MECHANICS
Vol. 28 No. 1 2009

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MULTI-CHANNEL VIRTUAL-MICROPHONE FEEDBACK MINIMUM-VARIANCE ACTIVE NOISE CONTROL SYSTEM

SUMMARY

Classical active noise control systems are designed to minimise, in a sense, the residual signal at the microphone providing information about acoustic noise and secondary sound interference. The secondary sound operates at the same time on the acoustic noise at other positions including user ears. In the worst case this can result in sound reinforcement at those positions. In many applications placing another microphone directly at the ears is not accepted. It is then justified to make efforts to design a dedicated system. The purpose is to minimise the mean-square value of the noise at the desired location while performing measurements of sound interference results at another location. This can be done by employing the general idea of Virtual-Microphone Control systems. However, for many active noise control applications the use of a single pair of microphone and loudspeaker does not suffice to obtain satisfactory performance, i.e. generate a zone of quiet of acceptable dimensions. Moreover, for some applications, presence of an obstacle, e.g. the head in an active headrest system, constitutes a barrier for the zone of quiet at one side to propagate to the other side. Therefore, more microphones and loudspeakers are often necessary. In the most general case a coupling between subsequent pairs should be taken into account resulting in a multi-channel system. In previous papers of the author adaptive systems have been designed and analysed. In this paper a fixed-parameter multi-channel Virtual-Microphone Control system, optimal in the mean-square sense, is designed. It includes non-minimum phase property of the system channels. Both the case of more inputs than outputs as well as the case of more outputs than inputs are considered. Factorisation techniques have been used for the design. The system is verified by successfully controlling noise for the active headrest system.

***Keywords**: multi-channel control, active noise control, feedback control, optimal control, inner-outer factorization, causal-anticausal decomposition, active headrest system*

WIELOKANAŁOWY MINIMALNOWARIANCYJNY UKŁAD REDUKCJI HAŁASU Z MIKROFONAMI WIRTUALNYMI I SPRZĘŻENIEM ZWROTNYM

Klasyczne układy aktywnej redukcji hałasu projektowane są w celu minimalizacji według określonego kryterium sygnału mierzonego przez mikrofon dostarczający informacji o efektach interferencji hałasu i generowanego dźwięku wtórnego. Jednak dźwięk wtórny propaguje również w inne miejsca w przestrzeni (np. do uszu użytkownika), gdzie interferuje z obecnym tam hałasem, co może skutkować wzmocnieniem dźwięku w tamtych miejscach. W przypadku wielu aplikacji umieszczenie w nich mikrofonów nie jest możliwe. Wówczas należy zaprojektować dedykowany układ sterowania, którego celem jest redukcja dźwięku w zadanym położeniu, podczas gdy pomiar przeprowadzany jest w innym miejscu. W tym celu można wykorzystać ideę tzw. mikrofonów wirtualnych. Zastosowanie jednej pary mikrofon-głośnik często nie wystarcza do otrzymania zadowalających efektów redukcji hałasu, np. wygenerowania strefy ciszy odpowiednich rozmiarów. Ponadto, obecność przeszkód na drodze dźwięku wtórnego może uniemożliwiać właściwe działanie układu. Ma to np. miejsce w aktywnym zagłówku fotela, dla którego głowa użytkownika stanowi przeszkodę dla propagacji dźwięku do obydwu uszu. Wówczas niezbędne jest zastosowanie większej liczby mikrofonów i głośników. W ogólnym przypadku należy uwzględnić wzajemne interakcje poszczególnych torów i traktować układ, jako wielowymiarowy. W poprzednich pracach autora zaprojektowano szereg adaptacyjnych algorytmów sterowania z mikrofonami wirtualnymi. W niniejszej publikacji prezentowany jest wielokanałowy układ regulacji z mikrofonami wirtualnymi, o stałych parametrach, redukujący poziom ciśnienia akustycznego dźwięku w zadanych położeniach. Uwzględnia on nieminimalnofazowość kanałów obiektu. Rozpatrzony jest przypadek większej liczby wejść niż wyjść układu, jak również przypadek odwrotny. Przy projekcie posłużono się metodami faktoryzacji. Zaproponowany układ zweryfikowany jest w kontekście redukcji hałasu w aktywnym ochronniku słuchu.

