($Z(s)/s$) and hybrid ($W(s)Z(s)/s$) models are shown: 1 – magnitude and phase of hybrid model of HAS. Figure 7 shows the magnitudes and phases of $\varphi(s)$ for narrow band noise excitation (NBN) and two values of λ and Ω. In Tables 1, 2 and 3 the numerical values of parameters of equations (5.22), (5.30) and (5.35) have been calculated and presented for chosen values of λ and Ω.

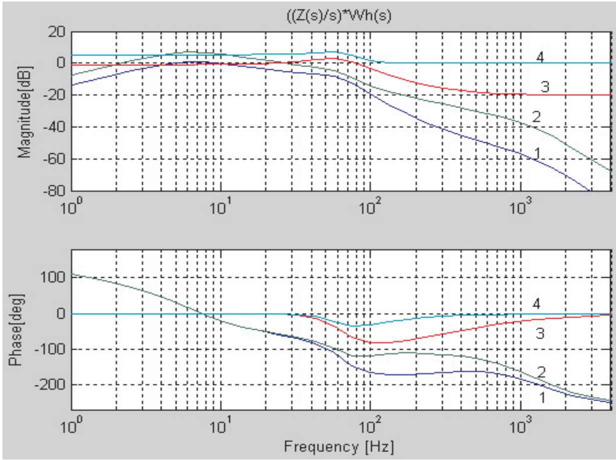

Fig. 6. The magnitudes and phases of passive ($Z(s)/s$) and hybrid ($W(s)Z(s)/s$) models: 1 – magnitude and phase of hybrid model of HAS, 2 – magnitude and phase of hybrid model of HAS including mass of the handle, 3 – magnitude and phase of passive biomechanical model of HAS, 4 – magnitude and phase of passive biomechanical model of HAS including mass of the handle

Table 1

Parameters A and B

λ [s^4]	A [s^2]	B [s]
0.0001	0.01	0.14142
10000	100	14.142

Table 2

White Noise (WN) excitation. Parameters K and L are calculated for two values of λ

λ [s^4]	K [s]	L [–]
0.0001	0.1414	1,0
10000	14,14	1,0

Table 3

Narrow-Band Noise (NBN) excitation. Parameters M, N, P and Q are calculated for pairs of parameters (Ω, λ)

Q [s^{-1}]	λ [s^4]	M [s^2]	N [s]	P [–]	Q [s^{-1}]
6	0,0001	0,0147288	0,1973831	1,84852	6
6	10000	2,3846926	0,21440804	85,852	6
60	0,0001	0,00264035	0,01727833	9,48520	60
60	10000	0,23597768	0,02138626	849,520	60

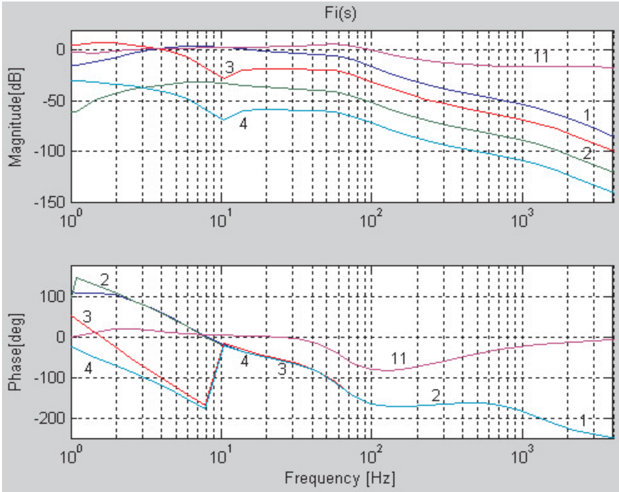

Fig. 7. Magnitudes and phases of φ(*s*) for the hybrid model of HAS and the narrow-band noise acceleration excitation: 1 – magnitude and phase of φ(*s*) calculated for the parameters shown in the first row of Table 3, 2 – magnitude and phase of φ(*s*) calculated for the parameters shown in the second row of Table 3, 3 – magnitude and phase of φ(*s*) calculated for the parameters shown in the third row of Table 3, 4 – magnitude and phase of φ(*s*) calculated for the parameters shown in the fourth row of Table 3, 11 – exemplary magnitude and phase of φ(*s*) calculated for the passive biomechanical model of HAS ($W_h = 1$) and parameters shown in the first row of Table 3

8. CONCLUDING REMARKS

The synthesis of an optimum vibration isolation system presented in this paper can be extended to an arbitrary vibration isolation HAS model described by its driving-point impedance and frequency weighting curve W_h. The final results will depend on structure of chosen passive HAS model and also on the forms of the criterion and excitation. As has been shown in Figure 7, the magnitudes of the optimal φ(*s*) for hybrid models are lower than the magnitude of φ(*s*) corresponding to passive model of HAS. It signifies that the force transmitted to the hybrid model of HAS by the optimum vibration isolation system is lower than the force transmitted to the passive model of the HAS. The differences increase with increasing values of the Lagrangian multiplier λ. Similar calculations can be led for more complex forms of criteria of vibration isolation and more complex biomechanical models.

