

## ANALYSIS OF POSITIONING ACCURACY OF THE EFFECTOR OF A TRIPLANAR MANIPULATOR

### SUMMARY

This paper presents an analysis of positioning of the effector of a spatial parallel manipulator of the triplanar type. Computer simulation software for examining positioning accuracy and the simulation results have been described herein. The paper also contains verification of the examination through stand measurements of a manipulator of a selected configuration.

**Keywords:** planar relative base, parallel manipulators, positioning accuracy

### ANALIZA DOKŁADNOŚCI POZYCJONOWANIA EFEKTORA MANIPULATORA TYPU TRIPLANAR

Artykuł prezentuje analizę dokładności pozycjonowania efektora manipulatora przestrzennego typu triplanar. Omówione zostało oprogramowanie przygotowane do wyznaczania dokładności pozycjonowania. W artykule zamieszczono również weryfikację przeprowadzonych badań na laboratoryjnym stanowisku pomiarowym z wybraną konfiguracją manipulatora.

**Słowa kluczowe:** względna płaszczyzna odniesienia, manipulator równoległy, dokładność pozycjonowania

### 1. EXAMINATION OBJECT AND DEFINITION OF EFFECTOR POSITIONING ACCURACY

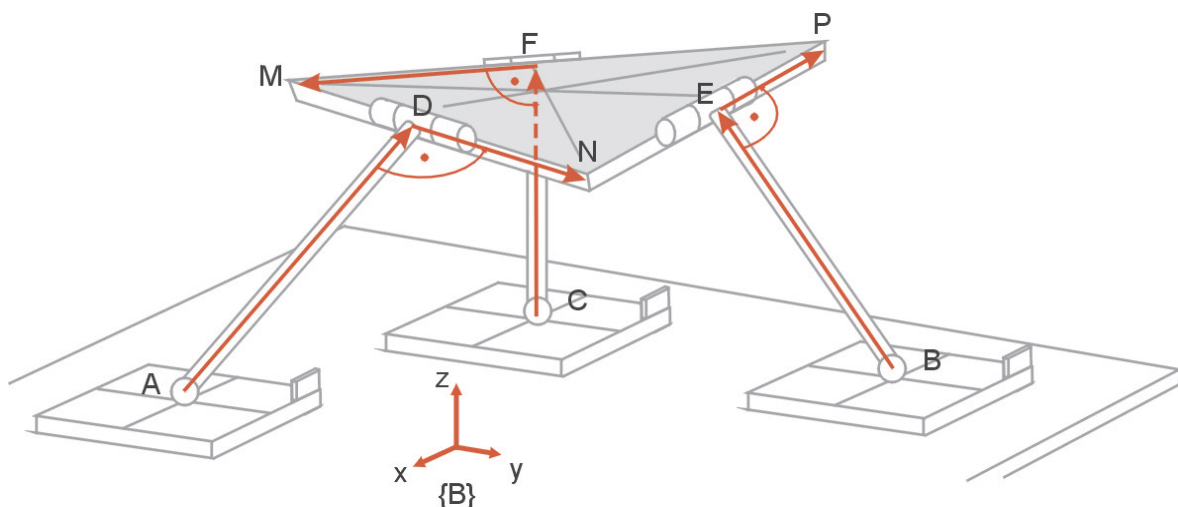
Accuracy is a measure of the manipulator's ability to attain a programmed position of the working unit. It is the difference between the programmed and actual position of the effector. The examination object was a parallel spatial manipulator of the triplanar type (Fig. 1). It is a mechanism with one passive unit having 6 degrees of freedom. The tree arms connecting the passive unit (effector) with the mobile drive bearing form – along with – the effector – a V category rotary pair V (D, E, F) and a III category spherical pair (A, B, C). Each bearing moves over a stationary base in two perpendicular axes forming a IV category planar kinematic pair.

The analysis of positioning accuracy of the drive bearings comes down to determining the effect of the effector

position changes on the bearing position change depending on the location in the working space. The accuracy of the drive bearing position changes is a function of the minimum effector position change and the manipulator's geometrical parameters.

