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Algorithm MG(F-K) of migration in model TTI anisotropy

Introduction

One of the most frequently encountered models in anisotropic medium is the model of Transverse Isotropy (TI). If the symmetry axis of model TI coincides with the vertical axis of right-angle coordinates, then we receive horizontal thin-layered system proposed by G. Postma [12] and called VTI (*Vertical Transversely Isotropic*). The application of algorithms of propagation and migration of „isotropic" waves in anisotropic medium VTI results in deformations and relocation of reproduced structures [6, 10], and the higher the anisotropic parameters are, the greater their extent is. (parameters by L. Thomsen [14]).

In the domain of wavenumbers and frequencies, the algorithm of migration MG(F-K) was presented in anisotropic medium type VTI [11]. This algorithm uses approximative version of vertical wavenumber [5] with reference to medium VTI. This article presents the algorithm of migration MG(F-K) in monoclinal medium marked as TTI (*Tilted Transversely Isotropic*) model, whose symmetry axis is tilted at θ angle to the vertical axis (Fig. 1).

Algorithmic solutions

In case of a thin-layered arrangement, arbitrarily oriented in relation to the Cartesian coordinate system *x*, *y*, *z,* it is appropriate to use general law of tensor rotation in Bond`s formulation [2, 3, 13] which allows to obtain relation between matrix $D^{\varphi\theta}$ of elastic modules in measuring coordinate system *x*, *y*, *z*, and appropriate matrix *C* of these modules in coordinate system *x'*, *y'*, *z'*.

The is the ensuing relation:

$$D^{\varphi\theta} = R_{(\varphi\theta)} C R^{T}_{(\varphi\theta)} \qquad (1)$$

where φ denotes the angle of rotation of system *x'*, *y'*, *z'* with regard to axis *z*, whereas θ is the tilt angle of the symmetry plane of isotropy TI.

The dimensions of matrices $R_{\varphi\theta}$ and $R^{T}_{\varphi\theta}$ are 6 × 6 and they transform the matrix of elastic modules *C* into symmetric matrix $D^{\varphi\theta}$. Matrices $R_{\varphi\theta}$ and $R^{T}_{\varphi\theta}$ transpose the vectors of stress and strain from system *x'*, *y'*, *z'* to system *x*, *y*, *z* by means of rotational matrix:

$$A = \begin{vmatrix} \cos\theta \sin\varphi & \cos\varphi & \sin\theta \sin\varphi \\ -\cos\theta \cos\varphi & \sin\varphi & -\sin\theta \cos\varphi \\ -\sin\theta & 0 & \cos\theta \end{vmatrix} \qquad (2)$$

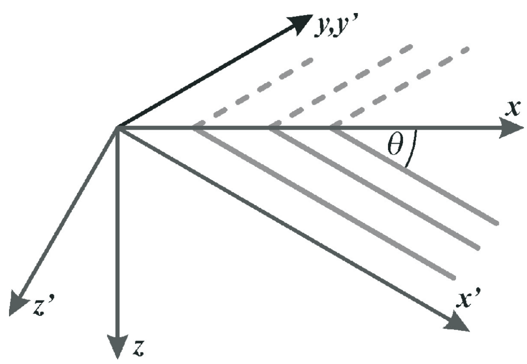

Fig. 1. Geometrical model of monoclinal thin-layered system, angle θ is the tilt angle between axis x' and horizontal plane. Rotation angle φ of system x', y', z' towards the axis is 90°

The subject of discussion will be the situation when the rotation angle $\varphi = 90^o$, i.e. when the coincidence of axis y and y' occurs (Fig. 1). Then, similarly as for the medium VTI [4] the shear waves of type SH may be separated from longitudinal waves P and shear waves SV. It means that displacements U_y of oscillating particles of the medium towards axis y are independent from displacements U_x and U_z in directions x and z respectively. Thus, only components U_x and U_z can be discussed, assuming that U_y and its derivatives equal zero.

