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OPERATION MODES OF FULL-BRIDGE VOLTAGE-SOURCE SERIES RESONANT INVERTER WITH PFM CONTROL FOR INDUCTION HEATING APPLICATIONS

SUMMARY

Possible operation modes of a full-bridge voltage-source series resonant inverter with PFM control used in induction heating have been dealt with in the paper. The mode the inverter is operated at is determined by the ratio of the used operating frequency to the frequency of free oscillations of the series resonant circuit created by the heating coil with workload and the series capacitor. The most important properties of the inverter have been presented like magnitudes of currents, voltages and power as well as conduction times of semiconductor elements, which depend on the selected operation mode. Fundamental requirements for thyristor and transistor inverters have been given.

Keywords: *electroheat, induction heating, power electronics, inverters*

STANY PRACY MOSTKOWEGO FALOWNIKA NAPIĘCIOWEGO Z SZEREGOWYM OBWODEM REZONANSOWYM DO NAGRZEWANIA INDUKCYJNEGO PRZY STEROWANIU CZĘSTOTLIWOŚCIOWYM

W artykule przedstawiono możliwe stany pracy sterowanego częstotliwościowo falownika rezonansowego szeregowego napięciowego pracującego w układzie mostkowym stosowanego do nagrzewania indukcyjnego, określone stosunkiem częstotliwości przełączania łączników do częstotliwości drgań własnych obwodu rezonansowego utworzonego przez wzbudnik ze wsadem i kondensator szeregowy. Przedstawiono najważniejsze właściwości falownika, takie jak wartości prądów, napięć i mocy oraz czasy przewodzenia elementów półprzewodnikowych, które są zależne od wybranego stanu pracy. Podano zasadnicze wymagania dla falowników tyrystorowych oraz tranzystorowych.

Słowa kluczowe: *elektrotermia, nagrzewanie indukcyjne, energoelektronika, falowniki*

1. INTRODUCTION

The object of the analysis is a full-bridge series resonant inverter for induction heating applications operated using PFM control. A circuit of the inverter is shown in Figure 1. The inverter is fed by a DC voltage source U_d and consists of four unidirectional power electronic switches $S_1 \div S_4$ with antiparallel diodes $D_1 \div D_4$, heating coil with workload represented by resistance R_o and inductance L_o connected in series and capacitor C_s also connected in series. Switches $S_1 \div S_4$ can be SCR thyristors or controllable semiconductor devices. At PFM control the switches are alternately ON and OFF in pairs at the operating frequency $f_s = \dfrac{\omega_s}{2\pi}$ with a duty cycle of 50%. When the switches conduct current, the energy is drawn from the supply source and when the diodes conduct current – the energy is returned to the capacitor C_d connected at the input of the inverter where $C_d \gg C_s$ (Fig. 1).
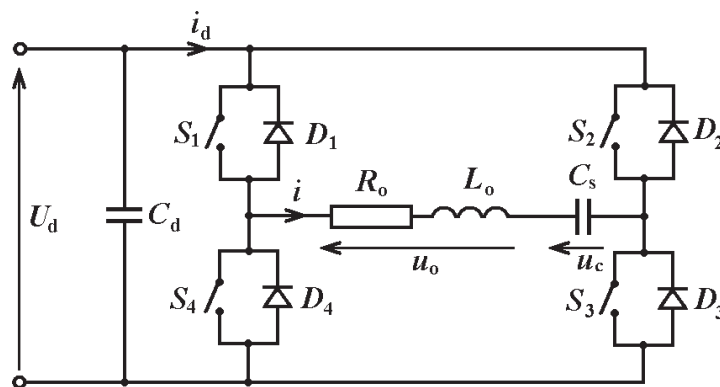

Fig. 1. Basic circuit of full-bridge voltage-source series resonant inverter for induction heating

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Current and voltage waveforms and the power dissipated in the load depend strongly on the operating frequency f_s. Five different operation modes of the inverter can be distinguished, where two of them are only possible when using controllable switches.

