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Using Reflectorless Total Stations in Surveying of Industrial Chimney Inclinations**

1. Introduction

The industrial chimneys are influenced by numerous factors which act as loads on their structures. The following influences may be distinguished: mechanical, thermal or physiochemical. A well measurable effect of the structure effort is a change in its geometry, and therefore periodic surveys of changes to the inclination and shape of the axis of the chimney structure are conducted. The most widely used method for measuring the shape of the axis of the chimney structure is the trigonometric method [2] (Fig. 1).

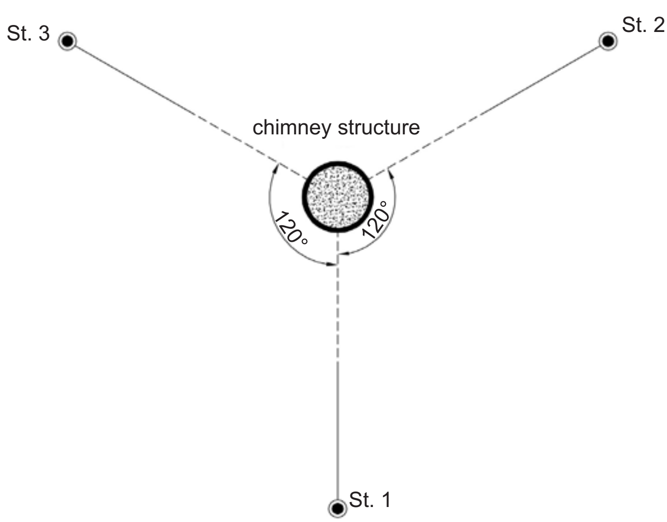

Fig. 1. Optimal layout of observation positions in trigonometric method

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** The work has been carried out as part of statutory studies no. 11.11.150.005

We apply this method by measuring the direction of tangents to the structure at fixed levels from a minimum of two positions. However, to be certain about the determined results, we have to perform the surveying from three positions arranged in such a manner that the object is located in the centre of gravity of the area defined by means of control network points (Fig. 1).

Looking at the development of modern reflectorless total stations [1] as well as the results of surveying experiments aiming to assess their accuracy [6, 8], surveying methods using their enormous potential should be developed. Therefore we might achieve shorter surveying time while ensuring the necessary accuracy of the obtained results.

2. Methods for Measuring Inclinations and Shape of the Axis of Industrial Chimney Structure with the Use of Reflectorless Total Stations

In order to propose alternative surveying methods, it is to be remembered what the purpose of a survey is and how to achieve it with the use of instruments enabling to measure the distance to the points representing the analysed object. According to the author, the aim is to determine the coordinates of the centre of the chimney layer stack in pre-determined levels of observation as a result of elaboration of the field material, and then use it to determine the inclination of the structure with respect to the lowest level.

The methods fulfilling the above assumption are as follows:

- measuring multiple points on the perimeter,
- measuring directions of tangents and a single point on the bisector,
- measuring directions of tangents and multiple points at the observed level.

2.1. Method for Measuring Multiple Points on the Perimeter

The simplest method in terms of a concept, but the most demanding during surveying procedures, involves surveying of a set of points at each of the observed levels of the chimney structure. For this purpose it is necessary to establish surveying control network with an accuracy greater than the expected accuracy of the inclination surveys. The positions of the instrument should be arranged in such a manner that the measuring points were distributed evenly along the entire perimeter. The survey should preferably be carried out from three positions (Fig. 1), however, using two positions, it is also possible to arrange the points appropriately and to obtain a satisfactory geometry of the task (Fig. 2).

