MECHANICS
Vol. 27 No. 4 2008

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MODELING AND SIMULATION OF PIEZOELECTRIC ELEMENTS – COMPARISON OF AVAILABLE METHODS AND TOOLS

SUMMARY

The paper presents an overview of modeling techniques of piezoelectric elements and a comparison of a software for simulation mechanical and electro-mechanical systems with a piezoelectric transducer. The described models are applied to simulate a two degrees-of-freedom mechanical system with passive damping, a two-layer and multi-layer piezoelectric bimorph. The results of simulations preformed by the use of the presented software packages are being discussed.

***Keywords**: piezoelectrics, smart materials, electro-mechanical systems, mechatronics, virtual prototyping*

MODELOWANIE I SYMULACJA ELEMENTÓW PIEZOELEKTRYCZNYCH – PORÓWNANIE DOSTĘPNYCH METOD I NARZĘDZI

W pracy dokonano przeglądu metod modelowania elementów piezoelektrycznych. Porównano oprogramowanie umożliwiające symulację układów mechanicznych i elektryczno-mechanicznych zawierających przetwornik piezoelektryczny. Omówione modele użyto do symulacji układu o dwóch stopniach swobody z biernym tłumikiem drgań, dwuwarstwowej i wielowarstwowej belki piezoelektrycznej. Przeprowadzono dyskusję wyników symulacji wykonanych z użyciem zaprezentowanego oprogramowania.

***Słowa kluczowe**: piezoelektryki, materiały inteligentne, układy elektryczno-mechaniczne, mechatronika, wirtualne prototypowanie*

Piezoelectric elements are commonly use in mechatronic products design. To make integration of such elements into a product easier, a design procedure including virtual prototyping of piezoelements integrated with a structure should be developed. It allows inexpensive and fast tests on alternative solutions of the product being designed to be carried out without the need to build a physical prototype. This technique allows the creation of a better product by performing more iterations of several design solutions, which helps to make the design phase for the product shorter.

The mechatronic design approach is focused on the parallel development of individual elements of the structure, which each have different physical natures. This fact creates special requirements for the simulation of an interdisciplinary system. Cosimulating these pieces together with different levels of accuracy enables the checking of energy and information flow between them and helps to prevent incompatibilities that may occur during the synchronous design process. Thus techniques that enable simultaneous simulation of different domains of the designed system are very important in the mechatronic approach. In this paper the simulation software packages enabling the simulation of piezoelements behavior are listed and tested on a simple example.

1. INTRODUCTION

The piezoelectric phenomenon was discovered in 1880 by the Curie brothers in some naturally occurring materials. However, the first serious application of piezoelectrics appeared during World War I, when the submarine detector was built. Following this successful product, piezoelectric crystals were used to create ultrasonic transducers, sonars and other applications as microphones and accelerometers. The development of piezoceramics during and after World War II extended the application of piezoelements. Piezoceramics were discovered as side-effect of developing materials with very high dielectric constants for the construction of capacitors. Man-made polycrystalline ceramic materials can be processed to exhibit significant piezoelectric properties stronger than those of naturally occurring materials (Moheimani and Fleming 2006). The strong coupling between the electrical and mechanical domains made piezoceramics widely used as sensors and actuators.

Nowadays, piezoelectric elements find applications in precise positioning systems. The Canon Company incorporates ultrasonic motors for automatic focusing mechanisms in their cameras. Instead of classical mechanical spring type camera shutters, Minolta uses a piezo-bimorph solution (Uchino 1994).

Piezoelectrics are widely used in hard disc drive systems, as a secondary actuation mechanism beside the voice coil motor (Hatch 2006). Micro-robotic designs incorporate piezoelectric actuators (Smiths and Dalke 1989).

Piezoceramics find applications in servo-valves. In common-rail systems they enable fine electronic control over the injection time and amount of fuel which is put into cylinder in combustion engine (Kneba and Makowski 2004). High bandwidth enables up to five injections per stroke. The Epson company developed the variable droplet technology that incorporates micro-piezoelements in their inkjet print heads. Thanks to precise voltage control of the valve, different sizes of droplets can be delivered, optimized for the detail in print (EPSON, Mirco Piezo Technology 2008).

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A wide application area for piezoelectric materials is vibration control. There are two approaches to this solution. The first is active damping, where part of the piezo elements attached to structure is performing sensing activity and the others are actuators. Active damping requires a closed loop controller and power supply (Moheimani and Fleming 2006). The other approach is passive damping, also called shunt damping, where the piezoelectric is a transducer that transforms part of the mechanical vibration energy into electrical energy that can be conveniently dissipated in circuit (Moheimani and Fleming 2006).

These solutions cover things from space and military applications to consumer products. Vibration control is used in military aircraft like the F-15 and FA-18, to prevent mechanical fatigue and improve flight parameters (Moheimani and Fleming 2006), Boeing's Smart Helicopter Rotor to reduce vibration and noise and improve aerodynamic performance (Boeing 2007).

2. OVERVIEW OF PIEZOMATERIAL MODELING TECHNIQUES

There are many different methods of modeling of piezoelements. Among them the following should be listed:

- equivalent circuit,
- spring model,
- thermal analogy,
- finite element method.

2.1. Equivalent circuits for piezoelectric transducers

The simplest model of a piezoelectric transducer for the thickness mode vibration is described in (Żyszkowski 1984) and can be obtained direct from the description of phenomena observed by the Curie brothers. They discovered that difference of potential is proportional to the strain of a piezoelectric crystal (Curie and Curie1880).

$$U = -kx \tag{1}$$

where:
U – voltage [V],
k – piezoelectric constant [V/m],
x – displacement [m].

Taking in account that sum of delivered and received mechanical and electrical energy in the system have to be equal zero: $Uq + Fx = 0$, thus $F = -(Uq)/x$ and taking into consideration (1) we get the following equation:

$$F = -kq \tag{2}$$

where:
F – force [N],
q – electric charge [C].