2. ANALYTIC DETERMINATION OF INVERTER CURRENTS AND VOLTAGES

To carry out a mathematical analysis of the inverter the following assumptions have been made:

– supply voltage U_d stays constant,
– the switches and diodes used are ideal elements,
– that active power is only dissipated in the resistance R_o of the receiver,
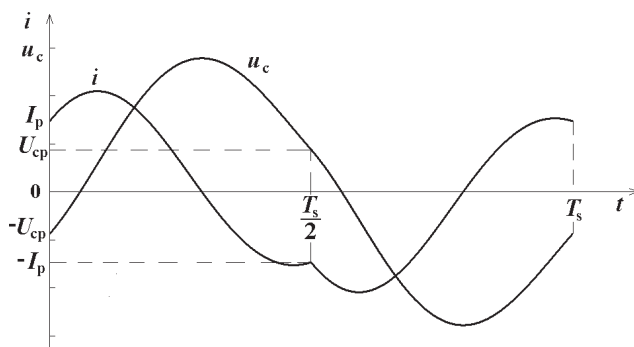– all parasitic inductances and capacitances as well as commutating reactors are negligible.

Fig. 2. Exemplary waveforms of load current i and voltage u_c across capacitor C_s

Figure 2 shows possible waveforms of current i of the receiver R_oL_o and voltage u_c across capacitor C_s. The waveforms are antisymmetrical so it is sufficient to determine the equations describing them for the half-cycle $\left(0 \div \dfrac{T_s}{2}\right)$. The equivalent circuit of the inverter in operational form valid for the above-given half-cycle is shown in Figure 3. The senses of initial values of coil current I_p and voltage U_{cp} across capacitor C_s have been so chosen that at $t = 0$ the values of I_p and U_{cp} are positive for waveforms from Figure 2. Thus $i(0) = I_p$ and $u_c(0) = -U_{cp}$.
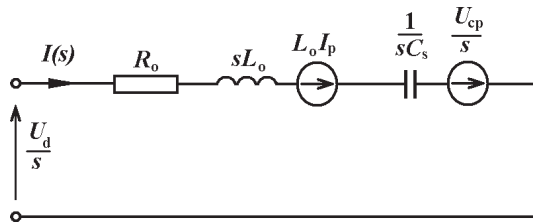

Fig. 3. Equivalent circuit of the inverter in operational form

After assuming that damped oscillations occur in the inverter, which corresponds to the condition $R_o < 2\sqrt{\dfrac{L_o}{C_s}}$, be-

ing fulfilled in case of induction heaters and furnaces, current $i(t)$ of the receiver R_oL_o and voltage $u_c(t)$ across capacitor C_s were determined in [1, 2]:

$$i(t) = e^{-\alpha_o t}\left[I_p \cos\omega_o t + \left(\frac{U_d + U_{cp}}{\omega_o L_o} - I_p\frac{\alpha_o}{\omega_o}\right)\sin\omega_o t\right] \tag{1}$$

$$u_c(t) = U_d +$$
$$-e^{-\alpha_o t}\left\{(U_d + U_{cp})\cos(\omega_o t) + \left[(U_d + U_{cp})\frac{\alpha_o}{\omega_o} - \frac{I_p}{\omega_o C_s}\right]\sin(\omega_o t)\right\} \tag{2}$$

Using relative values of current and voltage facilitates analysis of the inverter's properties. After assuming designations:

$$i_w = \frac{i}{\dfrac{U_d}{\omega_o L_o}}; \quad u_{cw} = \frac{u_c}{U_d}; \quad u_{ow} = \frac{u_o}{U_d}$$

$$\tag{3}$$

$$I_{pw} = \frac{I_p}{\dfrac{U_d}{\omega_o L_o}}; \quad U_{cpw} = \frac{U_{cp}}{U_d}$$

and replacing time t with variable $\alpha = \omega_s t$, the relative values of load current i_w, voltage u_{cw} across capacitor C_s and voltage u_{ow} across receiver R_oL_o are given by:

$$i_w(\alpha) = e^{-\frac{\alpha_o}{\omega_o}\frac{\omega_o}{\omega_s}\alpha} \times$$
$$\times\left[I_{pw}\cos\left(\frac{\omega_o}{\omega_s}\alpha\right) + \left(1 + U_{cpw} - \frac{\alpha_o}{\omega_o}I_{pw}\right)\sin\left(\frac{\omega_o}{\omega_s}\alpha\right)\right] \tag{4}$$