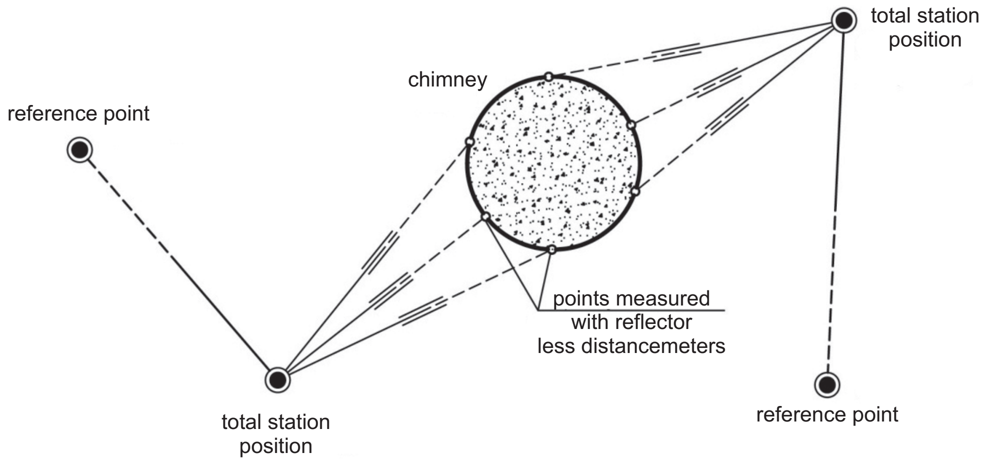

Fig. 2. Surveying scheme of multiple points on the perimeter at selected chimney level

The surveys at the reference point and at the points defining the studied object are carried out from each of the positions. A minimum of three points located at the same level should be surveyed. However, such a low number of surveying data does not allow detection of possible errors, and it is therefore recommended to measure more survey points.

Having conducted the surveys, geodetic coordinates of points defining each of the levels of surveying should be calculated, and then the coordinates of the centre of the chimney structure should be determined. The calculation of the coordinates of the circle centre which is an approximation of the chimney section may be conducted by numerous methods, however, one algorithm proposed by Wędzony [9] may confidently be recommend, as it is characterized by the fact that it does not require an iterative procedure.

As is known from analytic geometry, the equation of a circle may be defined by the following formula:

$$(x - X_0)^2 + (y - Y_0)^2 = R^2 \tag{1}$$

where:

X_0, Y_0 – coordinates of the circle centre,

R – radius of the circle,

x, y – coordinates of the point at which the circle intersects.

Upon explication we get:

$$x^2 - 2xX_0 + X_0^2 + y^2 - 2yY_0 + Y_0^2 - R^2 = 0 \tag{2}$$

Assuming the signs:

$$
\begin{aligned}
A &= -2X_0 \\
B &= -2Y_0 \\
C &= X_0^2 + Y_0^2 - R^2
\end{aligned}
\tag{3}
$$

the following equation may be written:

$$
Ax + By + C + x^2 + y^2 = 0 \tag{4}
$$

Combining the above equation for the three points we obtain a system of three equations with three unknown quantities. Its solution are three auxiliary parameters *A*, *B* and *C*, whereby it is possible to calculate the searched coordinates of the circle centre of the chimney structure at the desired level:

$$
\begin{cases}
X_0 = -\dfrac{A}{2} \\
Y_0 = -\dfrac{B}{2} \\
R = \sqrt{X_0^2 + Y_0^2 - C}
\end{cases}
\tag{5}
$$

If there are more survey points than the unknown quantities of the equation, the principle of least squares of C.F. Gauss should be applied:

$$
\sum_i v_i v_i = \min \tag{6}
$$

For this purpose, the equation is written as follows:

$$
v_i = Ax_i + By_i + C + x_i^2 + y_i^2 \tag{7}
$$

and it is solved according to the following formula:

$$
\begin{bmatrix} A \\ B \\ C \end{bmatrix} = (A^T A)^{-1} A^T L \tag{8}
$$

where:

A – matrix of partial derivatives of equation (7),

L – vector of equation free terms (7).

Fitting accuracy may be assessed using the formula for residual variance estimator:

$$\sigma = \pm\sqrt{\frac{(AX-L)^T(AX-L)}{n-3}} \tag{9}$$

where n – number of survey points located at the analysis level.

The final inclination of the axis from the vertical line of the chimney structure is calculated using the following formulas:

$$\begin{aligned} w_X^j &= X_0^j - X_0^0 \\ w_Y^j &= Y_0^j - Y_0^0 \end{aligned} \tag{10}$$

where:

w_X^j, w_Y^j, – inclination of the j-th level of regarding the "0" level,

X_0^j, Y_0^j, – estimated coordinates of the centre of the j-th level.