If all the excitations in equations (1) and (2) are harmonic the equations become:

$$U = \frac{-k}{j\omega} v \tag{3}$$

$$F = \frac{k}{j\omega} I \tag{4}$$

Considering the piezoelectric transducer as a T-shape four terminal described by matrix

$$\begin{bmatrix} U \\ F \end{bmatrix} = \begin{bmatrix} Z_{e0} & -Z_{em} \\ Z_{em} & -Z_{m0} \end{bmatrix} \begin{bmatrix} I \\ v \end{bmatrix} \tag{5}$$

where:
Z_{e0} – electrical impedance [VA^{-1}],
Z_{m0} – mechanical impedance [kgs^{-1}],
Z_{em} – electrical-mechanical coupling impedance [Vsm^{-1}].

The equivalent circuit for piezoelectric transducer can be shown as on Figure 1 (Curie and Curie 1980).

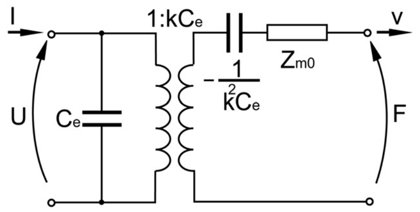

Fig. 1. Equivalent circuit for a piezoelectric transducer described by equations (3), (4) and (5) (Żyszkowski 1984)

A more precise equivalent circuit for a thickness mode piezoelectric transducer and the derivation of its model can be found in (Leech 1994). It describes the transducer as solid and is based on an analogy between Newton's law for continuous materials and transmission line equations. Thus the disc mode piezoelectric transducer can be modelled using the equivalent circuit shown on Figure 2.

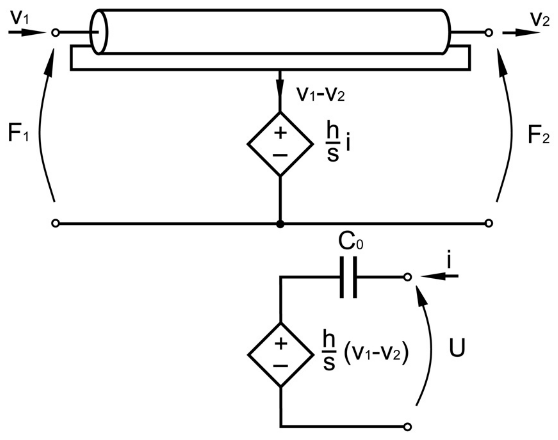

Fig. 2. Circuit model of a piezoelectric thickness mode transducer (Leech 1994)

The above mentioned models require the conversion of the mechanical part of the system into an electric circuit using electro-mechanical analogies. It allows the whole electro-mechanical system to be analyzed using circuit theory based methods.

2.2. Modeling a piezoelectric transducer as a mechanical system

The diagram of this model is shown in Figure 3. Where x_1 and x_2 are the displacement of lumped masses m_1 and m_2 respectively, k is the spring, F is the force exciting the mechanical part of the system, R is resistance, and L is inductance of electrical circuit, between mass m_1 and m_2 is piezoelectric transducer.

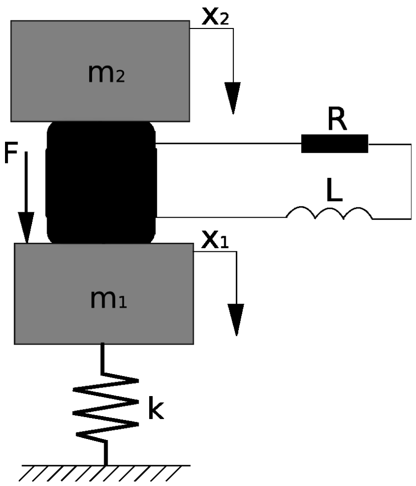

Fig. 3. Two degree of freedom mechanical system with piezoelectric element

Using constitutive equation for a linear piezoelectric material (ANSI/IEEE std 176 1987)

$$T = c^E S - eE \tag{6}$$

$$D = eS + \varepsilon^{\varepsilon} E \tag{7}$$

where:

T – stress [MPa],
S – strain,
E – electric field [V/m],
D – electric displacement [C m^{-2}],
c^E – elastic stiffness for constant electric field [MPa],
e – piezoelectric constant [C m^{-2}],
$\varepsilon^{\varepsilon}$ – dielectric permittivity for constant strain [F m^{-1}],

substituting:

$$T = \frac{-F_p}{A}, \quad U = \int_0^h E dz, \quad q = \int_A D dA, \quad S = \frac{\Delta x}{h} \tag{8}$$

where:

A – surface area of electrodes,
h – distance between electrodes,

into (6) and (7) , defining:

$$EM = \frac{e}{S}, \quad k_p = \frac{A}{h}\left(c + \frac{e^2}{S}\right) \tag{9}$$

$$C_0 = \frac{SA}{h} \tag{10}$$

and solving in terms of U and F gives the formulas describing the voltage generated by the piezoelectric, which is described as a voltage source and capacitance C_0 (10) and the force with which the piezoelement affects the mechanical part of the system, respectively (6) and (7).

EM and k_p (9) describe the electromechanical coupling coefficient and stiffness of piezoelectric respectively.

$$U = \frac{q}{C_0} - EM\Delta x \tag{11}$$

$$F_p = -k_p \Delta x + EMq \tag{12}$$

including (11) and (12) in the equations of motion derived for the system shown in Figure 3 the following state space equations are obtained:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-1}{C_p L} & \frac{-R}{L} & \frac{EM}{L} & 0 & \frac{-EM}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{EM}{m_1} & 0 & \frac{-k_p}{m_1} & 0 & \frac{k_p}{m_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-EM}{m_2} & 0 & \frac{k_p}{m_2} & 0 & \frac{-k+k_p}{m_2} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \tag{13}$$

$$\begin{gathered} \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{bmatrix}^T = \\ = \begin{bmatrix} I & q & v_1 & x_1 & v_2 & x_2 \end{bmatrix}^T \end{gathered} \tag{14}$$

The state space equations are very useful for the simulation and development of control system.

2.3. Modeling of piezoelectric bimorphs

The derivation of the model for a piezoelectric bender is described in (Smith and Dalke 1989). This is the constitutive equation for an antiparallel bimorph. Two piezoelectric elements are joined over their entire length and their polarizations are antiparallel to one another. An electric field is applied across the entire beam. The mechanical boundary conditions are: a mechanical moment M applied to the end of the beam and a force F applied perpendicularly to the tip of the beam. The displacement and the rotation are obtained for the free end of the bender.