or

$$i_w(\alpha) = I_{mw}\, e^{-\frac{\alpha_o}{\omega_o}\frac{\omega_o}{\omega_s}\alpha}\,\sin\left(\frac{\omega_o}{\omega_s}\alpha - \varphi_1\right) \tag{5}$$

$$u_{cw}(\alpha) = 1 - e^{-\frac{\alpha_o}{\omega_o}\frac{\omega_o}{\omega_s}\alpha} \times$$
$$\times\left\{(1 + U_{cpw})\cos\left(\frac{\omega_o}{\omega_s}\alpha\right) + \left[\frac{\alpha_o}{\omega_o}(1 + U_{cpw}) - I_{pw}\left[1 + \left(\frac{\alpha_o}{\omega_o}\right)^2\right]\right]\sin\left(\frac{\omega_o}{\omega_s}\alpha\right)\right\} \tag{6}$$

or

$$u_{cw}(\alpha) = 1 - \frac{I_{mw}}{\sin\gamma_o}\, e^{-\frac{\alpha_o}{\omega_o}\frac{\omega_o}{\omega_s}\alpha}\,\sin\left(\frac{\omega_o}{\omega_s}\alpha - \varphi_1 + \gamma_o\right) \tag{7}$$

$$u_{ow}(\alpha) = 1 - u_{cw}(\alpha) \tag{8}$$

where:

$$\alpha_o = \frac{R_o}{2L_o}; \quad \omega_o = \sqrt{\frac{1}{L_o C_s} - \alpha_o^2} \qquad (9)$$

$$I_{mw} = \frac{I_m}{\dfrac{U_d}{\omega_o L_o}} = \sqrt{I_{pw}^2 + \left(1 + U_{cpw} - \frac{\alpha_o}{\omega_o} I_{pw}\right)^2} \qquad (10)$$

$$tg\varphi_1 = \frac{-I_{pw}}{1 + U_{cpw} - \dfrac{\alpha_o}{\omega_o} I_{pw}} \qquad (11)$$

$$tg\gamma_o = \frac{\omega_o}{\alpha_o} \qquad (12)$$

Both current i_w and voltage u_{cw} are antisymmetrical functions of time, so after half-cycle their values equal:

$$i_w(\pi) = -I_{pw}; \quad u_{cw}(\pi) = U_{cpw} \qquad (13)$$

After taking account of the above given equalities in (4) and (6) and carrying out appropriate rearrangements we obtain [1, 2]:

$$I_{pw} = -\frac{\sin\left(\pi\dfrac{\omega_o}{\omega_s}\right)}{\cosh\left(\pi\dfrac{\alpha_o}{\omega_o}\dfrac{\omega_o}{\omega_s}\right) + \cos\left(\pi\dfrac{\omega_o}{\omega_s}\right)} \qquad (14)$$

$$U_{cpw} = \frac{\sinh\left(\pi\dfrac{\alpha_o}{\omega_o}\dfrac{\omega_o}{\omega_s}\right) - \dfrac{\alpha_o}{\omega_o}\sin\left(\pi\dfrac{\omega_o}{\omega_s}\right)}{\cosh\left(\pi\dfrac{\alpha_o}{\omega_o}\dfrac{\omega_o}{\omega_s}\right) + \cos\left(\pi\dfrac{\omega_o}{\omega_s}\right)} \qquad (15)$$

Substituting (14) and (15) in (11) yields

$$tg\varphi_1 = \frac{\sin\left(\pi\dfrac{\omega_o}{\omega_s}\right)}{e^{\pi\frac{\alpha_o}{\omega_o}\frac{\omega_o}{\omega_s}} + \cos\left(\pi\dfrac{\omega_o}{\omega_s}\right)} \qquad (16)$$

As mentioned above, Eqs. (4) and (5) describing current and Eqs. (6), (7) and (8) describing voltages are valid for $(0\div\pi)$ corresponding to the time interval $\left(0\div\dfrac{T_s}{2}\right)$ during

which switches S_1 and S_3 or diodes D_1 and D_3 conduct. During the time intervals when switches S_2 and S_4 or diodes D_2 and D_4 conduct, the functions describing the current and the voltages have the opposite sign to those described with Eqs. (4)÷(8).