The value of position errors in X-axis and Y-axis determined by means of partial differentiation for A bearing results from equations (1) and (2), according to Figure 1 denotations.

$$\begin{aligned} \Delta x_A = & \frac{\partial x_A}{\partial x_D} \Delta x_D + \frac{\partial x_A}{\partial y_D} \Delta y_D + \frac{\partial x_A}{\partial z_D} \Delta z_D + \\ & + \frac{\partial x_A}{\partial x_N} \Delta x_N + \frac{\partial x_A}{\partial y_N} \Delta y_N + \frac{\partial x_A}{\partial z_N} \Delta z_N \end{aligned} \quad (1)$$



Rys. 1. Triplanar type manipulator

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$$\begin{aligned} \Delta y_A = & \frac{\partial y_A}{\partial x_D} \Delta x_D + \frac{\partial y_A}{\partial y_D} \Delta y_D + \frac{\partial y_A}{\partial z_D} \Delta z_D + \\ & + \frac{\partial y_A}{\partial x_N} \Delta x_N + \frac{\partial y_A}{\partial y_N} \Delta y_N + \frac{\partial y_A}{\partial z_N} \Delta z_N \end{aligned} \quad (2)$$

where:

- $x_A, y_A$  – coordinates of drive bearing (point A) on a plane,
- $x_D, y_D, z_D$  – coordinates of the connecting point of the effector with the kinematic arm (point D),
- $x_N, y_N, z_N$  – coordinates of the effector vertex (point N).

If the required error values limits of the effector points are known, the bearing position error can be determined and so can the accuracy with which it has to be positioned for a particular manipulator configuration.

## 2. TRIPALNAR ACCURACY SOFTWARE FOR POSITION ACCURACY DETERMINING

The accuracy with which the bearing supports should be positioned depends on the manipulator's geometrical parameters, the assigned linear accuracy and the effector's orientation angular accuracy. With the assumed effector's position accuracy the error of the bearing positioning will be different in various manipulator configurations and various places of the working space. To examine the bearings positioning maximum error limit the author of this paper has developed Tripalnar Accuracy Software. The software makes it possible to examine both the positioning error for any given configuration parameter and the effect of the precision with which the manipulator's elements were made on the effector's accuracy performance.

The principal assumptions and tasks of the software:

- ability to examine the error limit of the drive bearing positioning for any given configurations of the manipulator,
- ability to examine the error for the whole range of any given parameter and for any given change of the examined parameter,
- arbitrary defining of the required effector's positioning in space,
- ability to independently define the required linear accuracy (position) and the angular accuracy (orientation) of the effector,
- arbitrary defining of the manipulator's geometrical dimensions,
- ability to examine the effect of the precision with which the effector and the unit connecting the effector with the mechatronic bases were made,
- presenting the error maximum and minimum values along with presenting a parameter value at which the extrema are reached,
- ability to export the calculated data to a text file.

According to the developed algorithm the software calculates the bearing positioning error limit for the whole range of the examined parameter (an arbitrarily selected one). For each of its values two bearing positions for extremal values of the effector's permissible positioning accura-

cy and orientation in space are calculated. The effector's linear position accuracy and its angular orientation accuracy can be presented independently. Supports position calculating is done by solving a converse problem of the manipulator's kinematics. The next step involves calculating the differences between the calculated values of the bearings positions, the value of the examined parameter being the same. Of the results obtained for the tree bearings, in two axes each, the minimum value is selected. The value is assumed to be the maximum error limit with which each bearing can be positioned lest the assumed minimum accuracy of the effector's position should be exceeded. The examinations are done for the effector position described in the global co-ordinate system by the position vector of  $(0, 0, H)$  co-ordinates, where  $H$  is the distance of the effector's center from the base. The positions relative to  $X$  and  $Y$  – axes of the global system do not affect the calculations result, therefore zero values were assigned to them. The effector's orientation is described by means of inclination angles relative to axis  $OX$  –  $\alpha$  and relative to axis  $OY$  –  $\beta$ . The angle of rotation relative to axis  $OZ$  –  $\theta$  – assumed zero. After doing calculations for the whole change range of the examined parameter, the software draws a graph and gives its maximum and minimum values along with their location. The presented above software has been used to examine the required positioning accuracy of the bearings supports of the triplanar type manipulator with various assumed accuracies of the effector in space.