Symmetrical matrix C, dimensions 6 × 6, represents components of tensor C_{ijlk} in the medium of transverse isotropy TI. In abbreviated notation by Voigt [14] this matrix can be presented in this way:

$$C = \begin{vmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{vmatrix} \quad (3)$$

Matrix $D^{\varphi=90^o,\theta} = D$ (omitting indices φ and θ) in discussed case may be presented in the following way:

$$D = \begin{vmatrix} d_{11} & d_{12} & d_{13} & 0 & d_{15} & 0 \\ d_{21} & d_{22} & d_{23} & 0 & d_{25} & 0 \\ d_{31} & d_{32} & d_{33} & 0 & d_{35} & 0 \\ 0 & 0 & 0 & d_{44} & 0 & d_{46} \\ d_{51} & d_{52} & d_{53} & 0 & d_{55} & 0 \\ 0 & 0 & 0 & d_{64} & 0 & d_{66} \end{vmatrix} \quad (4)$$

and the elements can be expressed by means of components of tensor C_{ijkl} (in Voigt`s notation) and the tilt angle θ as follows:

$$d_{11} = C_{11}\cos^4\theta + 2C_{13}\cos^2\theta\sin^2\theta + C_{33}\sin^4\theta + \\ + 4C_{44}(\sin\theta\cos\theta)^2 \quad (5)$$

$$d_{21} = d_{12} = C_{12}\cos^2\theta + C_{13}\sin^2\theta \quad (6)$$

$$d_{31} = d_{13} = (C_{11}\cos^2\theta + C_{13}\sin^2\theta)\sin^2\theta + (C_{13}\cos^2\theta + \\ + C_{33}\sin^2\theta)\cos^2\theta - 4C_{44}\sin^2\theta\cos^2\theta \quad (7)$$

$$d_{51} = d_{15} = [C_{13}\cos^2\theta + C_{33}\sin^2\theta - C_{11}\cos^2\theta - \\ - C_{13}\sin^2\theta + 2C_{44}(\cos^2\theta - \sin^2\theta - \\ - \sin^2\theta)]\sin\theta\cos\theta \quad (8)$$

$$d_{22} = C_{11} \quad (9)$$

$$d_{32} = d_{23} = C_{12}\sin^2\theta + C_{13}\cos^2\theta \quad (10)$$

$$d_{25} = d_{52} = (C_{13} - C_{12})\sin\theta\cos\theta \quad (11)$$

$$d_{33} = (C_{11}\sin^2\theta + C_{13}\cos^2\theta)\sin^2\theta + (C_{13}\sin^2\theta + \\ + C_{33}\cos^2\theta)\cos^2\theta + 4C_{44}\sin^2\theta\cos^2\theta \quad (12)$$

$$d_{35} = d_{53} = [C_{13}\sin^2\theta + C_{33}\cos^2\theta - C_{11}\sin^2\theta - \\ - C_{13}\cos^2\theta - 2C_{44}(\cos^2\theta - \sin^2\theta)]\sin\theta\cos\theta \quad (13)$$

$$d_{44} = C_{44}\cos^2\theta + C_{66}\sin^2\theta \quad (14)$$

$$d_{46} = d_{64} = (C_{44} - C_{66})\sin\theta\cos\theta \quad (15)$$

$$d_{55} = (C_{11} - 2C_{13} + C_{33})\sin^2\theta\cos^2\theta + C_{44}(\cos^2\theta - \sin^2\theta)^2 \quad (16)$$

$$d_{66} = C_{44}\sin^2\theta + C_{66}\cos^2\theta \quad (17)$$

and other remaining components are equal zero.

For small tilt angles, when $\theta \to {}^o0$ we have:

$$\begin{aligned} & d_{11} \to C_{11} \\ & d_{12} = d_{21} \to C_{12} \\ & d_{13} = d_{31} \to C_{13} \\ & d_{51} = d_{15} \to 0 \\ & d_{23} = d_{32} \to C_{13} \\ & d_{25} = d_{52} \to 0 \\ & d_{33} \to C_{33} \\ & d_{35} = d_{53} \to 0 \\ & d_{44} \to C_{44} \\ & d_{46} = d_{64} \to 0 \\ & d_{55} \to C_{44} \\ & d_{66} \to C_{66} \end{aligned} \quad (18)$$

So for small tilt angles $\theta \to 0$, as expected, matrix

$$\underset{\theta \to 0}{D} \to C \quad (19)$$

The initial point in the discussion on construction of algorithmic solutions will be Hook`s law – basic relation between the tensor of stress T_{ij} and strain E_{ij}, which results in a conclusion that each stress component is a linear function of strain, i.e.:

$$T_{ij} = d_{ijkl}E_{kl} = d_{ijkl}E_{lk} \quad (20)$$

While the tensor of strain is

$$E_{lk} = \frac{1}{2}\left(U_{l,k} + U_{k,l}\right) \quad (21)$$

Presenting the relation (20) in detail, we receive:

$$T_{ij} = d_{ij11}E_{11}+d_{ij22}E_{22}+d_{ij33}E_{33}+2d_{ij23}E_{23}+ \\ +2d_{ij13}E_{13}+2d_{ij12}E_{12} \quad (22)$$