The constitutive equations are:

$$\begin{bmatrix} \alpha \\ \delta \\ q \end{bmatrix} = \begin{bmatrix} \frac{3s_{11}^E L}{2wh^3} & \frac{3s_{11}^E L^2}{4wh^3} & \frac{-3d_{31}L}{4h^2} \\ \frac{3s_{11}^E L^2}{4wh^3} & \frac{3s_{11}^E L^3}{2wh^3} & \frac{-3d_{31}L^2}{8h^2} \\ \frac{-3d_{31}L}{4h^2} & \frac{-3d_{31}L^2}{8h^2} & \frac{Lw\varepsilon_{33}^T}{2h}\left(1-\frac{k_{31}^2}{4}\right) \end{bmatrix} \begin{bmatrix} M \\ F \\ V \end{bmatrix} \tag{15}$$

where:

α – rotation angle of the free end,
δ – displacement of the free end,
L – length of the beam [m],
w – width of the beam [m],
h – thickness of the beam [m],
k_{31} – piezoelectric coupling coefficient.

The model of a piezoelectric bimorph given by (15) allows simple calculation of the rotation angle, displacement of the free end and electrical charge. This model is an algebraic equation and does not describe the dynamics of the system.

2.4. Using thermal analogy for the modeling of piezoelements

The thermal analogy approach allows piezoelectric actuators to be modelled using classic FEM software like MSC.Nastran. The constitutive equation for the inverse piezoelectric effect is similar to the constitutive equation for the thermoelastic effect if the applied electric field is modeled as a thermal load (Dong and Meng 2004).

The inverse piezoelectric effect can be described by the following equation:

$$[T] = [c^E][S] - [e][E] \tag{16}$$

where the mechanical variables T and S denote stress and strain vector respectively, E is the electric field vector, c^E is the elastic stiffness coefficient matrix and e is the piezoelectric stress coefficient matrix.

Equation (16) can be rewritten as:

$$[T] = [c^E]([S] - [d][E]) \tag{17}$$

where d is the piezoelectric strain coefficient matrix.

If we consider the PZT material polarized along z-axis or 3-axis the piezoelectric constants matrix can be written as:

$$[d] = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

For the extension actuation, the electric field vector and polarization of the piezoelectric material are parallel, E_1 and E_2 are zero so the constitutive equation can be simplified to:

$$[T] = [c^E]\Big([S] - [d_{31} \quad d_{31} \quad d_{33} \quad 0 \quad 0 \quad 0]^T E_3\Big) \tag{19}$$

For the shear actuation, the electric field vector and poling direction of the piezoelectric material are perpendicular, i.e. E_1 and E_3 are zero so the constitutive equation can be simplified to:

$$[T] = [c^E]\Big([S] - [0 \quad 0 \quad 0 \quad d_{15} \quad 0 \quad 0]^T E_2\Big) \tag{20}$$

Above-mentioned equations (19), (20) are similar to MSC.Nastran MAT9 material (MSC.Nastran Manual), defined as:

$$[T] = [c^E]\Big([S] - (\tau - \tau_{REF})[A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6]^T\Big) \tag{21}$$

where τ denotes the temperature excitation, τ_{REF} is the reference temperature and A_x denotes the thermal expansion coefficients.

For the extension actuation comparing (19) with (21) and assuming that $\tau_{REF} = 0$ gives:

$$A_1 = A_2 = d_{31}; \quad A_3 = d_{33}; \quad \tau = E_3$$

and for shear actuation comparing (20) with (21) and assuming that $\tau_{REF} = 0$ gives:

$$A_1 = A_2 = A_3 = A_5 = A_6 = 0; \quad A_4 = d_{15}; \quad \tau = E_2.$$

Piezoelectric actuators are driven by a voltage source. Assuming that material is uniform, the electric field between electrodes is constant: and is given by equation:

$$E = \frac{U}{x} \tag{22}$$

where E is the electric field, U is the potential difference between electrodes, and x is the distance between electrodes.

It is worth noting that the thermal analogy allows only piezoelectric actuators to be modelled.

It is not possible to simulate the behavior of transducers using this method.

2.5. Modeling piezoelectrics using multiphysics FEM

A description of Finite element matrix formulation can be found in (Ansys.Multiphysics Manual; Abaqus Manual; Kagawa *et al.* 1996).

Establishing nodal solution variables and element shape functions over an element domain, which approximate the solution, leads to finite element discretization.

$$u_c = \left[N^u\right]^T u \tag{23}$$

$$V_c = \left[N^V\right]^T V \tag{24}$$

where:

u_c – displacement within element domain in the x, y, z dimensions,
u – vector of nodal displacement,
N^u – matrix of displacement shape functions,
V_c – electrical potential within element domain,
V – vector of nodal electrical potential,
N^V – matrix of electrical potential shape function.

Strain and electric field are related to displacement and electric potential respectively.

$$S = [B_u]u \tag{25}$$

$$E = [B_V]V \tag{26}$$

where:

$$B_u = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \tag{27}$$

$$B_V = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \left[N^V\right]^T \tag{28}$$

After the application of the variational principle finite element discretization, the coupled element matrix equation derived for a one element model is:

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{bmatrix} [\ddot{u}] \\ [\ddot{V}] \end{bmatrix} + \begin{bmatrix} [C] & [0] \\ [0] & [0] \end{bmatrix} \begin{bmatrix} [\dot{u}] \\ [\dot{V}] \end{bmatrix} + \\ + \begin{bmatrix} [K] & \left[K^Z\right] \\ \left[K^Z\right]^T & \left[K^d\right] \end{bmatrix} \begin{bmatrix} [u] \\ [V] \end{bmatrix} = \begin{bmatrix} [F] \\ [q] \end{bmatrix} \tag{29}$$

where:

$[M]$ – mass matrix,
$[C]$ – damping matrix,
$[K]$ – structural stiffness matrix,
$[K^d]$ – dielectric matrix,
$[K^Z]$ – piezoelectric coupling matrix,
$[F]$ – vector of nodal forces, structural forces and body forces,
$[q]$ – vector of nodal charges, surface charges and body charges.