***Słowa kluczowe**: wielowymiarowy układ sterowania, aktywna redukcja hałasu, regulacja, sterowanie optymalne, faktoryzacja, dekompozycja, aktywny zagłówek fotela*

1. INTRODUCTION

In active noise control a real microphone is usually used to provide information about results of noise reduction, i.e. the error signal. Such signal can be used to supervise adaptation of the control filter and/or be the input to a feedback controller. In many applications placing a microphone directly at the ears is not accepted. However, the zone of quiet generated around the real microphone may not satisfactorily propagate to the desired location. Moreover, sound reinforcement at such location may be perceived. To avoid this effect the general idea of Virtual Microphone Control

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(VMC) systems can be applied. They have been extensively studied by many researchers including the author for the last several years (see, e.g. (Pawełczyk 2006, Kestell *et al.* 2000, Tseng *et al.* 2002)). They are usually designed in a feedforward or feedforward-like architecture. A VMC system based on classical feedback approach has been presented in (Pawełczyk 2006). Although the structure considered in this paper is similar to that of (Tseng *et al.* 2002) the approach to control system design is completely different (Pawełczyk 2005).

Experiments demonstrate that the use of a single pair of microphone and loudspeaker does not frequently suffice to obtain satisfactory performance, i.e. generate a zone of quiet of acceptable dimension (Pawełczyk 2005, Elliott 2001). Moreover, for some applications, presence of an obstacle, e.g. the head for an active headrest system, constitutes a barrier for the zone of quiet at one side to propagate to the other side. Therefore, more microphones and loudspeakers are often necessary. In the most general case a coupling between subsequent pairs (channels) should be taken into account resulting in a multi-channel system referred to also as the multi-input multi-output (MIMO) system. In this paper a fixed-parameter optimal minimum-variance feedback VMC system is designed. The approach to minimum-variance feedback controller design using Diophantine equations is originally combined with the approach to feedforward Wiener filter design.

Let G be the number of real and virtual microphones (plant outputs), and I be the number of secondary sources (plant inputs). A sample plant with $G = I = 2$ is presented in Figure 1.

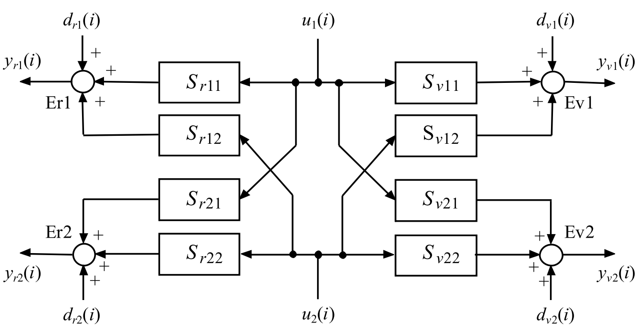

Fig. 1. A plant of two inputs, and two real and two virtual outputs

Rational transfer functions of the real and virtual paths can be grouped together in polynomial matrices, $\mathbf{S}_r(z^{-1})$ and $\mathbf{S}_v(z^{-1})$, respectively, of dimension $G \times I$, e.g.

$$\mathbf{S}_r(z^{-1}) = \begin{bmatrix} S_{r11}(z^{-1}) & S_{r12}(z^{-1}) & \cdots & S_{r1I}(z^{-1}) \\ S_{r21}(z^{-1}) & S_{r22}(z^{-1}) & \cdots & S_{r2I}(z^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ S_{rG1}(z^{-1}) & S_{rG2}(z^{-1}) & \cdots & S_{rGI}(z^{-1}) \end{bmatrix} \tag{1}$$

