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# SELF-TUNING DAMPING SYSTEM FOR CABLE VIBRATION

*SUMMARY*

*The paper presents calculations of the system giving the maximal dissipation of energy by using the self-tuning damper. The damper is applied to reduce vibrations of inclined cable. It is attached near the lower support of cable. In order to calculate the optimal damping coefficient the self-tuning subsystem is introduced. The optimal damping coefficient is calculated taking into account the criterion of maximal energy dissipated by the damper. Results of calculations indicate that the concept of self-tuning damper can be applied to effective damping of cable vibration under various external inputs.*

***Keywords**: cable motion, self-tuning damper, optimal damping*

*SAMODOSTRAJAJĄCY SIĘ UKŁAD TŁUMIENIA DRGAŃ LINY*

*W pracy przedstawiono obliczenia samodostrajającego się tłumika realizującego maksymalne rozpraszanie energii drgań ukośnie zamocowanej liny. Tłumik jest umieszczony w pobliżu dolnego zamocowania liny. Samodostrajający się układ został wprowadzony, aby podczas ruchu układu zapewnić optymalne tłumienie. Optymalny współczynnik tłumienia obliczany jest na podstawie kryterium maksymalizacji rozpraszania energii przez tłumik. Otrzymane wyniki pozwalają stwierdzić, że przedstawiona koncepcja jest dobrą propozycją efektywnego tłumienia drgań liny, niezależnie od przyczyn wywołujących jej ruch.*

***Słowa kluczowe**: ruch liny, tłumik samodostrajający się, optymalne tłumienie*

## 1. INTRODUCTION

Long span cables are commonly used in modern constructions such as bridges, masts or towers, cableways and above all in the overhead transmission lines. In adverse weather conditions like wind, rain or snow large vibrations of cables are observed (Pacheco and Fujino 1993, Poston 1998). Cables are susceptible to vibration because of their high flexibility, relatively small mass, considerable length and very low inherent damping (Maslanka *et al.* 2007). Dangerous, high-amplitude cables vibrations lead to failures of the cable due to material fatigue.

Because of high tension which can be applied to cables and their relatively small mass there are many engineering application of cables. Taking into account bridges, Sutong Bridge (Yangtze River, China 2008) has the longest span. The maximum length of main span is equal to 1088 m. The Tatara Bridge (Seto Inland Sea, Japan 1999), Pont de Normandie (Le Havre, France 1995) are the other bridges with long spans. The long cables are also used in mast constructions. Warsaw Radio Mast was the highest mast ever builds – (646.4 m, Konstantynow, Poland 1974). It was destroyed during renovation in 1991. The KVLY-TV television transmitting mast (628.8 m, Blanchard, North Dakota, USA) and KXJB-TV television transmitting tower (628 m, Galesburg, North Dakota, USA) are the other tall masts.

A lot of methods have been proposed to eliminate cable vibrations. Some of research has been focused on application of semi-active dampers (Krenk 2000, Weber 2005, Wang *et al.* 2005).

Magnetoreolgical (MR) dampers are often used as semi-active dampers. First installation of MR damper in cable vibration control system was introduced in 2002 on the Dongting Lake Bridge in China. The dampers used in this system worked as passive dampers. Currently many research groups work on application of MR dampers with appropriate control systems. Results of investigations are summarized by Weber (Weber *et al.* 2005).

In the study (Krenk 2000) an approximate formula to the optimal damping of a viscous damper placed at a small distance from the support was derived. Using this formula, the optimal damping for only one mode of cable vibrations can be calculated. The optimal damping for the selected mode is not optimal for the other modes. In practice, it is difficult to tune viscous damper using this formula because dominant vibration mode depends on various types of excitations (Maslanka *et al.* 2007). In this study the self-tuning damping system for cable vibration is considered. In order to determine the optimal damping coefficient the concept of maximal damping energy was used.