The relative values of the current and voltages (Eqs. (4)÷(8)) depend only on angle α and parameters $\dfrac{\omega_s}{\omega_o}$ and $\dfrac{\alpha_o}{\omega_o}$, which makes it easier to analyse the properties of the discussed inverter.

3. SPECIFICATION OF THE INVERTER OPERATION MODES

Five operation modes of the inverter can be distinguished depending on the ratio of the used operating angular frequency ω_s to the angular frequency ω_o (9) of free oscillations of the series resonant circuit $R_o L_o C_s$ [3]:

mode I	– at $\omega_s > \omega_o$	
mode II	– at $\omega_s = \omega_o$	– continuous load current,
mode III	– at $0.5\omega_o < \omega_s < \omega_o$	
mode IV	– at $\omega_s = 0.5\omega_o$	– boundary of continuous load current,
mode V	– at $\omega_s < 0.5\omega_o$	– discontinuous load current.

The operation modes of the inverter have been characterized shortly and waveforms of current i_w and voltage u_{cw} in function of angle $\alpha = \omega_s t$ determined based on equations given in the previous chapter have been shown for each mode. The intervals during which odd elements (S_1 and S_3 or D_1 and D_3) conduct have also been marked.

Mode I – operating frequency higher than frequency of free oscillations, $\dfrac{\omega_s}{\omega_o} > 1$ $\left(\dfrac{T_s}{T_o} < 1\right)$.

The characteristic feature of this mode is turning the switches off during their conduction time – only controllable switches can be used in this case. Waveforms of current i_w and voltage u_{cw} in mode I are shown in Figure 4.
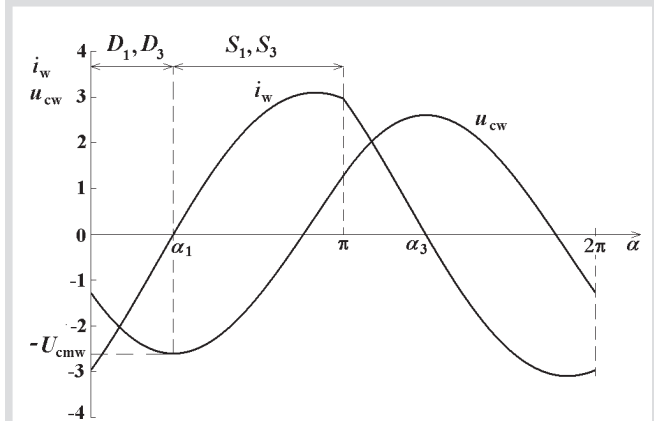
Fig. 4. Waveforms of load current i_w and voltage u_{cw} across capacitor C_s in mode I for $\dfrac{\alpha_o}{\omega_o} = 0.1$ and $\dfrac{\omega_s}{\omega_o} = 1.20$

At angle $\alpha = 0$ the switches S_2 and S_4, which have been conducting so far are turned off, which causes a voltage rise across inductance L_o and, consequently, diverting the load current to diodes D_1 and D_3. They conduct during the interval $(0 \div \alpha_1)$ until their current reaches zero. The current is then diverted to switches S_1 and S_3, which conduct until their turn-off at angle π. The current is taken over by diodes D_2 and D_4, and then (angle α_3) by switches S_2 and S_4.

In this operation mode of the inverter there is a natural commutation of the current from the diode to the switch in each inverter arm. Consequently, the switches are turned on at very advantageous conditions – at zero current and nearly zero voltage. At switch turn off the conditions are disadvantageous – full load current and full supply voltage.

Mode II – operating frequency equals frequency of free oscillations, $\dfrac{\omega_s}{\omega_o} = 1 \left(\dfrac{T_s}{T_o} = 1 \right)$.