2.2. Method for Measuring Tangents and a Single Point Located on the Bisector of Tangent Directions

Other solutions which does not require extensive surveying includes measuring tangents and one point on the bisector of each of the surveying sections (Fig. 3). This method does not require either a control network or reference points, however, thanks to their use, the surveys are oriented in a coordinate system, which is quite helpful in interpreting the direction of the inclination of the studied object. For each level, tangential directions and distance to a point located in the bisector intersection of tangential directions and chimney structure are measured.

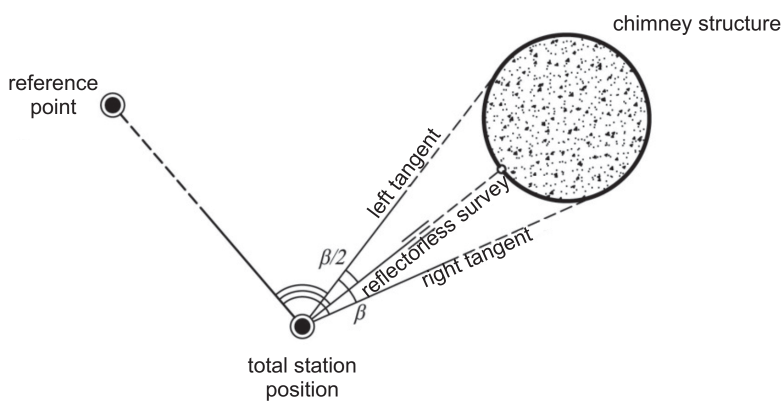

Fig. 3. Surveying scheme of the tangents and the point on the bisector

The calculations of the coordinates of the measuring section centres are conducted using the following algorithm.

If a survey orienting at the adjacent control network point was carried out, it is possible to calculate the azimuth zero mark of the horizontal limb:

$$Az_{Stn-Naw} = \operatorname{arctg} \frac{\Delta Y_{Stn-Naw}}{\Delta X_{Stn-Naw}} \tag{11}$$

$$Az_0 = Az_{Stn-Naw} - Hz_{Naw} \tag{12}$$

where:

$\Delta X_{Stn-Naw}$ – coordinate increase between the position of the total station and the reference point,

$\Delta Y_{Stn-Naw}$ – azimuth of the section between the position and the reference point,

$Az_{Stn-Naw}$ – reading of the horizontal limb at the reference point,

Az_0 – azimuth of "zero" of the horizontal limb.

In the further calculations it is necessary to determine the azimuth value of bisector of tangential directions in reference to the chimney structure, as well as to calculate the radius at each of the analysed levels:

$$Az_L^j = Az_0 + Hz_L^j \tag{13}$$

$$Az_P^j = Az_0 + Hz_P^j \tag{14}$$

$$Az_{sr}^j = \frac{1}{2}(Az_L^j + Az_P^j) \tag{15}$$

$$\beta^j = Az_P^j + Az_L^j \tag{16}$$

$$R^j = \frac{d^j \cdot \sin\frac{\beta}{2}}{1 - \sin\frac{\beta}{2}} \tag{17}$$

where:

Hz_L^j, Hz_P^j – horizontal angle readings for tangential direction towards the chimney structure at the j-th level,

Az_L^j, Az_P^j – a tangential azimuth to the chimney structure at the j-th level,

Az_{sr}^j – azimuth of the section joining the position with the centre of the chimney at the j-th level,

d^j – horizontal distance to the chimney structure at the j-th level,

R^j – radius of the chimney structure at the j-th level,

β^j – vertical angle between the tangents towards the chimney structure.

Based on the above data it is possible to calculate the coordinates of the chimney centre at each of the measured cross sections:

$$X_0^j = X_{Stn} + (d^j + R^j)\cdot \cos Az_{sr}^j$$
$$Y_0^j = Y_{Stn} + (d^j + R^j)\cdot \sin Az_{sr}^j \tag{18}$$

Calculation of the components of the chimney structure inclination from the vertical line should be carried out based on the formula (10).

A survey from a single position does not ensure control of determining the inclination, therefore it is advisable to repeat the survey from an independent position, and as a final inclination, to adopt a mean value.

It should be noted that the above-mentioned method has been implemented in the software of Leica total stations as one of the eccentric surveying subfeatures [5].