## 2. CONCEPT OF SELF-TUNING DAMPING SYSTEM

In control engineering the feedback control with fixed controller is in common use. The physical properties of the object and the operating conditions are the base of a controller selection. However, due to the possible changing of object properties and existence of unknown disturbances, fixed controller may not give satisfactory results. Therefore, an adaptive subsystem is added to modify parameters of the controller.

In the system of cable with attached damper, the optimal damping coefficient depends on participation of modes. This participation is not the same during the motion. Thus the optimal damping coefficient should be changed. It is apparent that in this case the adaptive control approach due to its character can be used to control of damping system.

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Therefore the tuning subsystem is applied in damping system. This subsystem consists of two basic blocks: extrapolation block and optimization block (Fig. 1). Basing on state model and optimization criteria new parameters for controller are performed (Shaw 2000).

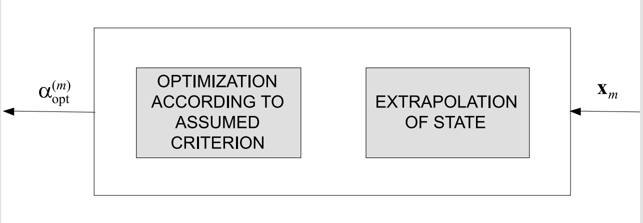


**Fig. 1.** Self-tuning subsystem

The block diagram of the system is shown in Figure 2. The system consists of a damper, a cable and a tuning subsystem. Cable and damper are time continuous subsystems whereas tuning subsystem works in certain instants. In these instances the tuning subsystem uses the extrapolated state to perform a new optimal damping coefficient $\alpha_{opt}^{(m)}$ ($m$ is a number of time instances). The cable block has two inputs (the damper force $F_d$ and the vector of disturbance forces **z**) and two outputs (the state vector $\mathbf{x}_m$ and the velocity $v_d$ at the point where the damper is attached).

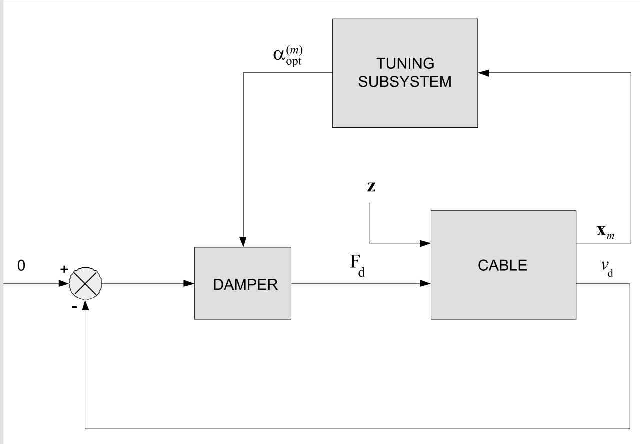


**Fig. 2.** Block diagram of the system

## 3. STATE EQUATION OF INCLINED CABLE

Cables used in engineering construction are usually attached to supports of different heights. Thus we consider an inclined cable with inclination angle θ (Fig. 3). The damper is attached to the cable at distance $\xi_d$ from the lower support. The equilibrium line of the cable is exactly described by catenary. When the tension of cable is large in relation to the weight of the cable the equilibrium line can be approximately described by the parabola. Assuming the sag-to-span ratio sufficiently small (Wang *et al.* 2005) the component of displacement in $w$ direction is significant, and remainig components are negligible small.

If the damper is placed at the node of any mode this mode cannot be control by the damper force. Therefore the position of the damper $\xi_d$ determines the controllability of the system.