## 3. EXAMINATIONS OF THE EFFECTOR POSITIONING ACCURACY

The examinations of drive bearings positioning accuracy have been done for an equilateral angle effector of 0.1 [mm] – the assumed required value of the linear accuracy of the effector position and 0.1 [°] – the angular accuracy of the effector orientation. The effect of the effector centre working point height –  $H$ , the effector side size –  $m$  and the length of the kinematic arm connecting the effector with the bases –  $k$  and the inclination angles relative to the global system of axes –  $\alpha$  i  $\beta$  on the bearings positioning error limit have been investigated. The examinations have been done for the whole range of the manipulator's working space.

### 3.1. Value of bearings positioning error limit for particular $S_H$ spaces

Examinations of angle  $\alpha$  and angle  $\beta$  changes corresponding to working space  $S_H$ ,  $\delta = f(\alpha, \beta)$  have been done for value  $H = 20, 40, 80$  and 100 [mm].

After examining the working spaces in terms of the value of the effector positioning error limit, the following conclusions have been reached:

- The value of error limit for 90% of area of a given  $S_H$  space does not exceed 50% of the assumed value of the effector position and orientation. It means that the drive bearings positioning accuracy of should be no more than twice as big as the assumed accuracy of the effector position and orientation. Assuming that the effector position accuracy should be 0.1 [mm], the bearings positioning accuracy should be 0.05 [mm].

- The value of the bearings positioning error limit increases from 2 to 4 times on the edges of the effector's working space. When the effector comes near to the limit values of the angular inclination, the accuracy with which the bearings should be positioned decreases. The study shows that the required accuracy of the bearings position should be the same or no more than twice as small as the required accuracy of the effector position. The bearings positioning precision, when the effector is on the edge of a working space, can even be twice as small as when the effector is within a working space.
- The area of the error drafted graph strictly corresponds to the area of the working zone for a given configuration of the manipulator.

The above described situations are shown in the graphs of error limits (Fig. 2).

The demonstrated properties of the positioning error limit do not depend on the manipulator geometrical parameters. The graphs are similar for various sizes of the effector ( $m$  – dimension) regardless of the length of the unit connecting the effector with a bearing ( $k$  – dimension). The changes of the  $m/k$  ratio have been found insensitive to the changes of the positioning error limit.

It is noticeable that the error value increases in the border area of the working space. The precision of bearing positioning can be smaller for the height  $H$  approaching the maximum (when  $H \rightarrow k$ ).

### 3.2. The effect of the effector size and the height of the working point on the value of positioning error limit

As a result of the examinations of the error limit in the  $m$  parameter function (effector side length) for the whole range of the working point ( $H$  parameter) a uniform course of positioning error changes have been noted. It is noticeable that the positioning error value for a given  $H$  height maintains a constant value for the whole range of  $m$  parameter changes.

The error limit value is bigger for manipulators of longer units connecting the effector with the manipulator ( $k$  parameter). The conclusion can be drawn that the bigger the height of the kinematic arm is, the easier the effector positioning in space is. The graphs in Figure 3 show the described error dependence in the function of  $m$  and  $H$  parameters ( $\delta = f(m, H)$ ).

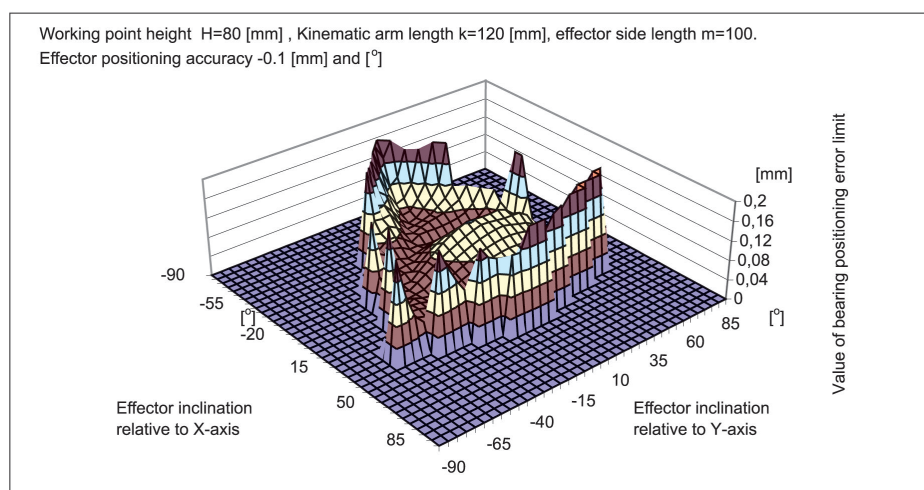


Fig. 2. Graph of bearing positioning error limit in the whole range of angle  $\alpha$   $\beta$  inclination for working point  $H = 80$  [mm],  $m = 100$  [mm],  $k = 120$  [mm]