These models are implemented in commercially available software.

3. SOFTWARE FOR SIMULATION OF MODELS OF PIEZOELEMENTS

On the software market there is not much software which allows direct simulation of piezoelectric transducers. Multiphysics FEM software products such as ANSYS. Multiphysics developed by ANSYS Inc. and Abaqus developed by SIMULA have this function.

For simulation of the piezoelectric phenomenon ANSYS has predefined element types PLANE13, PLANE233 – 2D solid elements, 4-node quadrilateral and 8-node quadrilateral respectively, SOLID5, SOLID98, SOLID226, SOLID227 – 3D solid elements 8-node brick, 10-node tetrahedron, 20-node brick and 10-node tetrahedron respectively. The electromechanical constitutive equations for linear model behavior are the same as (6) and (7). The matrices of the model containing piezoelectric elements are given by (29). For proper definition of the material it is necessary to give the piezoelectricity matrix, the electrical permittivity matrix and the anisotropic stiffness matrix. The matrices describing material properties in ANSI/IEEE standard of piezoelectricity need to be reordered. The order of rows and columns in ANSYS matrices in tensor notation is 11, 22, 33, 12, 23, 13. The ANSYS element library also contains the predefined element type CIRCU94. It allows simulation of a linear electrical circuit attached to piezoelectric transducer. This element, depending on the chosen parameters, can describe the behavior of a resistor, inductor, capacitor, voltage or current source. Possible analysis types for models containing piezoelectric elements are static, modal, prestressed modal, harmonic, prestressed harmonic and transient. The applicable analysis types for electric circuits are only full harmonic and transient (Ansys Multiphysics Manual).

Similar to ANSYS, Abaqus has also predefined 2D and 3D solid element types that allow simulation of piezoelectric materials. There are 5 two-dimensional different constant strain and constant stress element types containing 3, 4, 6 and 8 nodes and there are 7 three-dimensional elements containing 4, 6, 8, 10, 15 and 20 nodes.

The constitutive equations for the piezoelectric model are the same as given by (6) and (7) and the system equations for the model containing piezoelectric elements are similar to (29), but excluding the first time derivative part – structural damping. To define material properties it is essential to give the piezoelectric matrix and the electrical permittivity matrix and the stiffness matrix. The order of rows and column in Abaqus material properties matrices in tensor notation is 11, 22, 33, 12, 13, 23 and is different from the IEEE standard of piezoelectricity. It is not possible to model a full electro-mechanical system in Abaqus software, because it does not support simulation of electric circuits. The options available for analysis of models containing piezoelectric elements are static stress analysis, transient analysis, harmonic analysis and modal analysis. Piezoelectric elements can be used in general analysis steps, including both geometrically linear and nonlinear analyses (in nonlinear analysis, however, the piezoelectric constitutive response remains linear) (Abaqus Manual) The advantage of Abaqus software is a user friendly interface similar to CAD software. Every parameter of creating model can be easily found in the model tree.

In the previous section useful techniques for the simulation of piezoelectrics without using multiphysics FEM programs were reviewed.

The thermal analogy allows us to model a piezoelectric actuator. Thermal strain analysis is available in MSC.Nastran. This program is the oldest FEM software and is widely used in the automotive and aerospace industries (MSC.Software 2008).

Using equivalent circuits for the piezoelectric transducer and an electro-mechanical analogy for mechanical systems, the whole electro-mechanical system can be simulated in electrical CAD software such as PSpice. PSpice allows DC (static) analysis, AC (harmonic) analysis and transient analysis to be performed (Izydorczyk 1993). This program is dedicated to simulating sophisticated electrical and electronic circuits. There are many libraries containing models of electronic devices that can be easily included in the model, so all electrical or electronic parts of electro-mechanical system can be easily and precisely simulated. The main disadvantage is the necessity to convert the mechanical part of system into and electrical circuit, which is time consuming and error-prone.

Electro-mechanical systems can be also modeled in the multi-purpose simulation environment MATLAB-Simulink. The approach described in 2.2 allows the state equations of the whole system to be built. This form can be easily implemented in MATLAB-Simulink. The major advantages of using this software are a user friendly interface, a wide range of possibilities in terms of visualization results, the simple and fast development of sophisticated control algorithms, and signal processing algorithms using implemented, ready to use toolboxes provided with the software (Mathworks 2008) and the availability of many third party toolboxes. The main disadvantage is the need to manually write the state equations of the system. For large electromechanical systems this can be difficult, error-prone and time consuming.

4. COMPARISON OF SIMULATION RESULTS

To illustrate an application of the models that were described in paragraph 2, six simulations of two different mechanical systems with piezoelectric transducer are given here.

Simulations were performed of a simple mechanical system containing a piezoelectric transducer, presented on Figure 3, and FEM simulations of a piezoelectric bimorph fixed at the end and excited by a potential difference, shown on Figure 15.

The first three show results of the frequency response and time response of the system shown in Figure 3. This system has two lumped masses and a piezoelectric transducer connected to the electric circuit between them. Every simulation was performed with exactly the same parameters for the mechanical part (both masses are 2 kg) of the system and the same value of resistance (1 kΩ) in the electrical circuit. The difference is the model of the piezoelectric transducer, and inductance in electric circuit computed from:

$$L = \frac{1}{\omega^2 C_p} \tag{30}$$

where:

ω – resonance frequency of mechanical part and open-circuited piezoelectric transducers,
C_p – capacitance of piezoelectric transducer,

and the frequency of the harmonic force excited in time analysis, which is equal to the resonance frequency of the whole electro-mechanical system. For simulation purposes PZT4 piezoelectric material reduced to two dimension was used. It was assumed that the material is polarized perpendicularly to *Y* axis, the stiffness matrix *c* [N m^{-2}], piezoelectric matrix *e* [C m^{-2}] and relative permittivity matrix *g* [n/n] are given respectively as:

$$c^E = \begin{bmatrix} 13.2 & 7.2 & 7.1 & 0 \\ & 11.5 & 7.3 & 0 \\ & & 13.2 & 0 \\ & & & 3.0 \end{bmatrix} 10^{10};$$

$$e = \begin{bmatrix} 0 & -4.1 \\ 0 & 14.1 \\ 0 & -4.1 \\ 10.5 & 0 \end{bmatrix}; \quad g = \begin{bmatrix} 804 & 0 \\ 0 & 656 \end{bmatrix}.$$