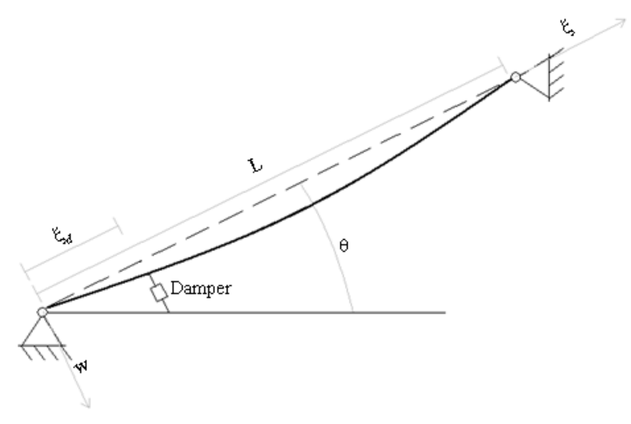


**Fig. 3.** Schematic diagram of inclined cable with attached damper

The transverse displacement of the cable $w(\xi, t)$ can be approximated by the sum:

$$w(\xi, t) = \sum_{k=1}^{n} X_k(\xi) q_k(t) \tag{1}$$

The functions $X_k(\xi) = \sin\left(\frac{k\pi\xi}{L}\right)$ where $L$ stands for distance between supports, satisfy the boundary conditions but they do not describe the natural modes of inclined cable. Using the function sequence $\{X_k(\xi)\}$ it is convenient to describe the motion of the cable. In this approach variables $q_k$ stand for the generalized coordinates of motion.

Using Ritz Galerkin method of discretization, the equation of cable motion can be written in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = -\mathbf{\Phi}(\xi_d)F_d(t) \tag{2}$$

where $\mathbf{q} = [q_1, q_2, ..., q_n]^T$ is the column vector containing generalized coordinates, **M** and **K** are mass and stiffness matrices derived e.g. in (Wang *et al.* 2005). They have the following form:

$$[\mathbf{M}]_{ij} = \int_0^L \mu X_i(\xi) X_j(\xi) d\xi \tag{3}$$

$$[\mathbf{K}]_{ij} = \int_0^L EIX_i''(\xi)X_j''((\xi)d\xi + \int_0^L T_0 X_i'((\xi)X_j'(\xi)d\xi + \\ + \frac{\lambda_s^2 T_0}{L^3}\int_0^L X_i(\xi)d\xi \int_0^L X_i(\xi)d\xi \tag{4}$$

where $\mathbf{\Phi}(\xi_d) = [X_1(\xi_d)\ X_2(\xi_d)\ ...\ X_n(\xi_d)]^T$ is vector depending on damper coordinate $\xi_d$. The designation μ is the linear mass density and $T_0$ is the cable tension. Taking into account the assumption of smal sag-to-span ratio the variability of tension along the cable can be neglected. Sag-extensibility parameter $\lambda_s^2$ is defined as the ratio of the elastic-to-catenary stiffness:

$$\lambda_s^2 = \left(\frac{\mu g L \cos\theta}{T_0}\right)^2 \frac{LEA}{T_0 L_e} \tag{5}$$

where $EA$ is extensional rigidity of the cable, and $L_e$ is the effective length of the cable expressed as:

$$L_e = \left(1 + \frac{(\mu g L \cos\theta)^2}{8T_0^2}\right)L \tag{6}$$

Finally the state equation of the cable can be written in the form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B_1}F_d + \mathbf{B_2}\mathbf{z}(t) \\ v_d(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{7}$$

where $\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$ is the state vector and $\mathbf{z}$ is the vector of disturbance forces. The matrices $\mathbf{A}$, $\mathbf{B_1}$, $\mathbf{B_2}$, $\mathbf{C}$ take the following form:

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M^{-1}K} & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} 0 \\ \mathbf{M^{-1}}\mathbf{\Phi}(\xi_d) \end{bmatrix}, \mathbf{B_2} = \begin{bmatrix} 0 \\ \mathbf{M^{-1}} \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 0 & \mathbf{\Phi}(\xi_d)^{\mathbf{T}} \end{bmatrix} \tag{8}$$

The numerical calculations were carried out for the parameters of the cable borrowed from (Lou *et al.* 2000). The distance between supports $L$ = 142 m, the inclination angle $\theta = 18^o$, the applied static force tension $T_0$ = 3711.8 kN, the linear mass density $\mu$ = 61.26 kg/m, the modulus of elasticity $E = 1.80 \times 10^{11}$ N/m$^2$ and damper position $\xi_d$ = 6.7 m. In calculations we assumed the number of modes $n$ = 30.