This mode (Fig. 5) is a peculiar case of the operation of the inverter – operating frequency equals frequency of resonant circuit free oscillations. The diodes do not conduct at all in this mode – each pair of switches conducts during time $\dfrac{T_o}{2}$. Switches S_1 and S_3 conduct during the interval $(0 \div \pi)$ and switches S_2 and S_4 – during $(\pi \div 2\pi)$.
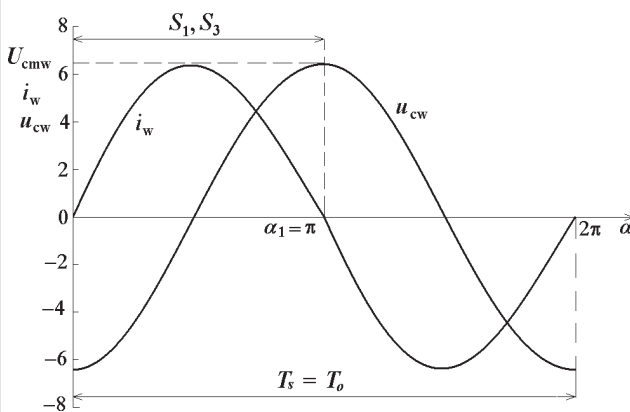

Fig. 5. Waveforms of load current i_w and voltage u_{cw} across capacitor C_s in mode II for $\dfrac{\alpha_o}{\omega_o} = 0.1$

This operation mode is only possible with controllable switches. Due to turning the switches on and off at zero current this is an advantageous operation mode. Mode II is the only mode without time intervals when energy is returned to capacitor C_d (Fig. 1).

It should be stressed that this operation mode occurs at operating frequency being exactly equal to frequency $f_o = \dfrac{\omega_o}{2\pi}$ of free oscillations of the series $R_oL_oC_s$ circuit and not to resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{L_oC_s}}$ of undamped circuit, which is often assumed in simplified analyses.

Mode III – operating frequency is less than frequency of free oscillations and $0.5 < \dfrac{\omega_s}{\omega_o} < 1 \left(1 < \dfrac{T_s}{T_o} < 2 \right)$.

In this operation mode of the inverter (Fig. 6) switches S_1 and S_3 are turned on at angle $\alpha = 0$ during conduction of diodes D_2 and D_4. When the current through the switches reaches zero (angles α_1, α_3 respectively), it is diverted in each arm to the antiparallel diode.
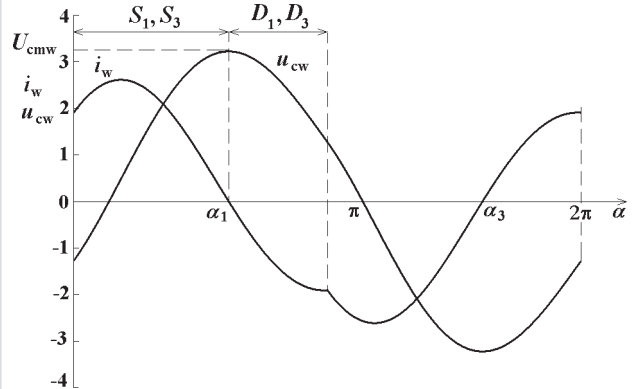

Fig. 6. Waveforms of load current i_w and voltage u_{cw} across capacitor C_s in mode III for $\dfrac{\alpha_o}{\omega_o} = 0.1$ and $\dfrac{\omega_s}{\omega_o} = 0.83$

Conduction time of each element is less than $\dfrac{T_o}{2}$: switches conduct from the instant of their turn-on at non-zero current until their current reaches zero, diodes – from the instant when the current crosses zero until turn-on of the other pair of switches.

In this operation mode of the inverter, like in modes IV and V described below, there is a natural commutation of the current from the switch to the diode in each inverter arm. Hence, the switches are turned off at very advantageous conditions – at zero current and nearly zero voltage. On the contrary, the switch turn-on takes place at very disadvantageous conditions – at full current and voltage equal to supply voltage. Additionally, high reverse-recovery diode current spikes flows also through the switches that have been turned-on (this is not seen in Fig. 6, because the elements have been assumed ideal in the analysis). This is very disadvantageous for transistor inverters operating at high frequencies; therefore this operation mode should be avoided if controlled switches (e.g. power MOSFETs) are used. However, the mode III is the fundamental operation mode for thyristor inverters.