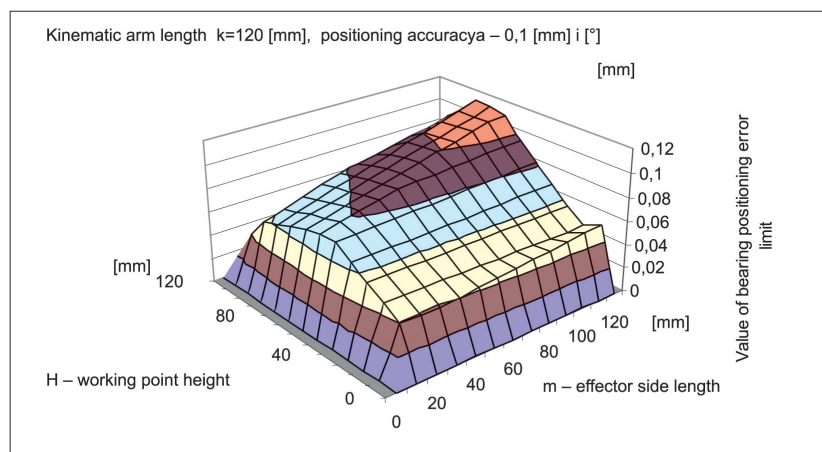


Fig. 3. Value of positioning error limit in the whole range of  $m$  and  $H$  parameters changes

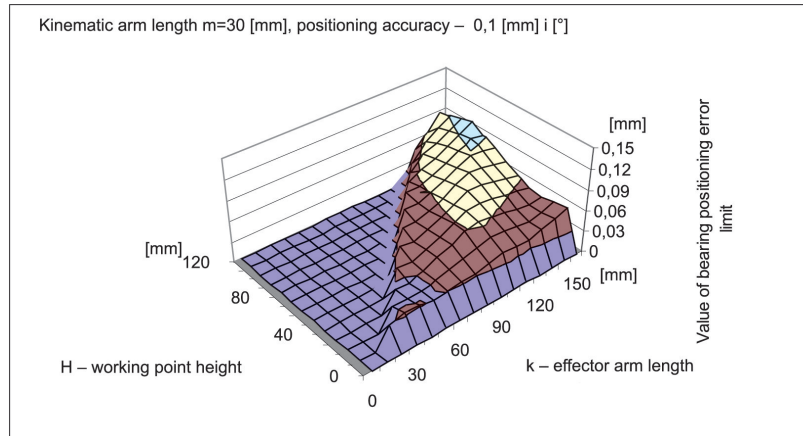


Fig. 4. Value of bearing positioning error limit depending on  $k$  length in the whole range of  $H$  height changes

