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MINIMIZATION OF THE MANUFACTURER'S DISTRIBUTION COSTS IN THE SUPPLY CHAIN – CASE STUDY

Abstract: The paper presents a linear programming model applied to solving a problem of planning vehicles routes. Although there is a lot of papers considering many different versions of classical vehicle routing problem, real life problems very often require special approach. In the paper the situation where the manufacturer orders transportation services from external company is taken into consideration. In the first part of the paper the basic definitions are introduced. Then possible models for finding solution to presented problem are discussed. In order to solve this issue two linear programming formulations based on the set partitioning model and on the vehicle routing problem have been developed. The last part of the paper describes application of the introduced model to planning routes for real life instance. Additionally the way of integrating presented models with the R3 system is discussed in details.

Keywords: zonal tariff system, distribution planning, outsourcing.

1. Introduction

Zonal system of calculation delivery costs is very often applied in distribution and the main cause of its popularity is its ease of use. Transportation costs depend on the cargo weight and the zone the delivery is intended to. Specification of delivery costs according to the described system requires only reading of the amount of the fee from an appropriately prepared table. In the case of LTL (less-than-truckload) deliveries, when the supplier can service more than one customer during one route, the supplier decreases the costs of deliveries through combining deliveries to more than one customer in one route.

The ever-growing specialisation of distribution services leads to situations where companies outsource deliveries to external transportation companies. The application of a zonal tariff in such cases is more advantageous for the transportation company because, as it has already been mentioned, an appropriate planning of routes

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permits servicing of several customers during one route, despite the fact that each cargo is paid for according to the zone tariff.

Due to the fact that transportation costs are one of the more important cost factors, companies try to introduce individual solutions in this area. An example of this can be mixed strategies consisting of using transportation companies and making settlements according to transportation tariffs of different types or according to tariffs more advantageous for the outsourcing company than for the transportation company.

2.   Zonal tariff taking into account the number of recipients

One of the simplest modifications of a standard zonal tariff consists of taking into consideration the farthest serviced zone, as well as the number of recipients to which the delivery is provided. In such case, the outsourcing company pays the standard zonal fare only for the delivery to the farthest customer and a flat-rate for each additional serviced client. (Fig. 1). Furthermore, the distance between locations serviced during one route is agreed upon.
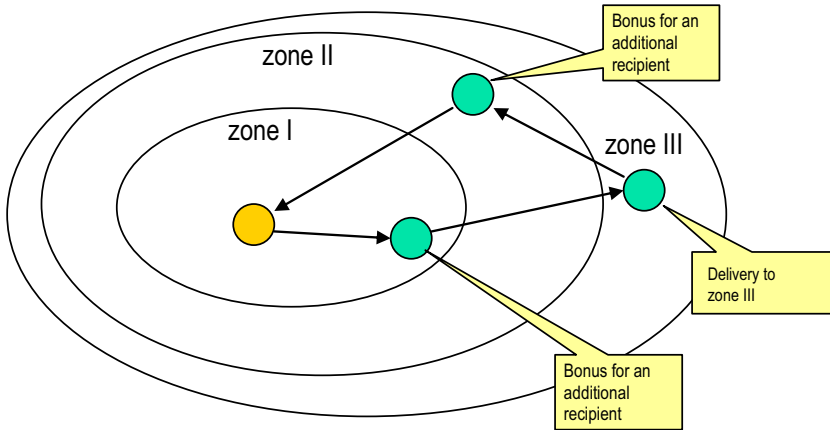

Fig. 1. *Modified zonal tariff*

The presented manner of settlement ensures, analogically to the case of classic zonal tariff, great facility in specifying delivery costs incurred by the recipients, as well as the possibility of direct association of income for realised deliveries with expenses for transportation companies. On the other hand, however, thus defined tariff requires a complicated manner of vehicle routing. The better the delivery plan, the lower the cost of their realisation; with a constant income (each recipient pays for transportation according to a constant rate, notwithstanding the way of realisation of delivery) this directly influences the company's profit.

The considerations proposed in the further part of this article do not pertain only to configurations like the association of the supplier with the transportation company and the customers presented above. They are also applicable in planning of deliveries by companies using external transportation.