Mode IV – operating frequency equal to half frequency of free oscillations, $\dfrac{\omega_s}{\omega_o} = 0.5 \left(\dfrac{T_s}{T_o} = 2 \right)$ – boundary of continuous load current.

The case of $\dfrac{\omega_s}{\omega_o} = 0.5$ is the other (besides mode II) peculiar operation mode of the inverter occurring at a strictly determined frequency. Waveforms of current i_w and voltage u_{cw} for this operation mode of the inverter are shown in Figure 7. Switches S_1 and S_3 start conducting immediately

after the current of diodes D_2 and D_4 have reached zero (angle $\alpha = 0$), and switches S_2 and S_4 – after the current of diodes D_1 and D_3 have reached zero (angle $\alpha = \pi$).
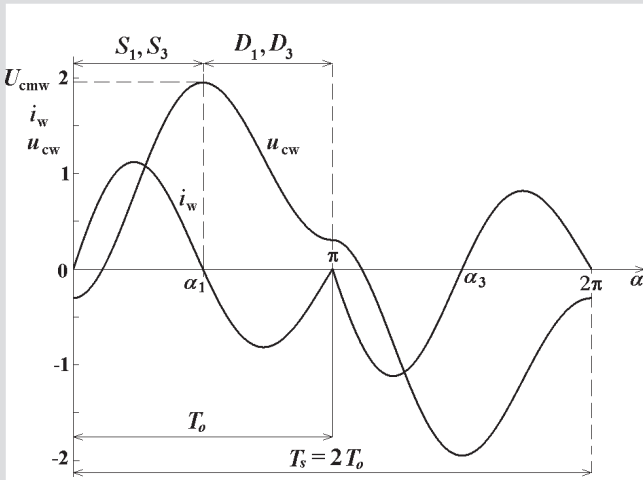

Fig. 7. Waveforms of load current i_w and voltage u_{cw} across capacitor C_s in mode IV for $\dfrac{\alpha_o}{\omega_o} = 0.1$

Mode V – operating frequency less than half frequency of free oscillations, $\dfrac{\omega_s}{\omega_o} < 0.5 \left(\dfrac{T_s}{T_o} > 2 \right)$ – discontinuous load current.

Mode V differs from mode IV in that the switches start conducting not immediately after the current of the other pair switch – diode have reached zero but with a time delay; thus there are time intervals with zero load current (Fig. 8). Conduction time of thyristors and diodes is the same as in mode IV and equals $\dfrac{T_o}{2}$. Instantaneous values of currents and voltages in mode V are the same as in mode IV, with the difference that during the time intervals with zero load current the voltage across capacitor C_s stays constant equal to initial value U_{cpw} (interval $\alpha_2 \div \pi$) or $-U_{cpw}$ (interval $\alpha_5 \div 2\pi$).
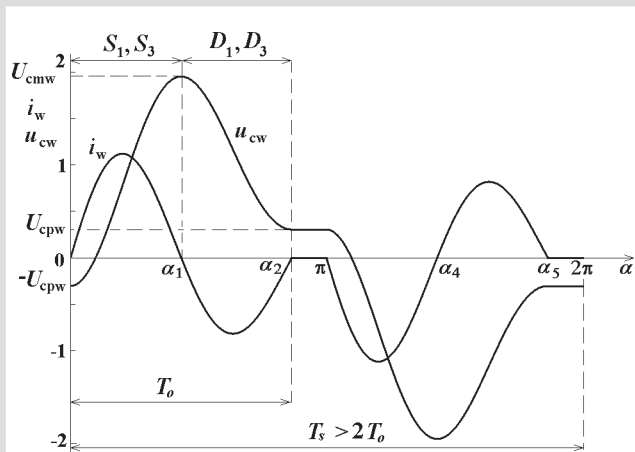

Fig. 8. Waveforms of load current i_w and voltage u_{cw} across capacitor C_s in mode V for $\dfrac{\alpha_o}{\omega_o} = 0.1$ and $\dfrac{\omega_s}{\omega_o} = 0.45$

Mode V is the only mode with discontinuous current. The length of time intervals with zero current depends on how much cycle T_s is larger than double period T_o. The inverter output power can be varied by changing the length of zero current time intervals.