2.3. Method for Surveying Tangents and any Points at the Same Level

The last method is a generalization of the previously defined ones. It is based on using the measurements of tangential directions as well as of the distance and angle to any point (or points) located at an analysed level of the chimney structure. This approach enables to survey several points at each section, which shall allow for partial elimination of possible surveying errors, and (most importantly) of the construction errors (Fig. 4).

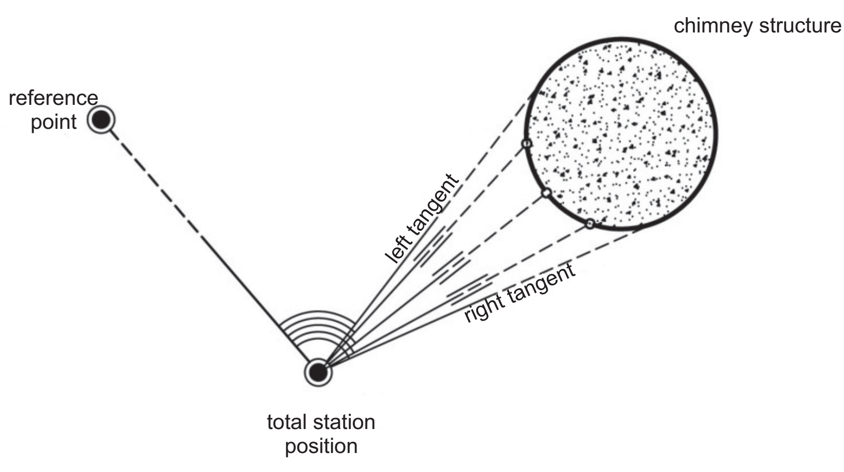

Fig. 4. Surveying scheme of the tangents and random points at the same level

Having completed the survey, which could be done both with reference to the control network points as well as in the position local system, a proper compilation of the field material should be made. The calculation starts with the orientation of a bundle of directions in the adopted coordinate system by determining the azimuth of "zero" of the horizontal limb ((11) and (12) formulas).

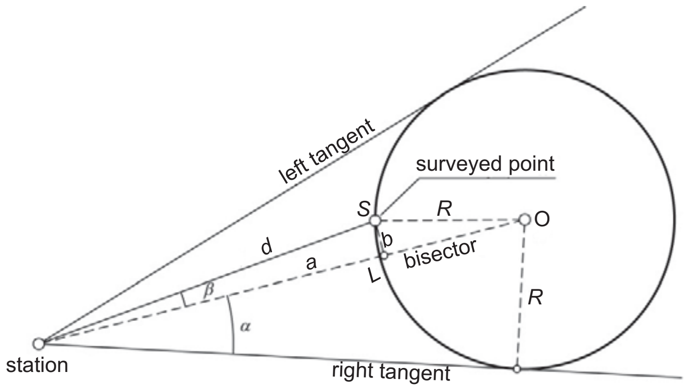

Fig. 5. Calculation scheme for a single point

In order to determine the coordinates of the central point of the chimney structure, it is necessary to determine the distance L calculated from the position of the instrument to the chimney centre along the bisector of tangential directions (Fig. 5):

$$\alpha^j = \frac{1}{2}(Hz_P^j - Hz_L^j) \tag{19}$$

$$\beta^j = \frac{1}{2}(Hz_L^j + Hz_P^j) - Hz_S^j \tag{20}$$

$$R^j = L^j \cdot \sin(\alpha^j) \tag{21}$$

$$a^j = d^j \cdot \cos(\beta^j) \tag{22}$$

$$b^j = d^j \cdot \sin(\beta^j) \tag{23}$$

Substituting (21)–(23) to the Pythagorean theorem, it was obtained:

$$(L^j \cdot \sin\alpha^j)^2 = (d^j \cdot \sin\beta^j)^2 + (L^j - d^j \cdot \cos\beta^j)^2 \tag{24}$$

and after simplification, quadratic equation is presented in the form:

$$(\sin^2\alpha - 1)\cdot(L^j)^2 + 2d^j\cos\beta\cdot L^j - (d^j)^2 \tag{25}$$

whose solution is:

$$L^j = \frac{-2d^j \cos\beta^j \pm \sqrt{(2d^j \cos\beta^j)^2 + 4\cdot(\sin^2 \alpha^j - 1)\cdot(d^j)^2}}{2\cdot(\sin^2 \alpha^j - 1)} \tag{26}$$

If more than one point on the section has been surveyed, the final length L is equal to the arithmetic mean.