Substituting $i,j = 1, 2, 3$, we obtain all the components of stress tensor: T_{11}, T_{22}, T_{33}, $T_{23} = T_{32}$, $T_{13} = T_{31}$, $T_{12} = T_{21}$. In the matrix notation this is as follows:

$$\begin{vmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{32} = T_{23} \\ T_{31} = T_{13} \\ T_{12} = T_{21} \end{vmatrix} = D \begin{vmatrix} E_{11} = E_{xx} = U_{x,x} \\ E_{22} = E_{yy} = U_{y,y} = 0 \\ E_{33} = E_{zz} = U_{z,z} \\ 2E_{23} = 2E_{yz} = U_{y,z} = 0 \\ 2E_{31} = U_{z,x} + U_{x,z} \\ 2E_{12} = U_{x,y} = 0 \end{vmatrix} \quad (23)$$

In relation (23) it has been considered that derivatives of the field with regard to coordinate y equal zero. Starting with the general law of movement (disregarding the external force)

$$T_{ij,j} = \rho \frac{\partial^2 U_i}{\partial t^2} \quad (24)$$

where ρ is the medium density, and t denotes time, let us write equations for the horizontal $U_x(U_1)$ and vertical $U_z(U_3)$ component

$$T_{11,1} + T_{13,3} = \rho \frac{\partial^2 U_1}{\partial t^2} \quad (25a)$$

$$T_{31,1} + T_{33,3} = \rho \frac{\partial^2 U_3}{\partial t^2} \quad (25b)$$

Using the matrix equation (23) with relation to equations (25), the following relations are received:

$$d_{11}U_{x,xx} + d_{55}U_{x,zz} + 2d_{15}U_{x,xz} + d_{15}U_{z,xx} + \\ +(d_{13}+d_{55})U_{z,zx} + d_{53}U_{z,zz} = \rho \frac{\partial^2 U_x}{\partial t^2} \quad (26a)$$

$$d_{51}U_{x,xx} + (d_{31}+d_{55})U_{x,xz} + d_{35}U_{x,zz} + d_{33}U_{z,zz} + \\ + 2d_{35}U_{z,xz} + d_{55}U_{z,xx} = \rho \frac{\partial^2 U_z}{\partial t^2} \quad (26b)$$

Adopting for small tilt angles θ $d_{15} = d_{51} \approx 0$ and $d_{53} = d_{35} \approx 0$ and applying Fourier`s transformation ($x \rightarrow k_x, z \rightarrow k_z, t \rightarrow \omega$) $z$ for equations (26) we obtain matrix equation analogical to Christoffel`s relation:

$$\begin{pmatrix} d_{11}k_x^2 + d_{55}k_z^2 - \rho\omega^2 & (d_{13}+d_{55})k_x k_z \\ (d_{31}+d_{55})k_x k_z & d_{33}k_z^2 + d_{55}k_x^2 - \rho\omega^2 \end{pmatrix} \begin{vmatrix} U_x \\ U_z \end{vmatrix} = 0 \quad (27)$$

In equation (27), k_x and k_z denote wavenumbers in horizontal and vertical direction, whereas ω is frequency.

It should be noted that matrix equation (27) for medium VTI, i.e. in the case when the tilt angle $\theta = 0$ transforms into analogical equation derived by Q. Han and R.S. Wu [5]. Equation (27) results in dispersion relation

$$b_0 k_z^4 + b_1 k_z^2 + b_2 = 0 \quad (28)$$

where:

$$b_0 = d_{33}d_{55} \\ b_1 = k_x^2\left(d_{11}d_{33} - d_{13}^2 - 2d_{11}d_{55}\right) - \rho\omega^2\left(d_{33}+d_{55}\right) \\ b_2 = k_x^4 d_{11}d_{55} - \rho\omega^2\left(d_{11}+d_{55}\right)k_x^2 + \rho^2\omega^4 \quad (29)$$

In general, equation (28) has four solutions corresponding to longitudinal wave qP and shear wave qSV (polarized in plane x-z) forward and backward propagation. Q. Han and R.S. Wu [5] concluded on the basis of numerous experiments that wave velocity qSV does not provide significant contribution in the quantity of vertical wavenumber of longitudinal wave qP, therefore it can be assumed that velocity of wave qSV equals zero [1]. On assumption that $b_0 = 0$, we receive:

$$b_1' = k_x^2\left(d_{11}d_{33} - d_{13}^2\right) - \rho\omega^2 d_{33} \\ b_2' = \rho^2\omega^4 - \rho\omega^2 d_{11}k_x^2 \quad (30)$$