The eigenvalue problem for inclined cable can be written in the form:

$$\mathbf{AX} = \lambda\mathbf{X} \tag{9}$$

where $\lambda$ is the eigenvalue and $\mathbf{X}$ is the eigenvector. The imaginary parts of eigenvalues are equal to natural frequencies. For the cable considered the first three natural frequencies are 0.867 Hz, 1.722 Hz and 2.583 Hz. The first and the second mode shapes are shown in Figure 4.

Since the tension of the cable is high the natural mode shapes are almost the same as the mode shapes of the string. Assuming viscous damping, it is easy to express the damper force $F_d$ on state vector $\mathbf{x}$:

$$F_d = \alpha\mathbf{Cx} \tag{10}$$

where $\alpha$ is damping coefficient of a viscous damper.

The state equation of the system with a viscous damper (Fig. 5) can be written as:

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}\mathbf{x}(t) + \mathbf{B_2}\mathbf{z}(t) \tag{11}$$

where:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M^{-1}K} & -\mathbf{M^{-1}P} \end{bmatrix} \tag{12}$$

and $\mathbf{P} = \alpha\mathbf{\Phi}(\xi_d)\mathbf{\Phi}^{\mathbf{T}}(\xi_d)$.

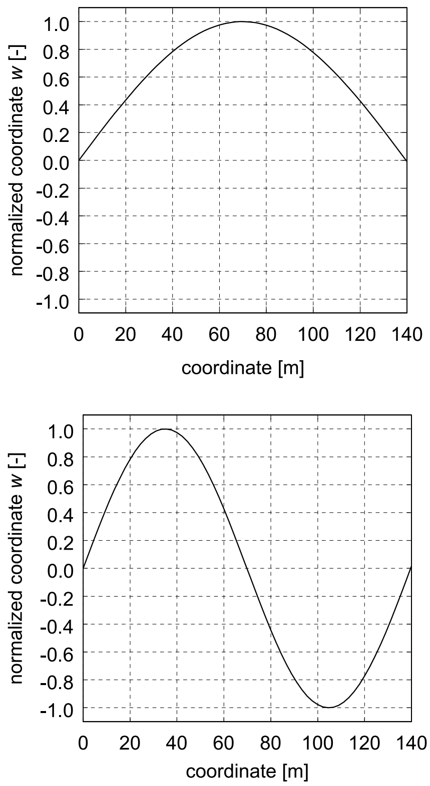


**Fig. 4.** Natural mode shapes

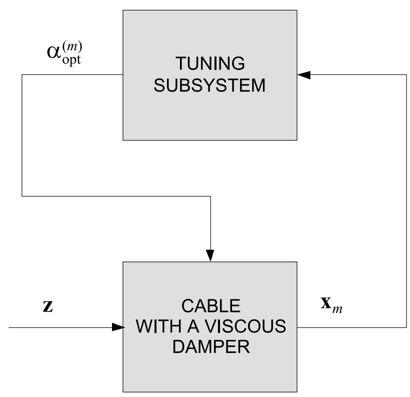


**Fig. 5.** System with a viscous damper

The eigenvalue problem for inclined cable with a viscous damper can be written in the form:

$$\tilde{\mathbf{A}}\mathbf{X} = \tilde{\lambda}\mathbf{X} \tag{13}$$

Eigenvalues and eigenvectors depend on $\alpha$. For $\alpha < \alpha_{critical}$ all eigenvalues are complex and they can be written as:

$$\tilde{\lambda}_k = \tilde{\alpha}_k + i \cdot \tilde{\omega}_k \tag{14}$$

where $\tilde{\alpha}_k$ is damping coefficient and $\tilde{\omega}_k$ is frequency. Index $k$ stands for the mode number. For $\alpha > \alpha_{critical}$ one mode is aperiodic and this means that for this mode $\tilde{\omega} = 0$.