Estimated models will be noted with hats. Control filters are grouped together in matrix $\mathbf{W}(z^{-1})$ of dimension $I \times G$, built in a similar way to the plant matrices, so that

$$\mathbf{W}(z^{-1}) = \begin{bmatrix} W_{11}(z^{-1}) & W_{12}(z^{-1}) & \cdots & W_{1G}(z^{-1}) \\ W_{21}(z^{-1}) & W_{22}(z^{-1}) & \cdots & W_{2G}(z^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ W_{I1}(z^{-1}) & W_{I2}(z^{-1}) & \cdots & W_{IG}(z^{-1}) \end{bmatrix} \tag{2}$$

It is assumed that the distance between the real and virtual microphones is much smaller than the smallest acoustic wavelength contributing to the noise to be reduced and the distance of the overall plant to the primary source is larger than the largest acoustic wavelength contributing to the noise. Then, corresponding disturbances at the real and virtual microphones can be considered equivalent. The disturbances, if they are stochastic and wide-sense stationary, can be modelled as uncorrelated wide-sense stationary white noise sequences with unity variances filtered by a shaping filters matrix (Elliott 2001), i.e.

$$\mathbf{d}(i) = \mathbf{F}(z^{-1})\mathbf{e}(i) \tag{3}$$

The matrix $\mathbf{F}(z^{-1})$ can be found by performing spectral factorisation of the matrix of the disturbance Power Spectrum Density (PSD) (Grimble and Johnson 1988)

$$\mathbf{S}_{dd}(z^{-1}) = \mathbf{F}(z^{-1})\mathbf{F}^T(z)\big|_{z^{-1}=e^{-j\omega T_S}} \tag{4}$$

where the matrix dimension is

$$\dim\left(\mathbf{F}(z^{-1})\right) = G \times G \tag{5}$$

provided $\mathbf{S}_{dd}(e^{-j\omega T_S})$ is analytic and positive definite for all ωT_S. For notational convenience it is assumed that the shaping filters in $\mathbf{F}(z^{-1})$ have finite impulse responses (FIR structure).

All signals are grouped in corresponding vectors, e.g.

$$\mathbf{d}(i) = \left[d_1(i), d_2(i), ..., d_G(i)\right]^T \tag{6}$$

In the above equations ω is the angular frequency, T_S is the sampling period, and z^{-1} is a complex variable if present in a polynomial or transfer function, or a one-step backward time-shift operator if present in a difference equation. Te variable/operator z^{-1} will be dropped in the sequel, where it will not lead to confusion.

Let the cost function be defined as

$$L = \operatorname{trace} E\left\{\hat{\mathbf{y}}_v(i)\hat{\mathbf{y}}_v^T(i)\right\} = MSE\left\{\hat{\mathbf{y}}(i)\right\} \tag{7}$$

where $\hat{\mathbf{y}}_v(i)$ is the vector of estimates of residual signals at the virtual microphones and the *MSE* symbol has been introduced to shorten equations in the sequel. Minimisation of such function corresponds to reduction of sound pressure levels at the virtual microphones. For the optimal control system design and analysis it is assumed that the control system is linear and time-invariant.

2. MULTI-CHANNEL VIRTUAL-MICROPHONE CONTROL SYSTEM

The structure of the feedback MVC system under consideration is presented in Figure 2. Models of the real paths are used to estimate noises at the real microphones:

$$\hat{\mathbf{d}}(i) = \mathbf{y}_r(i) - \hat{\mathbf{S}}_r\,\mathbf{u}(i) \tag{8}$$

Referring to the assumption that noises at the real and corresponding virtual microphones are equivalent, the residual signals at the virtual microphones can also be easily estimated as

$$\hat{\mathbf{y}}_v(i) = \hat{\mathbf{d}}(i) + \hat{\mathbf{S}}_v\,\mathbf{u}(i) \tag{9}$$