Let expressions b_1' and b_2' be represented by parameters analogical to Thomsen`s, i.e.:

$$\varepsilon = \frac{d_{11} - d_{33}}{2d_{33}} \quad (31)$$

$$\partial = \frac{\left(d_{13}+d_{55}\right)^2 - \left(d_{33}-d_{55}\right)^2}{2d_{33}\left(d_{33}-d_{55}\right)} \quad (32)$$

hence

$$q = 1 + 2\varepsilon = \frac{d_{11}}{d_{33}}$$

and for $d_{55} = 0$

$$1 + 2\partial = \frac{d_{13}^2}{d_{33}^2}$$

Assuming that the velocity of longitudinal wave $V_p = \left(\frac{C_{33}}{\rho}\right)^{1/2}$ a $d_{33} = C_{33}cos^2\theta$ we receive:

$$d_{33} = \rho V_p^2 \cos^2\theta = \rho V_{pp}^2 \quad (33)$$

Similarly, using parameters ε and $q=1+2\varepsilon$ and $\eta=2(\varepsilon-\delta)$ we have:

$$d_{11}d_{33}=(1+2\varepsilon)d_{33}^2=q\rho^2V_p^4\cos^4\theta=q\rho^2V_{pp}^4 \quad (34)$$

where: $V_{pp}=V_p\cos\theta$ denotes the velocity of longitudinal wave along the vertical axis

$$d_{13}^2=(1+2\partial)d_{33}^2=(1+2\partial)\rho^2V_{pp}^4 \quad (35)$$

$$d_{11}=(1+2\varepsilon)\rho V_{pp}^2 \quad (36)$$

From relation (29) and relations (30)-(35) we obtain the expression for vertical component of wavenumber k_z

$$k_z=\left(\frac{S_p^4\omega^4-qS_p^2\omega^2k_x^2}{S_p^2\omega^2-\eta k_x^2}\right)^{1/2} \quad (37)$$

where: $S_p=\frac{1}{V_{pp}}$ denotes vertical slowness.

It is not difficult to notice that parameters q and η which can be seen in expression (37) are represented by their values in model VTI, therefore for horizontally laminated medium

$$q=q_{VTI}\cdot\cos^2\theta$$
$$\eta=\eta_{VTI}\cdot\cos^2\theta \quad (38)$$

where: q_{VTI} and η_{VTI} denote the values of these parameters in VTI medium.

Thus relation (37) for medium TTI is analogical to the relation derived by Q. Han and R.S. Wu [5] for the horizontally laminated medium VTI. In order to use it in the process of propagation and migration of compressional waves one should know parameters q and η, the velocity of P waves along the axis and the tilt angle – θ, the plane of isotropy. We will apply received wavenumber k_z for migration MG(F-K) in the domain of wavenumbers and frequencies and spatial coordinates x and time t [8, 11]. Migration process performed in this way occurs in two stages. At the first stage, the relocation of the wave field takes place

$$U'(k_x,z_j+\Delta z,\omega)=e^{-ikz_0\Delta z}U(k_x,z_j,\omega) \quad (39)$$

from the level of z_j to $z_j+\Delta z$ by means of exponential operator with vertical wavenumber kz_0 corresponding to a homogeneous medium. At the second stage, correction of the wave field follows $U'(x, z_j+\Delta z, \omega)$ – Fourier transforms ($k_x \rightarrow$ x) of the field $U'(k_x, z_j+\Delta z, \omega)$ by way of spatial filter $F_j(x, \omega)=[1-i/2\Delta zM_{j(x)}]^{-1}$, which is the sum of Neumann's power series

$$M_{j(x)}=\Sigma k_{z_0}^{-1}\left(k_{z_0}^2-k_z^2\right)e^{ik_xx}dk_x \quad (40)$$

This relation may be represented in this way:

$$U_{zj+\Delta z}+=F_j(x,\omega)U'(x,z_j+\Delta z,\omega) \quad (41)$$

The correction positions the wave field in the function of spatial coordinates, taking into account the differences between parameters of a homogenous medium and parameters of heterogeneous medium in the function of lateral coordinates. For the prestack migration, the algorithm of extrapolation will be the product of corrective functions F related to the sources and receivers, while corrected field U' will be a function of coordinates of sources and receivers. With the zero-offset migration in relation (37) slowness S_p must be multiplied by 2. Further steps follow in an analogical way as in model VTI and it was discussed in detail in the article by A. Kostecki [11]. It should be noted that even when the elastic parameters are independent from the spatial coordinates, it is essential to take into consideration the differences in vertical wavenumbers as a result of different tilt angles of the laminated medium.

Artykuł nadesłano do Redakcji 17.09.2009. Przyjęto do druku 29.10.2009.

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