The other three simulations are static analyses made in FEM software of a piezoelectric bimorph excited by a voltage shown of Figure 15. The beam dimensions are: 2×8×50 mm. For these simulations PZTS1 material was used. It is polarized in *Z* axis, the stiffness matrix *c* [N m^{-2}] piezoelectric matrix *e* [C m^{-2}] and relative permittivity matrix *g* [n/n] are given respectively as:

$$c^E = \begin{bmatrix} 16.8 & 11.1 & 10.1 & 0 & 0 & 0 \\ & 16.0 & 10.1 & 0 & 0 & 0 \\ & & 12.3 & 0 & 0 & 0 \\ & & & 3.0 & 0 & 0 \\ & & & & 3.0 & 0 \\ & & & & & 2.8 \end{bmatrix} 10^{10};$$

$$e = \begin{bmatrix} 0 & 0 & -2.80 \\ 0 & 0 & -2.80 \\ 0 & 0 & 14.72 \\ 0 & 9.84 & 0 \\ 9.84 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$g = \begin{bmatrix} 1130 & 0 & 0 \\ 0 & 1130 & 0 \\ 0 & 0 & 914 \end{bmatrix}.$$

4.1. Simulation of piezoelements in MATLAB-Simulink environment

The Simulink model for state equations given by (13) is shown in Figure 4. Red connections are the displacement of mass m_1, green are the displacement of mass m_2 and blue are the charge in the circuit. The model of the piezoelectric transducer exclude the mass of the object, thus the singular mass of transducer is equally distributed across the lumped mass part of the mechanical system.

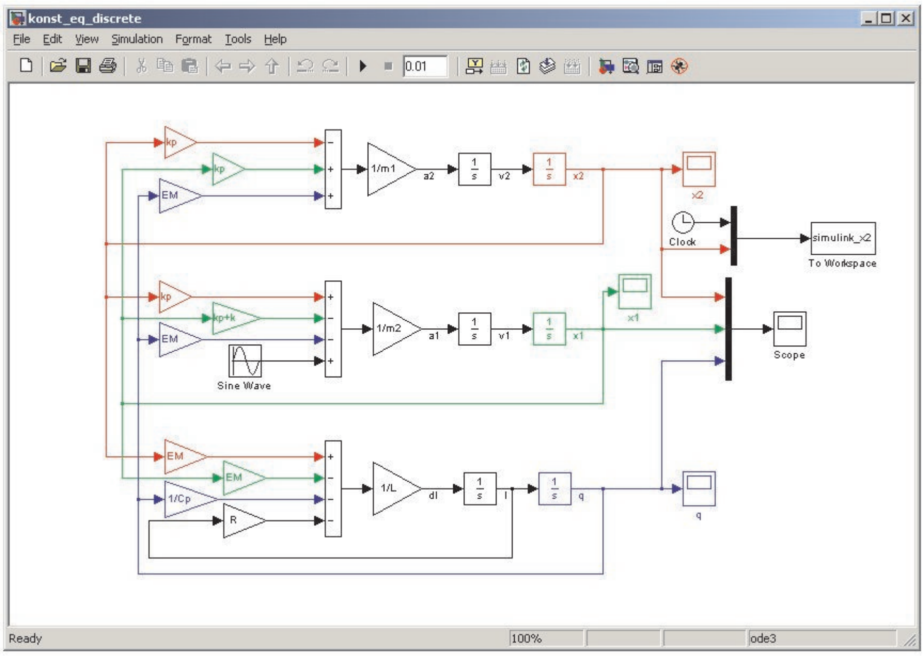

Fig. 4. Simulink model of electro-mechanical system with piezoelectric transducer

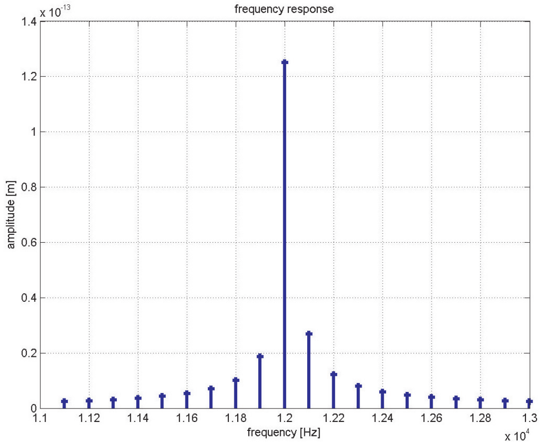

Fig. 5. Displacement of mass m_2 vs frequency for mechanical part of the system

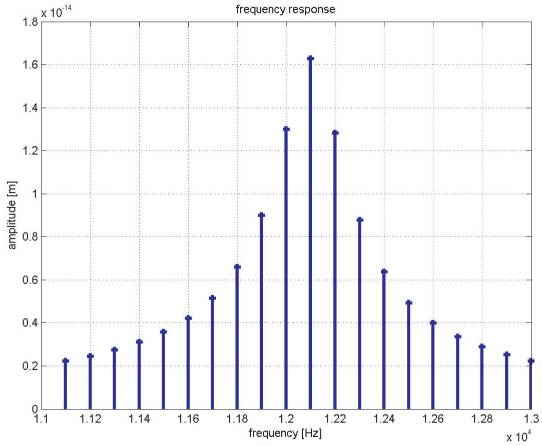

Fig. 6. Displacement of mass m_2 vs frequency for full electro-mechanical system

The harmonic response of the system was computed as a Fourier transform of the impulse response of the system. Figures 5 and 6 show the harmonic response of the mechanical part of the system, where EM – coupling coefficient is zero, and for the full system respectively.

Figure 7 shows the time response of the system excited by a harmonic force with amplitude 1 N and frequency 1.2 kHz.