Results of calculations are shown in Table 1 (for $\alpha$ = $= 5 \times 10^3$ Ns/m) and Table 2 (for $\alpha = 7 \times 10^3$ Ns/m).

**Table 1.** Frequencies and modal damping coefficients for $\alpha = 5\times10^3$ Ns/m

| Mode number | Frequency [Hz] | Modal damping coefficient [-] |
|---|---|---|
| 1 | 0.868 | 0.0019 |
| 2 | 1.722 | 0.0078 |
| 3 | 2.585 | 0.0168 |

**Table 2.** Frequencies and modal damping coefficients for $\alpha = 7\times10^3$ Ns/m

| Mode number | Frequency [Hz] | Modal damping coefficient [-] |
|---|---|---|
| 1 | 0.868 | 0.0027 |
| 2 | 1.723 | 0.0108 |
| 3 | 2.587 | 0.0234 |

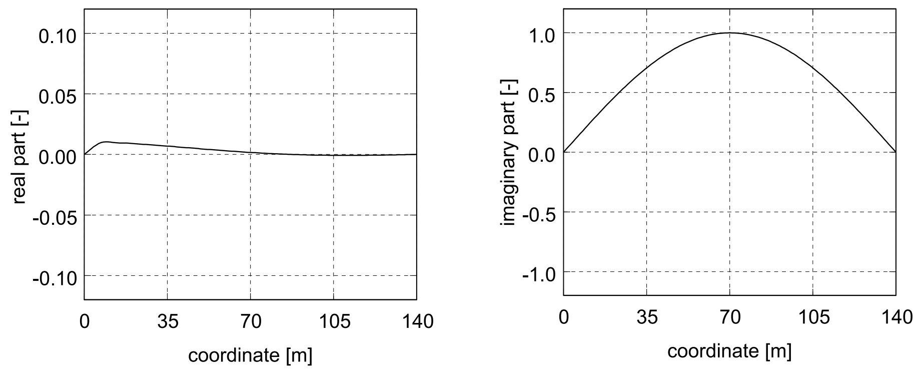


**Fig. 6.** Real part and imaginary part of the first mode

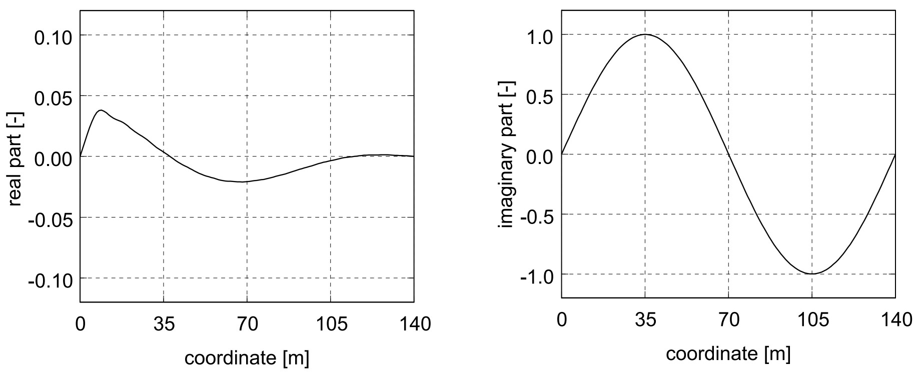


**Fig. 7.** Real part and imaginary part of the second mode

The first and second complex modes for $\alpha = 7\times10^3$ Ns/m are shown in Figures 6 and 7.

## 4. EXTRAPOLATION OF THE STATE IN THE RANGE BETWEEN TUNING INSTANTS

Tuning subsystem determines the optimal damping coefficients $\alpha_{\text{opt}}^{(m)}$ at tuning instants. In order to calculate $\alpha_{\text{opt}}^{(m)}$ it is necessary to extrapolate the state vector in the range $(t_m, t_{m+1})$ between tuning instants. We assume that the difference $t_{m+1} - t_m = t_d$ is constant for all $m$.