These signals constitute also the control filter inputs. For the closed-loop system the following relation can be derived:

$$\hat{\mathbf{y}}_v(i) = -\mathbf{SW}\,\hat{\mathbf{y}}_v(i) + \mathbf{Fe}(i) \tag{10}$$

where the following notation has been introduced:

$$\mathbf{S} = \left[\hat{\mathbf{S}}_r - \mathbf{S}_r\right] - \hat{\mathbf{S}}_v = \left[\left(\hat{\mathbf{S}}_r - \hat{\mathbf{S}}_v\right) - \left(\mathbf{S}_r - \mathbf{S}_v\right)\right] - \mathbf{S}_v \tag{11}$$

Thus, the polynomial matrix **S** can be expressed in terms of modelling errors of the real paths as well as in terms of modelling errors of the difference between the real and virtual paths.

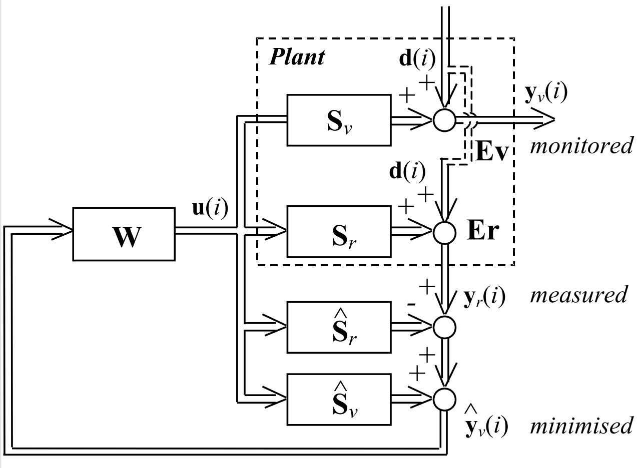

Fig. 2. The MIMO VMC system

Because control filters in matrix **W** are without delay, to the delay in each $\{j, l\}$-th element of **SW** contribute exactly all elements of the j-th row of matrix **S**. Therefore, let the following Diophantine equation be applied

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{Z} \otimes \mathbf{F}_2 \tag{12}$$

where **Z** is a matrix of backward-shift operators:

$$\mathbf{Z} = \begin{bmatrix} z^{-k_1} & z^{-k_1} & \cdots & z^{-k_1} \\ z^{-k_2} & z^{-k_2} & \cdots & z^{-k_2} \\ \vdots & \vdots & \ddots & \vdots \\ z^{-k_G} & z^{-k_G} & \cdots & z^{-k_G} \end{bmatrix} \tag{13}$$

$$k_j = \min_l k_{j\,l} \tag{14}$$

and k_{jl} is the discrete time delay of the the S_{jl}-th element of matrix **S**. This defines degrees of elements of the matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ as

$$\begin{cases} \deg F_{1,j\,l} = k_j - 1 \\ \deg F_{2,j\,l} = \dim F_{j\,l} - k_j \end{cases} \tag{15}$$

Combining (10) and (12) gives

$$\hat{\mathbf{y}}_v(i) = \left[-\mathbf{SW}\,\hat{\mathbf{y}}_v(i) + \mathbf{Z} \otimes \mathbf{F}_2\mathbf{e}(i)\right] + \left[\mathbf{F}_1\mathbf{e}(i)\right] \tag{16}$$

where the two terms in square brackets are uncorrelated. Taking into consideration that the second term cannot be controlled, minimisation of the cost function corresponds to minimisation of the first term in the mean-square sense (to avoid rewriting long equations an informal notation will be used below)

$$\min L \equiv \min MSE\left\{-\mathbf{SW}\,\hat{\mathbf{y}}_v(i) + \mathbf{Z} \otimes \mathbf{F}_2\mathbf{e}(i)\right\} \tag{17}$$