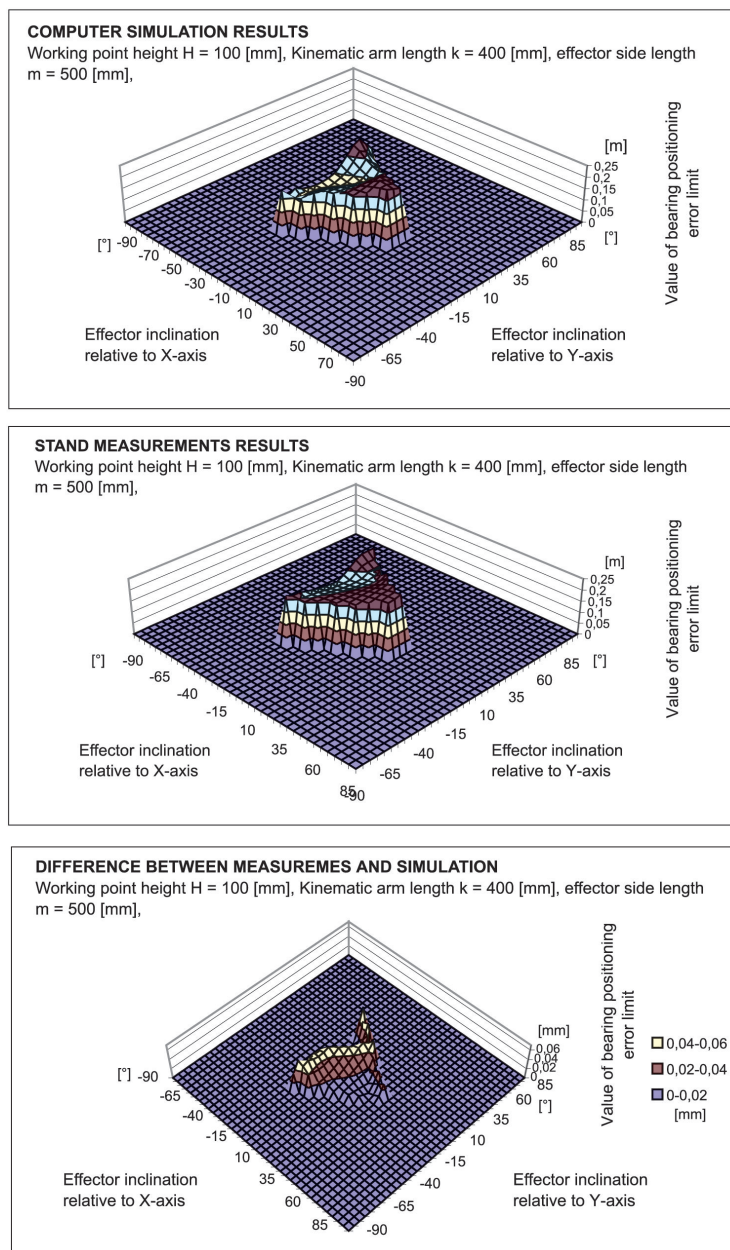


Fig. 5. Results of drive bearing positioning error limit determined by means of a) computer simulation; b) stand measurements and c) their differences for the examined height of the working point  $H = 100$  [mm]

### 3.3. The effect of the kinematic arm length and the working point height on the error limit value

The graphs in Figure 4 show the values of the manipulator drive bearing positioning error limit depending on the length of the unit connecting the effector with a support ( $k$  parameter) in the whole range of the effector working point height changes ( $H$  parameter) ( $\delta = f(k, H)$ ). They also present the examination results for a manipulator of two different effector side lengths (for  $m = 30$  [mm] and  $m = 100$  [mm]).

The graphs show that as the height of the effector operations increases, the value of bearing positioning error increases, too. The effect is even more noticeable when the measurement point approaches the border of the working space.

The above accuracy examinations allow forming the conclusion that the precision of the manipulator bearings positioning is relatively uniform and must be twice as big as the assumed required accuracy of the effector position description in space when the effector operates within the working space. When it approaches the working space border, the change of the bearings position has a smaller and smaller influence on the effector position. Manipulating can be less precise on the edges of the working space and yet the effector will be placed within the required positional tolerance.

### 4. COMPARISON OF SIMULATION WITH STAND MEASUREMENTS OF BEARING POSITIONING ACCURACY

Figure 5 shows graphs of differences in accuracy error measurements for the particular effector centre heights obtained as a result of computer simulation and stand examinations of positioning accuracy for a selected manipulator configuration.

The difference in the error value is bigger in the centre area of the working space and smaller on its edges. It results from the fact that the defects in manipulator structural elements engineering and the clearance in the kinematic pairs on the edges neutralize each other when the elements take

their edge /border positions. The difference is bigger inside the space where the clearances do not only neutralize each other but also add up.

The stand measurements have confirmed that the biggest error limit is within the borders of the working space and in the upper range of the effector working point height and it becomes smaller as the effector working point height diminishes when it approaches the inside of the working space  $S_H$ .

### 5. SUMMARY

Error limit determined by means of stand measurements has always been bigger than the one calculated by means of computer simulation. It means that if the simulation calculated minimum error limit is observed while manipulator's operating, it is certain that the effector will always fit in with the assigned area of positional tolerance in real operations.

The analysis of the obtained results has made it possible to develop working spaces of the manipulator in which the value of positioning error is approaching the assumed one. Further the spaces have allowed optimizing the algorithms of mechatronic bearing steering on a plane. The research into the effect of manipulator elements positioning accuracy allows developing optimal guidelines for constructing the triplanar type manipulators designed to operate in a complicated working space.

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