Extrapolation of the state can be performed using modal analysis of the system. In this analysis it is convenient to transform the state eqation into canonical form:

$$\dot{\mathbf{y}} = \mathbf{\Lambda y} \tag{15}$$

where $\mathbf{\Lambda} = \mathbf{T}^{-1}\tilde{\mathbf{A}}\mathbf{T}$ is a diagonal matrix containing eigenvalues $\lambda_k$. The matrix $\mathbf{T}$ is the transformation matrix. Since the eigenvalues are distinct, columns of matrix **T** contain the eigenvectors corresponding to the eigenvalues placed in matrix **Λ**. The solution of state equation Eq. (15) is:

$$\mathbf{y} = e^{\mathbf{\Lambda} t}\mathbf{y}_m \tag{16}$$

where $\mathbf{y}_m$ is the vector of initial values expressed in principal coordinates. Taking into account the transformation equation $\mathbf{y}_m = \mathbf{T}^{-1}\mathbf{x}_m$ the state vector in the range between tuning instants can be written in the form:

$$\mathbf{x} = \mathbf{T}e^{\mathbf{\Lambda} t}\mathbf{T}^{-1}\mathbf{x}_m \tag{17}$$

where $\mathbf{x}_m$ is the state vector at time $t = t_m$.

## 5. ENERGY DISSIPATION

The energy of the cable vibrations is dissipated by the damper. Using viscous damper the force acting on the cable is proportional to the velocity $v_d$ of the point where the damper is attached. The energy dissipated in the range of time $(t_m, t_{m+1})$ is equal to the work of damper force:

$$E_r = \int_{t_m}^{t_{m+1}} \left(-F_d v_d\right) dt \tag{18}$$

Using Eq. (10) and taking into account that $v_d = \mathbf{Cx}$, the dissipative energy can be experssed in the form:

$$E_r = \alpha \int_{t_m}^{t_{m+1}} \left(-\mathbf{x}^{\mathbf{T}}\mathbf{W}\mathbf{x}\right) dt \tag{19}$$

where $\mathbf{W} = \mathbf{C}^{\mathbf{T}}\mathbf{C}$.

In order to evaluate the integral in Eq. (19) we are looking for Lyapunov function in the following quadratic form:

$$V(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\mathbf{Q}\mathbf{x} \tag{20}$$

where **Q** is unknown matrix. The first derivative of Lyapunov function with respect to time along the state trajectories can be expressed as:

$$\frac{dV}{dt} = \mathbf{x}^{\mathbf{T}}(\tilde{\mathbf{A}}^{\mathbf{T}}\mathbf{Q} + \mathbf{Q}\tilde{\mathbf{A}})\mathbf{x} \tag{21}$$

Equating the derivative in Eq. (21) with integrand in Eq. (19), the following equation for matrix **Q** can be written:

$$\tilde{\mathbf{A}}^{\mathbf{T}}\mathbf{Q} + \mathbf{Q}\tilde{\mathbf{A}} + \mathbf{W} = \mathbf{0} \tag{22}$$

The above equation is known as Lyapunov equation. It can be solved numerically using the appropriate procedure of MATLAB. Finally taking into account Eqs. (19), (21) and (22) the absolute value of dissipative energy can be calculated from the following equation:

$$\left|E_r\right| = \alpha[\mathbf{x}_m^{\mathbf{T}}\mathbf{Q}\mathbf{x}_m - \mathbf{x}_{m+1}^{\mathbf{T}}\mathbf{Q}\mathbf{x}_{m+1}] \tag{23}$$

where: $\mathbf{x}_m$ is state vector at $t = t_m$ (initial conditions) and $\mathbf{x}_{m+1}$ is state vector at $t = t_{m+1}$, calculated from Eq. (17).

The optimization algorithm determines the maximum of the absolute value of energy $\left|E_r\right|_{\max}$ and optimal damping coefficient $\alpha_{opt}$.