Substituting for $\hat{\mathbf{y}}_v(i)$ from (10) results in

$$\min L \equiv$$
$$\equiv \min MSE\left\{-\mathbf{SW}\left[\mathbf{I}_{GxG} + \mathbf{SW}\right]^{-1}\mathbf{Fe}(i) + \mathbf{Z} \otimes \mathbf{F}_2\mathbf{e}(i)\right\} \tag{18}$$

where $\mathbf{I}_{GxG}$ stands for a unity matrix of dimension equal G. The white noise signal vector can be omitted in the analysis because it is present at both components. Since, the cost function is defined as the mean-square value, it can receive non-negative values only. Thus, the above minimisation will give the same effect as:

$$\min L \equiv \min MSE\left\{-\mathbf{SW}\left[\mathbf{I}_{GxG} + \mathbf{SW}\right]^{-1} + \mathbf{Z} \otimes \mathbf{F}_2\mathbf{F}^{-1}\right\}$$
$$\equiv \min MSE\left\{-\mathbf{SW} + \mathbf{Z} \otimes \mathbf{F}_2\mathbf{F}^{-1}\left[\mathbf{I}_{GxG} + \mathbf{SW}\right]\right\}. \tag{19}$$

Rearranging and using the Diophantine equation again leads to:

$$\min L \equiv$$
$$\equiv \min MSE\left\{\left[(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}-\mathbf{I}_{GxG}\right]\mathbf{SW}+(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{20}$$

Simplifying $\mathbf{F}$ and $\mathbf{F}^{-1}$ gives:

$$\min L \equiv \min MSE\left\{-\mathbf{F}_1\mathbf{F}^{-1}\mathbf{SW}+(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{21}$$

Models of the acousto-electric paths are non-minimum phase including delay. Therefore, making the expression in the curly brackets equal zero would lead to non-causal- stable control filters. To avoid such problem the cost function could be extended by including weighting of control signal variance or non-minimum phase factor of the plant. Then, the overall design should be changed accordingly as presented, e.g. in (Niederliński *et al.* 1995). In this paper another approach is proposed. It takes advantage of the idea of Diophantine equation and combines it with an approach successfully used for feedforward or feedforward-like control. This approach utilises inner-outer factorization and causal-anticausal decomposition (Ahlen and Sternad 1994, Zhang and Freudenberg 1992). Moreover, for MIMO systems the cases of more plant outputs than plant inputs and vice versa should be considered separately.

2.1. Design for more plant outputs than inputs

In this subsection the case of $G \geq I$ is considered. Let the following substitution and inner-outer factorization be used (matrix dimensions are explicitly provided):

$$\mathbf{S}_1(z^{-1})_{GxI} = \mathbf{F}_1(z^{-1})_{GxG}\,\mathbf{F}^{-1}(z^{-1})_{GxG}\,\mathbf{S}(z^{-1})_{GxI} =$$
$$= \mathbf{S}_1^{(i)}(z^{-1})_{GxI}\,\mathbf{S}_1^{(o)}(z^{-1})_{IxI} \tag{22}$$

where the inner matrix satisfies

$$\left[\mathbf{S}_1^{(i)}(z)_{GxI}\right]^T \mathbf{S}_1^{(i)}(z^{-1})_{GxI} = \mathbf{I}_{IxI} \tag{23}$$

Then, (21) can be written as

$$\min L \equiv \min MSE\left\{-\mathbf{S}_1^{(i)}\mathbf{S}_1^{(o)}\mathbf{W}+(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\}$$
$$\equiv \min MSE\left\{-\mathbf{S}_1^{(o)}\mathbf{W}+\left[\mathbf{S}_1^{(i)}(z)\right]^T(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{24}$$

The control filters are required to be causal. Therefore, causal-anticausal decomposition technique should be additionally applied. Taking additionally into account that according to (22) the outer matrix is square results in a feasible sub-optimal solution:

$$\mathbf{W}_{opt+}(z^{-1}) = \left[\mathbf{S}_1^{(o)}(z^{-1})\right]^{-1}\cdot$$
$$\cdot\left\{\left[\mathbf{S}_1^{(i)}(z)\right]^T\left[\mathbf{F}(z^{-1})-\mathbf{F}_1(z^{-1})\right]\right\}_+ \mathbf{F}^{-1}(z^{-1}) \tag{25}$$

where $\{.\}_+$ stands for the causal part of {.}.