3. Linear formulation of the delivery planning problem

The described problem is similar to two general problems analysed very thoroughly in the literature. The first one is the general assignment problem and the second one is the vehicle routing problem. The adaptation of those questions to the presented situation requires, however, several modifications. In the case of the first formulation one has to take into account vehicle load capacity and modify the objective function.

As to the second, a modification of the objective function is indispensable. In addition, one has to bear in mind that the magnitude of solved problems for the second formulation is significantly limited to no more than about 100 locations. Due to the fact that at the initial stage of the research it is difficult to state beyond doubt which approach would permit to obtain the best results further on, simultaneous use of both presented approaches has been considered.

Modification of the general assignment problem

First, we will consider a formulation based on a modification of the general assignment problem. The concept of its use consists of assigning all recipients to routes (the classical part), in addition, it has to be ensured that the vehicle load capacity shall not be exceeded in any route and guaranteed that the value of the objective function shall be calculated in compliance with the conditions specified in the problem description (Fig. 2).
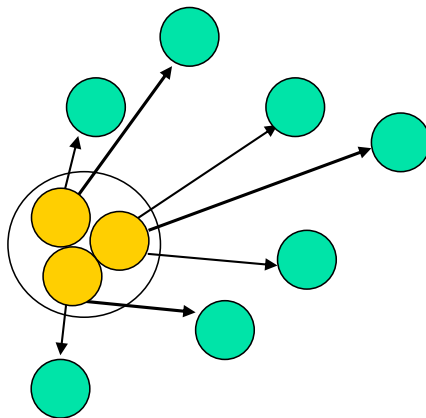

Fig. 2. *Planning of deliveries in zones as an assignment problem*

In the model described by formulas (1)–(6) two groups of decision variables (x_{ij} and z_j) are used. The first one is binary and specifies if the customer i is serviced in the route j, while the second one is the cost of service of the farthest customer in the route j. Additionally, the following parameters have been adopted: c_i is the cost of delivery to the customer i, q_i is the demand of the customer i, d_{ik} is the distance between the locations i and k and d_{max} is the maximum distance between customers in one route:

$$\min \sum_{j=1}^{m} z_j \tag{1}$$

$$z_j - c_i x_{ij} \geqslant 0 \qquad \text{for each } i, j \tag{2}$$

$$\sum_{i=1}^{n} q_i x_{ij} \leqslant Q \qquad \text{for each } j \tag{3}$$

$$\sum_{j=1}^{m} x_{ij} \geqslant 1 \qquad \text{for each } i \tag{4}$$

$$d_{ik} \left(x_{ij} + x_{kj} - 1 \right) \leqslant d_{\max} \qquad \text{for each } i, k, j \tag{5}$$

$$x_{ij} \in \{0, 1\} \qquad \text{for each } i, j \tag{6}$$

The objective function (1) with the constraint (2) ensures that the cost of realisation of all deliveries will be equal to the sum of delivery costs to the farthest location in each route. The constraint (3) ensures that the vehicle load capacity will not be exceeded in any route. The constraint (4) guarantees that each customer will be serviced exactly one time. The constraint (5) has been introduced to the model in order to ensure that the distance between each pair of customers in one route will not exceed the assumed maximum distance. Lastly, the constraint (6) guarantees that the decision variable x_{ij} is binary.

The presented formulation has been used to solve random test problems. The largest solved problem consisted of 16 cities and the time required for finding the solution was 550 seconds. Furthermore, the course of the optimisation process made it impossible to shorten the required time, as in all solved problems a significant difference with the linear approximation of the integer problem was observed. No attempt of solving the real number problem using the presented formulation was made.

Modification of the vehicle routing problem

Secondly, a formulation based on a modification of the vehicle routing problem will be considered. The concept of its use consists of generating delivery routes (analogically as in the basic form of the vehicle routing problem), but with a changed form of the objective function.

The route service cost should be equal to the cost of service of the zone containing the farthest customer in a route (Fig. 3).
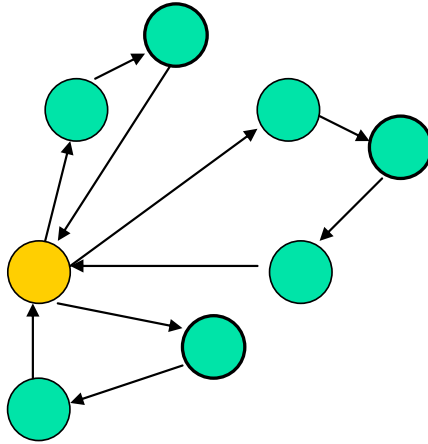

Fig. 3. *The zonal vehicle routing problem as a problem of generating delivery routes*

In a problem of delivery routes generation the value of the objective function is the sum of lengths of connections used in the solution. This causes that the previously planned modification of only the objective function from the original model proved impossible.