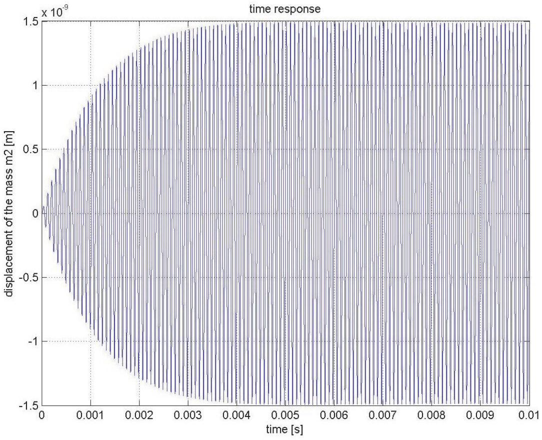

Fig. 7. Displacement of mass m_2 vs time for full electro-mechanical system, excited by harmonic force, frequency 12 100 Hz and amplitude 1 N

4.2. Simulation of the piezoelement model based on electro-mechanical analogies as an equivalent circuit in PSPICE

The equivalent circuit of the analyzed system is shown on Figure 8. Where VinF is a voltage source analogous to an exciting force. F_1 is a current source dependent on current through V_1, related by parameter hC_0, F_2 is a current source dependent on current through V_2 related by parameter h, F_3

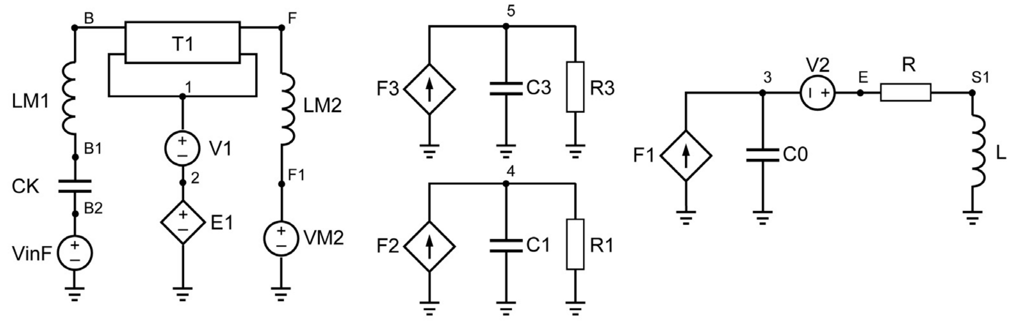

Fig. 8. Equivalent circuit for electro-mechanical system shown in Figure 4

is a current source dependent on current through V_L related by parameter 1. E_1 is a voltage source dependent on the voltage between node 4 and ground. The circuit containing source F_3 is attached to measure the displacement of mass LM_2, as equivalent to voltage on capacitor C_3.

Figure 9 and Figure 10 show the harmonic (AC) simulation results for the mechanical part and whole electro-mechanical system. The output value is the voltage on capacitor C_3 and is analogous to displacement of the mass 2.

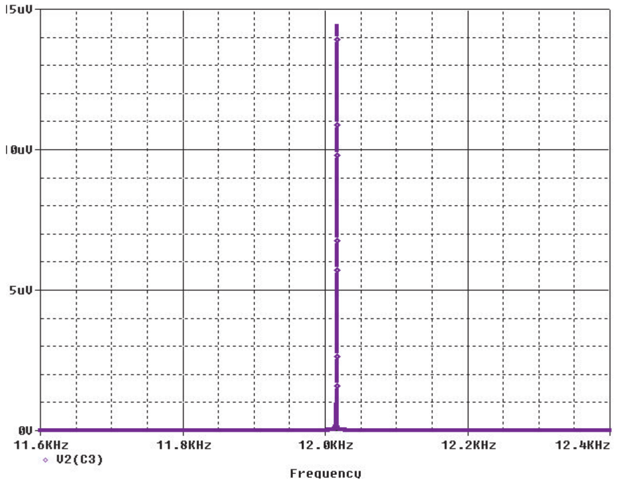

Fig. 9. Displacement of mass m_2 vs frequency for mechanical part only

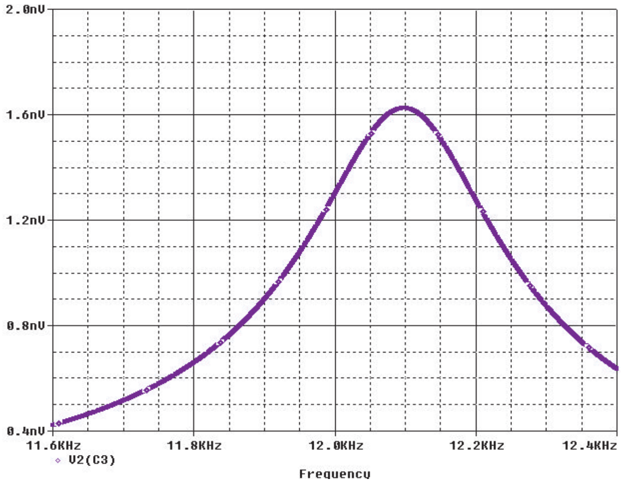

Fig. 10. Displacement of mass m_2 vs frequency for whole electro-mechanical system

Figure 11 shows the time response for excitation from a harmonic voltage source with amplitude 1 V and frequency 1.21 kHz which is equivalent to force with amplitude 1 N.