The values of the central point coordinates of the j-th section of the chimney are calculated using the following formulas:

$$\begin{aligned} X_0^j &= X_{Stn} + L^j \cdot \cos(Az_P^j - \alpha^j) \\ Y_0^j &= Y_{Stn} + L^j \cdot \sin(Az_P^j - \alpha^j) \end{aligned} \tag{27}$$

and the chimney structure inclination components may be determined using the formula (10).

3. Verification of the Method on Experimental Object

In order to verify the proposed surveying method as well as calculation algorithms, a measuring experiment was conducted involving the survey of the chimney structure inclination by classic method and using the reflectorless distancemeters. A multi-flue reinforced concrete industrial chimney of the height of 250 m was selected as the test object (Fig. 6).

Fig. 6. Experimental facility – cylindrical reinforced concrete industrial chimney of $H = 250$ m

Thanks to the courtesy of Mr Tadeusz Nadowski, three modern reflectorless total stations were used for the surveying:

- Leica FlexLine TS06 power – 5'',
- Leica FlexLine TS06 ultra – 5'',
- Leica TCRP1205.

The used total stations measure angles with an accuracy of ±5'' and they measure the distance for the natural purpose with the standard deviation of 2 mm 2 ppm. The only distinguishing feature of these total stations is their range of reflectorless surveying. The total station TPS1200 which was used by us is equipped with the R300 distancemeter of a maximum range of 500 m [4]. However, during the field surveying, it was possible to measure distances of up to 700 m, which reflects well on the producer. The instruments which are structurally newer models of total stations are the ones of the FlexLine series. The Power model is equipped with a distancemeter with a maximum range of up to 400 m, while the Ultra model, as it is specified by the producer [3], measures the distance up to 1000 m.

During the surveying experiment, a high speed and range of the Ultra and R300 series were confirmed, however, the second model of the FlexLine series with a theoretical range of 400 m did not permit the surveying of the distance to the chimney which was located about 210 m away, and it was used exclusively to measure the tangential directions.

The survey was carried out from a surveying control network, executed specially for that purpose, which was in reference to a realisation grid of an industrial plant. The mean accuracy of the point location was ±5 mm. The shape of the control network and the layout of observation stations to survey the chimney structure have been presented in figure 7.

To have a sufficient quantity of field material for the analysis of accuracy, the full surveying from all of the three observation points were carried out four times. Having collected the field material, the calculations of the chimney inclinations were made using the following methods:

- trigonometric (classical) – because it is a method commonly applied, it was considered a base for assessing the accuracy of the proposed methods;
- surveying of multiple points on the perimeter of each level – performed it in two variants involving the acceptance of all data points surveyed from two total station positions (150 and 153), and only from one survey point (153);
- surveying of tangents and any random points located at the same level – in this case two variants similar to the method above were also applied.