The model presented below was elaborated from the beginning and the only part it shares with the formulation of vehicle routing is the manner of defining the x_{ij} decision variable. In the original formulation (vehicle routing model), this variable specifies if the segment (i, j) was used in the optimal solution. It adopts the value of 1 when the vehicle uses the connection (i, j) in the solution and of 0 in other cases.

In the new formulation this variable is interpreted differently because connections and their weights are used to determine the value of the target function. In this formulation directed connections are considered. Two connection types have been distinguished. The first one are connections of the distribution centre with the customer with the assigned weight equal to the cost of servicing of the zone of the given customer. The second type are connections between customers. To the connections of this type the weight equal to the cost of servicing an additional location in a route is assigned. Exactly one connection must end at the location of each customer, which means that a customer will be serviced exactly once. Moreover, from customers connected directly with the distribution centre connections to other recipients within the distance smaller than dmax from the customer and pertaining to the same zone as a given customer or to a closer one.

Such formulation of the model guarantees that the cost of servicing a route will be equal to the cost of servicing of the farthest located customer in a route (Fig. 4).
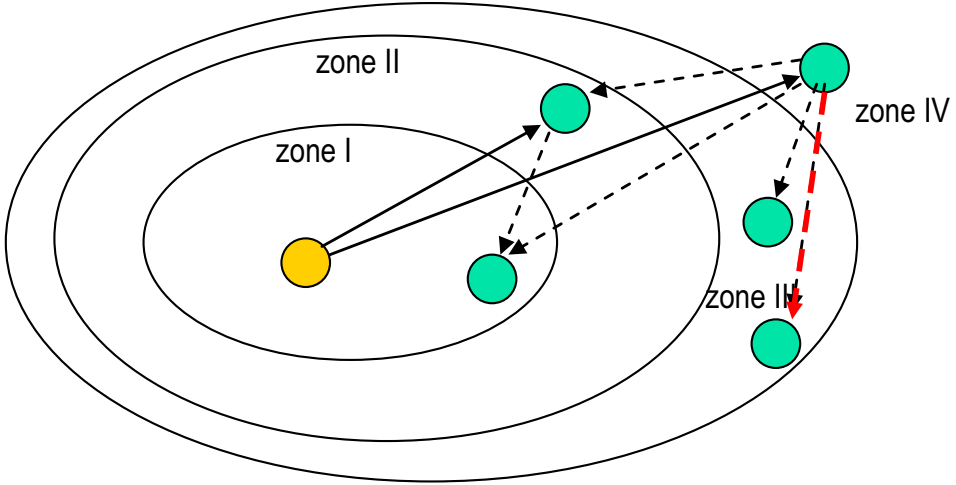
Fig. 4. *The model formulated in a way permitting determination of the value of the target function*

As mentioned above, in the model presented by the formulas (7)–(11) the x_{ij} decision variable is used. It adopts the value of 1 when the segment $< i, j >$ is used in the solution and of 0 in other cases. Additionally, the following parameters have been adopted: c_i is the cost of delivery to the customer i, q_i is the demand of the customer i and L is the maximum number of customers serviced in one route.

In this formulation of the model the parameter d_{max} is not used directly (as it was the case in the previous approach), because it is used during preparation of the model. If the distance between customers exceeds d_{max}, such connection is not taken into account during the optimisation process.

Such approach causes reduction of the size of the model, which can greatly impact the possibilities of solving it:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{7}$$

$$\sum_{i=0}^{n} x_{ij} = 1 \qquad \text{for each } j > 0 \tag{8}$$

$$\sum_{j=1}^{n} x_{ij} \leqslant L x_{0i} \qquad \text{for each } i > 0 \tag{9}$$

$$\sum_{j=1}^{n} q_j x_{ij} + q_i x_{0i} \leqslant Q \qquad \text{for each } i > 0 \tag{10}$$

$$x_{ij} \in \{0, 1\} \qquad \text{for each } i, j \tag{11}$$

The value of the objective function (7) is the sum of weights of the segments used in the solution. The manner of defining connections and their weights presented above guarantees that the value of the target function is compliant with the problem conditions. The constraint (8) ensures that each customer (except the distribution center marked with the index 0 in the model) will be serviced exactly one time.