2.2. Design for more plant inputs than outputs

In this subsection the case of $G \leq I$ is considered. Let the following substitution and co-inner-outer factorization be used (matrix dimensions are explicitly provided):

$$\mathbf{S}_1(z^{-1})_{GxI} = \mathbf{F}_1(z^{-1})_{GxG}\,\mathbf{F}^{-1}(z^{-1})_{GxG}\,\mathbf{S}(z^{-1})_{GxI} =$$
$$= \mathbf{S}_1^{(co)}(z^{-1})_{GxG}\,\mathbf{S}_1^{(ci)}(z^{-1})_{GxI} \tag{26}$$

where the co-inner matrix satisfies

$$\mathbf{S}_1^{(ci)}(z^{-1})_{GxI}\left[\mathbf{S}_1^{(ci)}(z)_{GxI}\right]^T = \mathbf{I}_{GxG} \tag{27}$$

Then, (21) can be written as

$$\min L \equiv \min MSE\left\{-\mathbf{S}_1^{(co)}\mathbf{S}_1^{(ci)}\mathbf{W}+(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{28}$$

Since according to (26) the co-outer matrix is square, the following is valid:

$$\min L \equiv \min MSE\left\{-\mathbf{S}_1^{(ci)}\mathbf{W}+\left[\mathbf{S}_1^{(co)}\right]^{-1}(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{29}$$

Taking advantage of the co-inner matrix property, (27), gives

$$\min L \equiv$$
$$\equiv \min MSE\left\{-\mathbf{W}+\left[\mathbf{S}_1^{(ci)}(z)\right]^T\left[\mathbf{S}_1^{(co)}\right]^{-1}(\mathbf{F}-\mathbf{F}_1)\mathbf{F}^{-1}\right\} \tag{30}$$

The control filters are required to be causal. Thus, applying the causal-anticausal decomposition technique leads to a sub-optimal solution:

$$\mathbf{W}_{opt+}(z^{-1}) =$$
$$= \left\{\left[\mathbf{S}_1^{(ci)}(z)\right]^T\left[\mathbf{S}_1^{(co)}(z^{-1})\right]^{-1}\left[\mathbf{F}(z^{-1})-\mathbf{F}_1(z^{-1})\right]\right\}_+ \cdot \tag{31}$$
$$\cdot\mathbf{F}^{-1}(z^{-1})$$

where $\{.\}_+$ stands for the causal part of {.}.

2.3. General properties

It follows from Figure 1 that the residual signals at the virtual microphones, which are of utmost interest in this work, can be expressed as

$$\mathbf{y}_v(i) = \mathbf{d}(i) + \mathbf{S}_v u(i) \tag{32}$$

After some matrix algebra the following expression can be obtained:

$$\mathbf{y}_v(i) = \left[\mathbf{I}_G + \left(\mathbf{S} + \mathbf{S}_v\right)\mathbf{W}\right]\left[\mathbf{I}_G + \mathbf{SW}\right]^{-1}\mathbf{d}(i) \tag{33}$$

Stability of the system is determined by properties of the polynomial matrix $\mathbf{I}_G$ + **SW** and can be analysed using the criteria described, e.g. in (Maciejowski 1989).