An advantage of inverter operation in mode IV or V is that all devices are turned ON and OFF at zero current, and the drawback is that an important amount of energy is returned to the supply source (capacitor C_d) during conduction time of diodes and, consequently, the output power is small.

The load current is distorted (Figs 4–8) and its shape is the least different from sinusoidal in mode II.

4. CURRENTS AND VOLTAGES IN PARTICULAR OPERATION MODES OF THE INVERTER

Relationships between selected inverter quantities and relative operating frequency (independent variable $\dfrac{\omega_s}{\omega_o}$) for various values of parameter $\dfrac{\alpha_o}{\omega_o}$ are dealt with below. It has been assumed that $\dfrac{\omega_s}{\omega_o}$ varies within the range 0.5÷1.5, corresponding to modes from IV to I, while the information about behaviour of a given quantity for $\dfrac{\omega_s}{\omega_o}$ over the range beneath 0.5 (mode V) will be given as a comment. The range for $\dfrac{\alpha_o}{\omega_o}$ has been assumed to be 0.01÷0.6, corresponding to quality factor Q of the resonant circuit being approximately within the range of 1÷50, which includes quality factors Q of induction heaters and furnaces used in practice.

Figure 9 represents graphs of relative value of initial current I_{pw} (14). The initial value I_p of current equals zero in modes II, IV and V (Figs 5, 7, 8), is positive in mode III (Fig. 6 – it is the switches that are turned on at $\alpha = 0$) and is negative in mode I (Fig. 4 – it is the diodes that start conducting at $\alpha = 0$). The value of I_{pw} depends very strongly on the damping factor of the circuit – it is the higher, the lower is load resistance R_o and the corresponding value of parameter $\dfrac{\alpha_o}{\omega_o}$. In view of switching loss minimization it is advantageous to operate the inverter at a small value of the initial current.

The expression for voltage initial value U_{cpw} (15) becomes simplified in mode II to the form

$$U_{cpw} = U_{cpw2} = \frac{\sinh\left(\pi \dfrac{\alpha_o}{\omega_o} \right)}{\cosh \left(\pi \dfrac{\alpha_o}{\omega_o} \right) - 1} \tag{17}$$
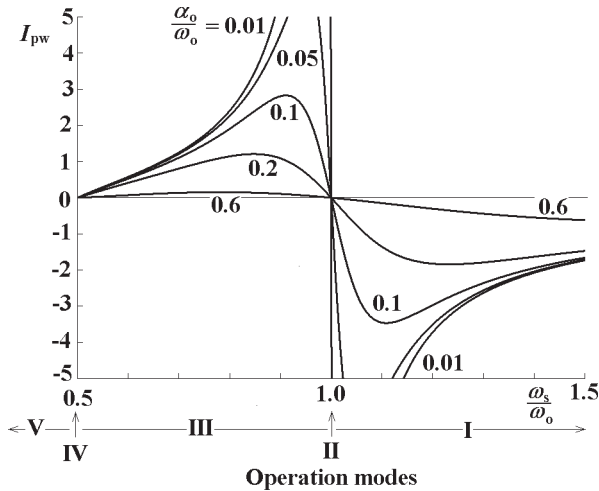
Fig. 9. Plots of $I_{pw} = f\left(\dfrac{\omega_s}{\omega_o}\right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

and in mode IV it yields

$$U_{cpw} = U_{cpw4} = \frac{\sinh\left(2\pi\dfrac{\alpha_o}{\omega_o}\right)}{\cosh\left(2\pi\dfrac{\alpha_o}{\omega_o}\right)+1} \tag{18}$$