## 6. NUMERICAL CALCULATIONS

In order to illustrate the result of optimization the various initial state were assumed:

$\mathbf{x}_m^{(1)} = [0.1,\ 0,\ 0,\ ...,\ 0]$,

$\mathbf{x}_m^{(2)} = [0.1,\ 0.05,\ 0,\ 0,\ ...,\ 0]$,

$\mathbf{x}_m^{(3)} = [0.1,\ 0.05,\ 0.02,\ 0,\ ...,\ 0]$.

In calculation the time interval $t_d = 8T_p$ where $T_p$ is a period of undamped first mode vibration and the disturbances force **z** is equal to zero. The plots of absolute value of dissipative energy for assumed initial conditions are shown in Figure 8. The values of optimal damping coefficient and dissipative energy are listed in Table 3.

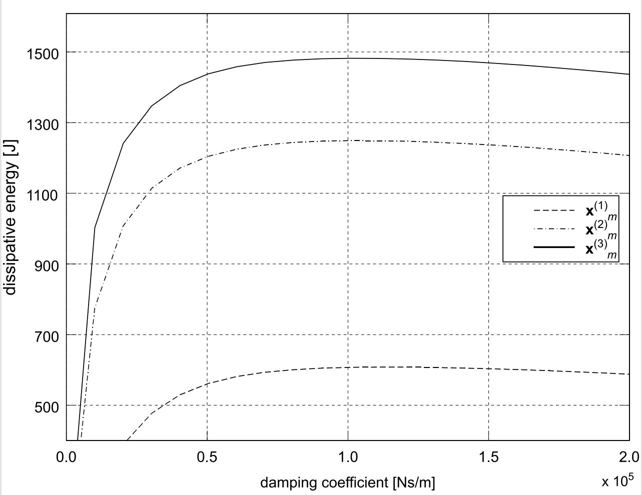


**Fig. 8.** Absolute value of dissipative energy vs. damping coefficient for various state vectors $\mathbf{x}_m$

**Table 3.** Results of calculations for various state vectors $\mathbf{x}_m$

| Initial state | Optimal damping coefficient [Ns/m] | Absolute value of dissipative energy [J] |
|---|---|---|
| $\mathbf{x}_m^{(1)} = [0.1, 0, 0, ..., 0]$ | $1.1109 \times 10^5$ | $6.0813 \times 10^2$ |
| $\mathbf{x}_m^{(2)} = [0.1, 0.05, 0, 0, ..., 0]$ | $1.0099 \times 10^5$ | $1.2489 \times 10^3$ |
| $\mathbf{x}_m^{(3)} = [0.1, 0.05, 0.02, 0, ..., 0]$ | $1.0099 \times 10^5$ | $1.4822 \times 10^3$ |

**Table 4.** Results of calculation for various time intervals $t_d$

| Time instances | Optimal damping coefficient [Ns/m] | Absolute value of dissipative energy [J] |
|---|---|---|
| $t_d^{(1)} = 6T_p$ | $1.0099 \times 10^5$ | $1.2013 \times 10^3$ |
| $t_d^{(2)} = 4T_p$ | $0.9098 \times 10^5$ | $1.0904 \times 10^3$ |
| $t_d^{(3)} = 2T_p$ | $0.9086 \times 10^5$ | $8.2507 \times 10^2$ |

To illustrate the influence of $t_d$ on the result of optimization the various $t_d$ are considered: $t_d^{(1)} = 6T_p$, $t_d^{(2)} = 4T_p$, $t_d^{(3)} = 2T_p$.

In calculations the initial state vector was assumed as $\mathbf{x}_m = [0.1, 0.05, 0, 0,..., 0]$ and the disturbance vector $\mathbf{z}$ was equal to zero. The plots of absolute value of dissipative energy are shown in Figure 9. The values of optimal damping coefficient and dissipative energy are listed in Table 4.