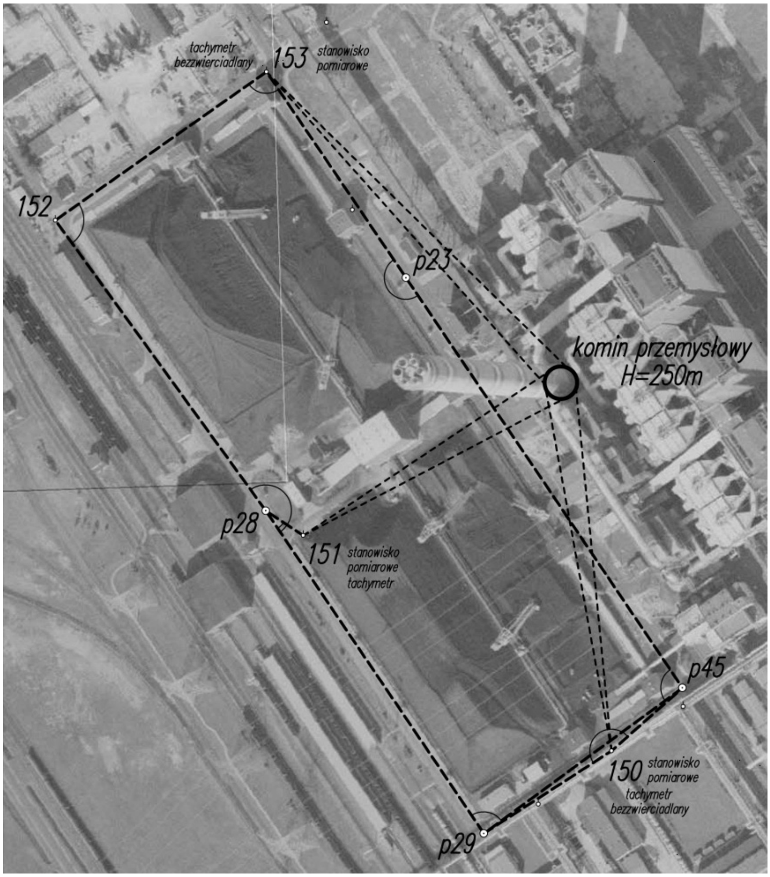

Fig. 7. Layout of total station positions at surveying the inclination of the chimney of $H = 250$ m

On the basis of the determined inclinations, internal accuracy of the applied surveying methods was calculated (analysis of variation from the mean values), which has been contained in table 1.

Table 1. Evaluation of repeatability of surveying methods

No.	Method	σ_{wx} [mm]	σ_{wy} [mm]	σ_{w} [mm]
1	Classical – trigonometric	8.7	3.3	9.3
2	surveying of multiple points on the perimeter of each section (positions 150 and 153)	18.8	7.6	20.3
3	surveying of multiple points on the perimeter of each section (position 153)	6.5	9.2	11.3
4	surveying of tangents and any random points at the same level (positions 150 and 153)	5.9	3.4	6.8
5	surveying of tangents and any random points at the same level (position 150)	5.9	5.3	7.9

The ultimate goal of the experiment is to compare the results of methods that use reflectorless distancemeters with regard to the trigonometric method. Mean values of absolute differences between the methods have been presented in table 2.

Table 2. Comparison of inclination values in terms of classical method

No.	Method	Δ_{wx} [mm]	Δ_{wy} [mm]	$\sqrt{\Delta^2_{wx} + \Delta^2_{wy}}$ [mm]
1	surveying of multiple points on the perimeter of each section (positions 150 and 153)	22.5	6.9	23.5
2	surveying of multiple points on the perimeter of each section (position 153)	18.8	19.3	27.0
3	surveying of tangents and any random points at the same level (positions150 and 153)	17.8	9.1	20.0
4	surveying of tangents and any random points at the same level (position 150)	12.0	21.1	24.2

4. Conclusions

On the basis of the conducted surveying experiment it might be concluded that the proposed methods for the determining the inclinations and the shape of the axis of the industrial chimney structure meet the accuracy requirements imposed by the (Ministry of Heavy Industry, 1976), which require the inclinations of tower structures to be determined with an accuracy of better than ±0.05 m.

Both in the analysis of repeatability of the results and of the compliance with the trigonometric method, the best in terms of accuracy appears to be a method in which the tangents and any random points located at the analysed level of the chimney structure were surveyed. In addition, it might be noticed that carrying out surveying from a greater number of positions does not significantly improve accuracy. It follows that it is sufficient to carry out surveying from a single survey position, and in order to achieve greater reliability of the final result, a more than one point at each of the surveying sections should be surveyed.

According to the author, three points are optimal (one central and two lateral). This is not a flawless method, though. Undoubtedly, the source of errors which are hard to detect is lack of alignment of the reflectorless distancemeter beam with the axis of the optical telescope. Even minor deviations will have a major impact on the value of the inclinations, especially in the case of single-flue chimneys – due to their smaller diameter.

It might be concluded that the proposed method of using modern reflectorless total stations for the purpose of determining chimney structure inclinations satisfies the imposed accuracy requirements and is significantly faster than the classical method, as it requires a smaller number of surveying positions.

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