References

Australian Standard AS 2993.1-1987, *Vibration and shock dynamic characteristics of the human body*, Part 1: *Driving Point Impedance of the Human Body*, pp. 1–14.

Basista Z. 2002, *The influence of a vibration-insulated handle on the ergonomic comfort of a percussive tool.* (in Polish), Zeszyty Naukowe Politechniki Rzeszowskiej, Nr 197, Mechanika 60, Rzeszów, pp. 19–26.

Basista Z. 2006, *Simulation investigation of an active vibration protection system of an operator of a hand-held percussive tool*. Engineering Transactions, 54, 3, Polish Academy of Science, pp. 173–187.

Basista Z., Książek M.A. 2004, *Estimation of comfort parameters of an active vibration isolation system of handle of percussive power tool*. Proceedings of the 33th International Congress Inter-Noise 2004, Prague, pp. 8–16.

Bovenzi M., Petronio L., Marimo F.Di. 1979, *Hand-Arm Vibration in Shipyard Caulkers*. International CISM-IFToMM Symposium: Man under Vibration, Suffering and Protection, Udine, Italy, April 3–6, pp. 151–165.

British Standards 6842, 1987b.

Bystroem B.O., Nilsson A., Olsson E. 1978, *Chain Saw Vibration, Investigation of Hand Vibration and Development of Mechanical Hand Model*. Technical Report 1978; 39T, Universitat Lulea.

Daikoku M., Ishikawa F. 1990, *Mechanical Impedance and Vibration Model of Hand-Arm System*. Proceedings of the 5th International Conference on Hand-Arm Vibration, Kanazawa, May 23–26, Kyoei Press Company Limited, Kanazawa, pp. 167–171.

Griffin M.J. 1990, *Handbook of Human Vibration*. Academic Press, London.

Gupta S.C., Hasdorf L. 1981, *Fundamentals of Automatic Control.* John Wiley and Sons, Inc., New York.

ISO 5349, International Standards 1986, *Mechanical Vibration – Guidelines for the Measurement and the Assessment of Human Exposure to Hand – Transmitted Vibration.*

Książek M. 1996, *Analiza istniejących modeli biomechanicznych układu ręka-ramię pod kątem wibroizolacji drgań pochodzących od narzędzi ręcznych*. Czasopismo Techniczne, Z.2-M/1966, Wydawnictwo Politechniki Krakowskiej, pp. 87–114.

Książek M.A. 1997, *Synthesis of optimal isolation systems of hand-transmitted vibration*. „Shock and Vibration", vol. 4, Issue 4, 1997, pp. 281–291, USA.

Książek M.A. 1999, *Modelowanie i optymalizacja układu człowiek – wibroizolator – maszyna.* Monografia 244, Kraków 1999, Wyd. Politechniki Krakowskiej.

Książek M., Basista Z. 1998, *Comparison of Hand-Arm Vibration Isolation Systems*. Materiały XI Międzynarodowej Konferencji Zwalczania Hałasu, Noise Control 98, 2–4 czerwca, Krynica.

Książek M.A., Tarnowski J. 2003, *Influence of frequency excitation and force pressure on the transient vibration responses of the handle – hand-arm system*. Structures – Waves – Human Health, vol. XII, No 1, Structural Acoustics and Waves in Environment, pp. 55–65.

Książek M.A., Tarnowski J., 2003, *Measurements of non-stationary vibrations of a handle – hand-arm system.* Proceedings of the 38th Conference on Human Response to Vibration, Institute of Naval Medicine, Alverstoke, Gosport, England, pp. 213–229.

Książek M. *et al.* 1995, *Analiza i optymalna wibroizolacja układu ręka-ramię*. Nowe metody eliminacji drgań przenoszonych na ludzi i konstrukcje, Monografia 185, Seria Mechanika, Wyd. Politechniki Krakowskiej, Kraków 1995, pp. 9–38.

Lee A.Y., Salman M.A. 1989, *On the design of active suspension incorporating human sensitivity to vibration*. Optimal Control Applications and Methods, vol. 10, (1989), pp.189–195

Reynolds D.D., Keith R.H. 1977, *Hand-Arm Vibration, Part I; Analytical Model of the Vibration Response Characteristics of the Hand*. Journal of Sound and Vibration, vol. 51, pp. 237–253.

Reynolds D.D., Jetzer T.C. 1999, *Use of air bladder technology in anti-vibration gloves.* Proceedings of 34th UK Group Conference on Human Response to Vibration, September 22–24, Southampton, England, pp. 153–160.

Rakheja S., Wu J.Z., Dong R.G., Schopper A.W. 2002, *A comparison of biodynamic models of the human hand-arm system for applications to hand-held power tools.* Journal of Sound and Vibration, 214, 1, pp. 165–182.