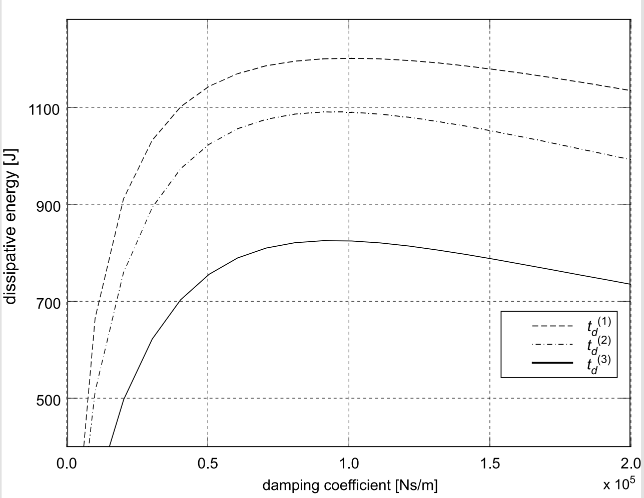


**Fig. 9.** Absolute value of dissipative energy vs. damping coefficient for various time intervals $t_d$

For the dominant first vibration mode (the state vector $\mathbf{x}_m = [0.1, 0, 0, ..., 0]$) the optimal damping coefficient is equal to $1.1109\times10^5$ Ns/m. This result can be compared to the damping coefficient ($\alpha = 1.0098\times10^5$ Ns/m) calculated by using approximated formula derived in (Krenk 2000). The difference is negligible.

In the next calculations the simple simulation of the cable motion is presented. At the beginning of the simulation of the cable motion, extrapolation of state is done on the basis of initial conditions associated with disturbance $\mathbf{z}$ at $t = 0$ (for time $t > 0$ influence of disturbances $\mathbf{z}$ is neglected). Then the extremum of objective functions is calculated. The simulation starts with optimal parameter $\alpha_{opt}$ corresponding to the initial conditions. During the simulation at appropriate instants, tuning block performs new optimal parameters.

As result of the simulation for $\mathbf{x}_0 = [0.1, 0.5, 0.02, ..., 0]$ (associated with disturbances $\mathbf{z}$ at $t = 0$) the displacement of the cable at $\xi$ = 71 m is shown in Figure 10. Result is of criterion 1 (proposed by Krenk) is compared with criterion 2 (propose by authors). It is apparent that for cases with more then one significant mode, criterion 3 gives better results then criterion 1.

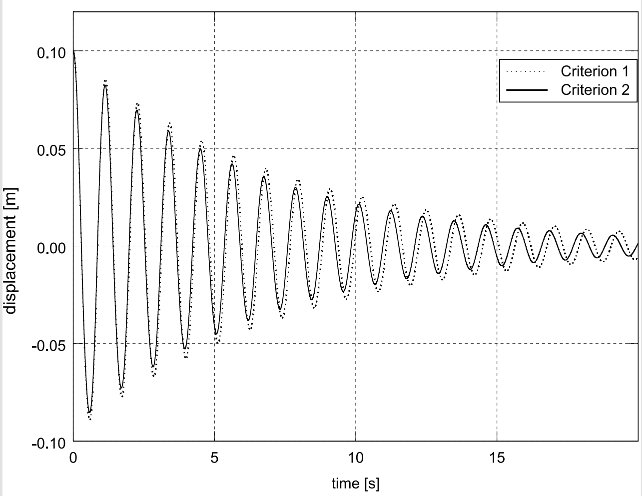


**Fig. 10.** Displacement of the cable at $\xi$ = 71 m

## 7. CONCLUSIONS

The study presents the self-tuning damping system assuring the maximal energy dissipated by the damper attached to the cable.

It is easy to show that the single-mode design control method may not give satisfactory damping ratios for arbitrary motion of the cable. The objective function gives optimal damping coefficient for various mode ratios. On the basis of simulations which has been conducted to prove the effectiveness of the presented control scheme, it is possible to say that it gives satisfactory results. In all considered control cases, algorithm has proved its ability to give optimal damping coefficient.

It seems that this self-tuning damping system for cable vibrations can be applied to real system. The presented system assumes the full information about the state and it might be difficult to realize in practice.

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