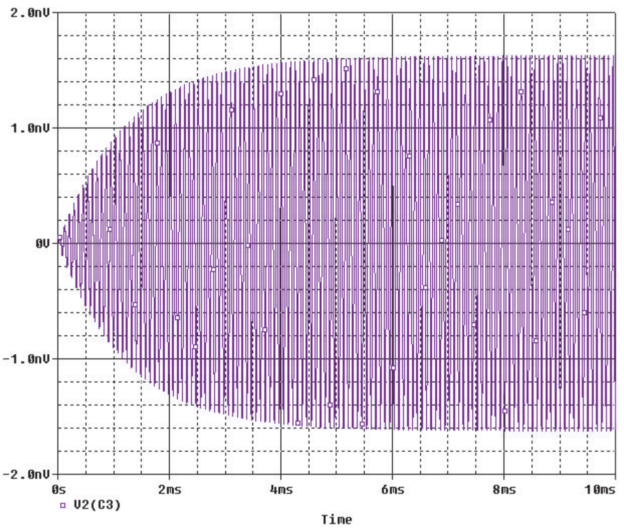

Fig. 11. Displacement of mass m_2 vs time for whole electro-mechanical system, excited by harmonic force, frequency 12 100 Hz and amplitude 1 N

4.3. Simulation of piezoelement in ANSYS Multiphysics software

The two mechanical masses are modeled as rigid bodies. They are made up of one 4 node element type PLANE42. The distances between nodes are constrained. The bottom spring is predefined element COMBIN14. The piezoelectric transducer is made up of element type PLANE13 and was discretized to 25 elements. The top and the bottom lines of nodes perpendicular to the *Y* axis are considered to be electrodes and have constrained VOLT and UY degrees of freedom. The electric circuit is made of CIRCU98 element type.

Figure 12 shows the results of harmonic analysis of the mechanical part of the system only, Figure 13 shows the results of harmonic analysis of mechanical part shunted by electric circuit, Figure 14 shows the time response for the whole electro-mechanical system excited by a harmonic force with frequency 12 010 Hz and amplitude 1 N.

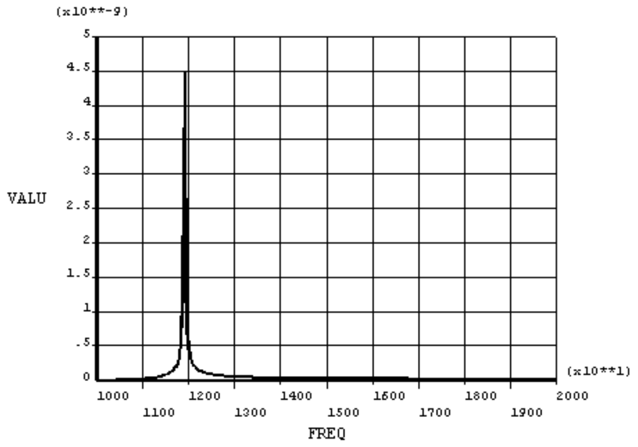

Fig. 12. Displacement of mass m_2 vs frequency for mechanical part only

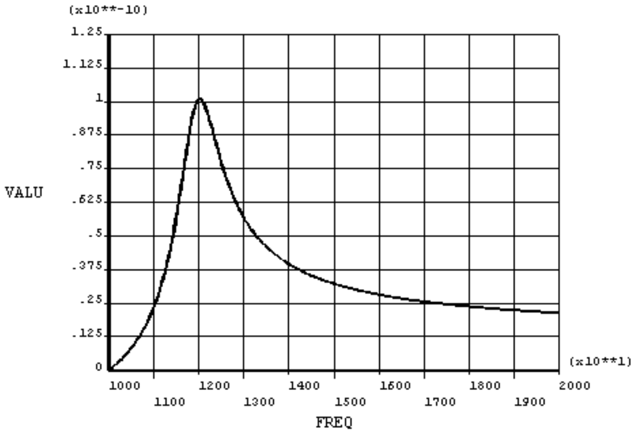

Fig. 13. Displacement of mass m_2 vs frequency for whole electro-mechanical system

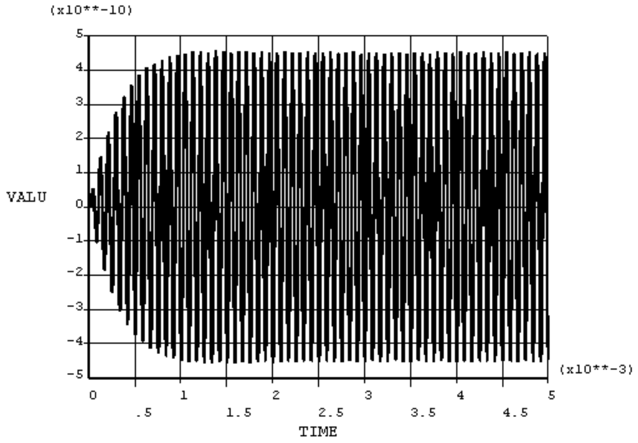

Fig. 14. Displacement of mass m_2 vs time for whole electro-mechanical system, excited by harmonic force, frequency 12 010 Hz and amplitude 1 N

4.4. Simulation of a piezoelectric bimorph in FEM software

A static simulation of a piezoelectric bimorph using different FEM software was also performed. The setup is presented on Figure 15.

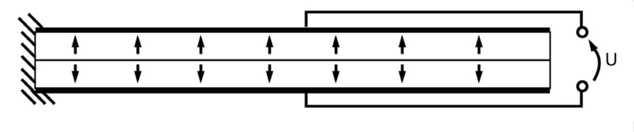

Fig. 15. Boundary conditions of analyzed piezoelectric bimorph

The antiparallel bimorph is fixed at one end, the top and bottom layers of nodes that model electrodes, have combined degree of freedom describing nodal potential. In MSC.Nastran the excitation is modeled directly by body thermal load as analogous to electric field, so modeling electrodes is unnecessary.

In ANSYS and Abaqus software the model geometry is input in meters, in MSC.NASTRAN because of numerical problems model has to be input in millimeters, and all material coefficients and voltage load have to be adjusted to the scale of one millimeter. The antiparallel polarity of the piezoelectric layers is modeled in multiphysics software by meshing model in flipped coordinate systems. Using a thermal analogy the two different materials has to be defined with opposite signs of *e* (*A*) matrix. The static analysis results of a piezoelectric bimorph excited by 1 V potential difference are shown on Figures 16, 17 and 18, as performed in ANSYS, Abaqus and MSC.NASTRAN respectively. The summary of results obtained from simulations is presented in Table 1.