The constraint (9) causes that connections beginning from customer (interpreted as service of an additional location in a route) are admissible only when they begin from customers serviced directly from the distribution centre (connection x_{0i}). Furthermore, this constraint ensures that the number of additionally serviced locations will not exceed L.

The constraint (10) guarantees that the vehicle load capacity will not be exceeded in any route. Lastly, the constraint (11) ensures that the decision variable x_{ij} is binary.

The presented formulation has been tested on randomly generated problems. The model perfectly manages to solve problems consisting of even 100 cities. The time required to solve such large problems amounted to about 10 minutes. Moreover, it has been determined that the course of the optimisation process permitted the shortening of the time required to find the solution.

In all solved problems it has been observed that the difference between the linear approximation of the integer problem is very small already at the beginning of the solution process. Setting the optimality tolerance of solution to 1% has caused problems containing even 120 customers to be solved in a few seconds.

4.   Real life instance

The later model was used to solve a real number problem consisting of 110 customers. The solution with the optimality tolerance set to 1% was obtained in 2 seconds, which constitutes a significant saving of time in comparison to a working day of two people responsible for vehicle routing. In the solution found, the cost of realised deliveries is 2% smaller than in the solution proposed by the company.

Such small difference is most probably caused by a subjective approach to the limitation of the maximum distance to the recipients serviced in one route and the possibility of transferring recipients from future periods to the planned routes (i.e. an earlier realisation of the delivery agreed upon with the recipient).

In the discussed model, the condition is fulfilled for each route, but un reality people who route vehicles has some possibility of lengthening this distance. Hence, a less strict approach to this limitation, as well as a possibility of transferring a recipient to a current period greatly facilitate the distribution planning process.

A precise assessment of the quality of the proposed solutions in comparison to those determined by the department of logistics requires an analysis of a greater number of distribution plans. The graph of the determined solution is presented in the Figure 5.
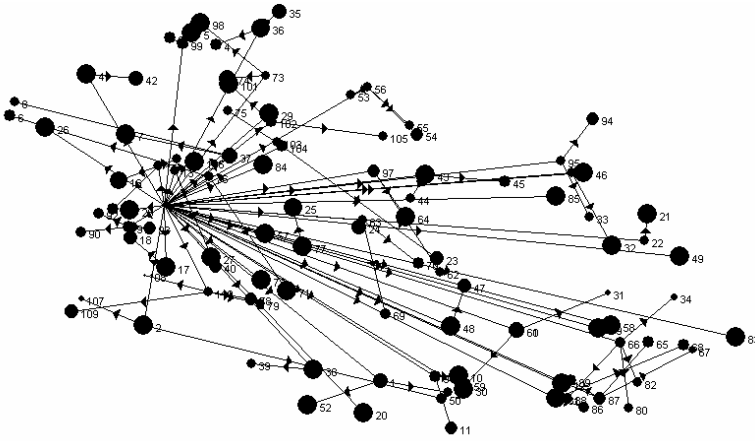
Fig. 5. *The graph of the solution of a real number problem of zonal delivery planning*

5.   Conclusions

The possibility of application of the proposed optimisation solutions very often depends on the possibility of programming and integration of the solution algorithm with the computer system extant in a company. In the discussed case the proposed model was implemented in a version using commercial CPLEX software and in a version operating with a free COIN-MP solver. At the implementation stage the company has not yet made the decision of purchasing the commercial software.

The software has been integrated with the SAP R3 system by means of standard methods as so-called user-exits and remote function call, permitting to call external programs. Such manner of integration ensured full comfort of use (the end-user does not know that external procedures are run on planning deliveries) and allowed to use professional optimisation software (CPLEX, COIN-MP). The presented research has shown, moreover, that in the field of optimisation models taking into account specific problem conditions allow to obtain close to optimal solutions to real number problems, while casting doubt on universal approaches of all types.

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