3. EXPERIMENTAL RESULTS

The control system designed in this paper was applied for reducing a transformer noise (see Fig. 3) in an active headrest. The noise was generated by a primary loudspeaker located in front of the headrest at the distance of 4 m. The active headrest aims at creating local zones of quiet at ears of a person occupying the chair where the headrest is installed. The headrest is equipped with two secondary loudspeakers, G1 and G2, and two real microphones, Er1 and Er2, placed close to the loudspeakers. The virtual microphones are at the ears, in the distance of 150 mm from corresponding real microphones for the nominal head position (see Fig. 4). Plant paths are defined between the digital inputs to loudspeakers and sampled outputs of corresponding microphones, and they include power and voltage amplifiers, A/D and D/A converters, analogue antialiasing and reconstruction 4th order Butterworth filters with 650 Hz cut-off frequencies, as well as the acoustic field. The sampling frequency is 2 kHz. All the paths are non-minimum phase including delays of 3 samples for real paths and 4 samples for virtual paths for the nominal head position. During the experiment the head was moved to different positions as presented in Figure 4 and attenuation at the right ear was only analysed. Although the paths in matrices $\mathbf{S}_r$ and $\mathbf{S}_v$ changed significantly, unchanged models in matrices $\hat{\mathbf{S}}_r$ and $\hat{\mathbf{S}}_v$ of FIR structure with 64 parameters were used. Results of noise control for the right ear changing its position are presented in Figure 4. Similar results, not presented here, were obtained for the left ear. Distribution of the zones of quiet and attenuation levels demonstrate that the proposed control system can be successfully applied for noise control at desired locations.

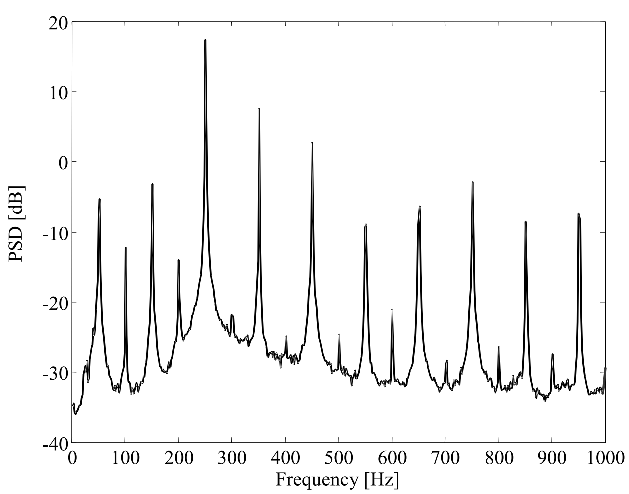

Fig. 3. PSD of the transformer noise

	3L	2L	1L	0	1R	2R	3R
f	0.6	1.6	4.4	5.1	5.8	5.0	2.7
e	0.8	1.9	4.7	5.0	9.3	4.6	2.1
d	2.3	3.1	6.0	7.5	10.1	6.9	3.0
c	3.8	4.4	8.5	14.3	16.7	8.2	4.1
b	4.7	6.3	10.4	18.2	18.1	11.5	5.0
a	4.6	7.8	8.4	8.1	7.0	3.2	2.3

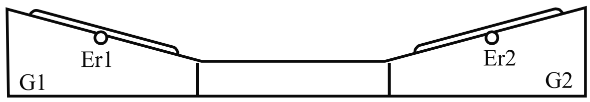

Fig. 4. Measurement scheme; each box, of dimensions 60 × 60 mm, represents the position of the centre of the head

4. SUMMARY

In this paper a feedback multi-channel minimum-variance virtual-microphone active noise control system has been designed. A combined approach has been applied which requires a Diophantine equation, spectral factori-zation of a matrix of the disturbance PSD, inner-outer factorization of a polynomial matrix, and causal-anticausal decomposition of a polynomial matrix. The cases of more plant inputs than outputs and vice versa have been consi-dered separately. As a result corresponding polynomial matrices of sub-optimal control filters have been obtained.

The control system has been applied for controlling a transformer noise in the active headrest system in order to enhance user's acoustic comfort. Obtained results have demonstrated satisfactory performance of the system.

Acknowledgement

Financial support from the state budget for science is gratefully acknowledged.

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