Nowadays, instead of uniform piezoceramics, multilayer transducers are being widely used because of better performance. Manufacturers claim to be able to make a single piezoelectric layer of thicknesses down to 25 μm (PI Ceramic, Manufacturing 2008). To approximately calculate the number of layers, we can assume that the capacitance of one piezoelectric layer between electrodes is given by (10). When we consider a transducer with *n* layers we can rewrite (10) as

$$C_0 = n\frac{\varepsilon A}{h} \tag{31}$$

We consider the *n* layers with electrodes connected parallel, so the measured capacitance of transducer is *n* times bigger than single layer, thus:

$$C = nC_0 = n^2\frac{\varepsilon A}{h} \tag{32}$$

solving (32) in terms of *n*, we receive the approximate number of layers in piezoelectric transducer given by:

$$n = \sqrt{\frac{Ch}{\varepsilon A}} \tag{33}$$

The simulation results of a twenty layer piezoelectric bimorph model with the same dimensions and boundary conditions as other simulated FEM models are presented in Figure 19. It is easy to notice that the total displacement of the free end is more than five times greater than that of the uniform two layer bimorph.

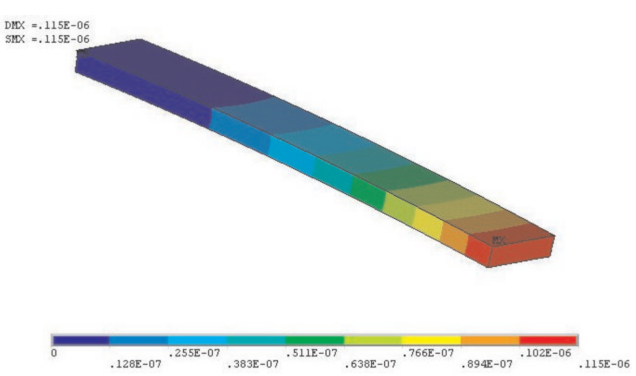

Fig. 16. Total displacement of piezoelectric bimorph excited by 1 V electric voltage. Static analysis made in ANSYS.Multiphysic software

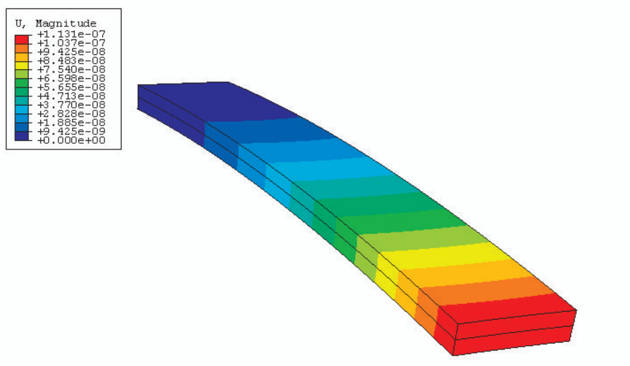

Fig. 17. Total displacement of piezoelectric bimorph excited by 1 V electric voltage. Static analysis made in Abaqus software

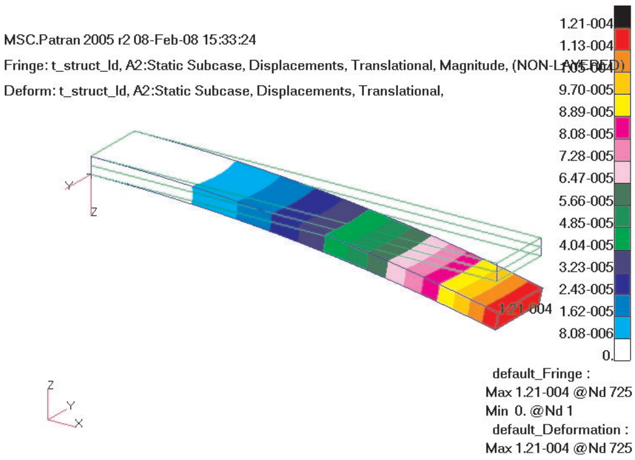

Fig. 18. Total displacement of piezoelectric bimorph excited by 1 V electric voltage. Static analysis made in MSC.Nastran software based on thermal analogy

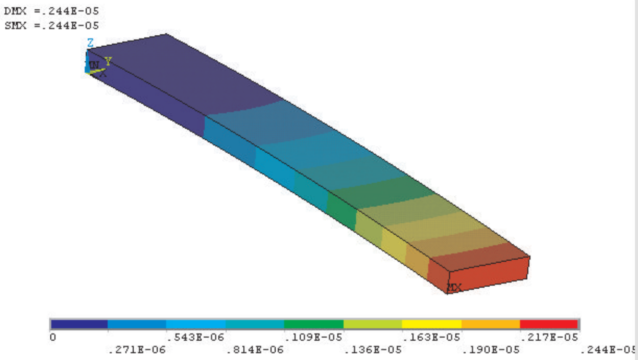

Fig. 19. Total displacement of 20-layers piezoelectric bimorph excited by 1 V electric voltage. Static analysis made in ANSYS.Multiphysics software

Table 1. Summary of simulation results

FEM software	Beam's tip displacement [m]
ANSYS.Multiphysics	1.15e-7
Abaqus	1.13e-7
MSC.Nastran	1.21e-7

5. CONCLUSIONS AND FINAL REMARKS

The models made in Simulink and PSPICE give the same frequency of vibration of electro-mechanical system, differing slightly from the FEM model in ANSYS, also the amplitude in time response are the same for PSPICE and Simulink models. However, those results are three times bigger then obtained from analysis run in ANSYS.

The ratios between amplitudes of vibration for the mechanical part and the shunted system are similar for Simulink and FEM model. Simulation in PSPICE gives completely different results.

The static simulation of bending a piezoelectric bimorph in FEM software gives almost the same results.

Different models of piezoelectric transducer were presented. It has been shown that beside multiphysics FEM software it is possible to model piezoelements in MATLAB-Simulink, PSPICE and MSC.NASTRAN.

The equivalent circuit method and modeling of a piezoelectric as a spring allows us to perform simulations for whole electro-mechanical systems for thickness mode vibration of transducer. The thermal analogy enables modeling actuators using FEM software, but does not allow modeling of sensing activity of piezoelectric elements. Abaqus software has facilities to model both piezoelectric actuators and sensors, but only in ANSYS.Multiphysics is possible to simulate the whole electro-mechanical system.

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