The graphs of U_{cpw} are shown in Figure 10. They reach maximum values for $\dfrac{\omega_s}{\omega_o}=1$ (mode II), which are the bigger, the smaller the value of parameter $\dfrac{\alpha_o}{\omega_o}$.
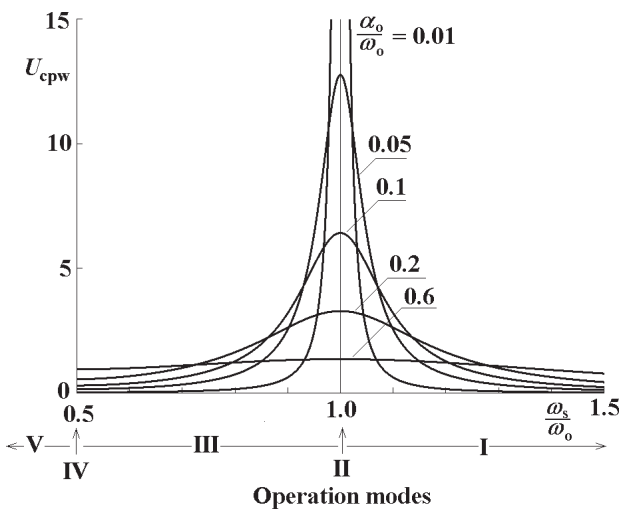

Fig. 10. Plots of $U_{cpw} = f\left(\dfrac{\omega_s}{\omega_o}\right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

When moving away from the operating point $\dfrac{\omega_s}{\omega_o}=1$ (both decreasing or increasing $\dfrac{\omega_s}{\omega_o}$) the value of U_{cpw} decreases, and the stronger, the smaller is the value of parameter $\dfrac{\alpha_o}{\omega_o}$. For $\dfrac{\omega_s}{\omega_o}$ less than 0.5 (operation mode V) the initial value U_{cpw} stays constant (independently of operating frequency) being equal to U_{cpw4} (18). U_{cpw} is always positive, which means that at the assumed sense of voltages (Fig. 3), the initial value of voltage $u_c(0)$ across the capacitor is always negative, independently of the inverter operating frequency (that means $U_{cp} > 0$, $u_c(0) = - U_{cp}$).

The expression for relative value of current I_{mw} (10) takes in modes II, IV and V the following form

$$I_{mw} = 1 + U_{cpw} \tag{19}$$

where the value of U_{cpw} is given by (17) or (18) respectively. The maximum value I_{maxw} of current i_w (5) was computed numerically basing on the knowledge of I_{mw} and shown in Figure 11.
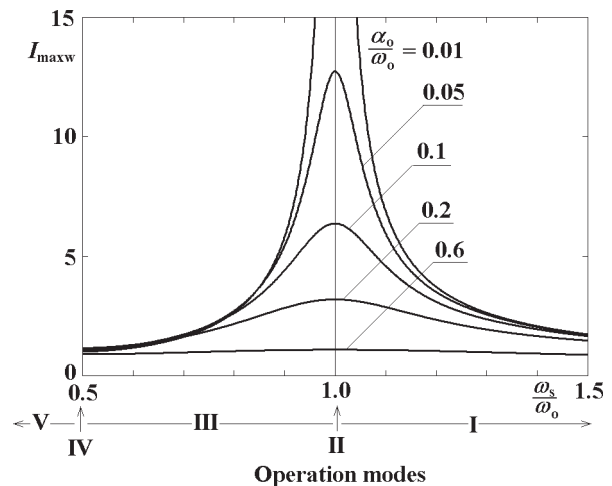

Fig. 11. Maximum receiver current $I_{\max w} = f\left(\dfrac{\omega_s}{\omega_o}\right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

Current I_{maxw} reaches highest values at $\dfrac{\omega_s}{\omega_o}=1$ and small values of parameter $\dfrac{\alpha_o}{\omega_o}$. The range of variation of I_{maxw} in terms of relative operating frequency depends very strongly on the damping factor of the circuit: the current I_{maxw} varies over a wide range in a circuit with low damping whereas in case of high damping the current variation is not big.

In operation mode V $\left(\dfrac{\omega_s}{\omega_o} < 0.5\right)$ the current $I_{\max}$ stays constant, its magnitude being equal to that occurring in mode IV.

Plots of angle φ_1 (16) as a function of relative operating frequency $\dfrac{\omega_s}{\omega_o}$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$ are shown in Figure 12. The zero value of angle φ_1 corresponds to zero value of current I_{pw} (Fig. 9) and the angle φ_1 also equals zero in mode V. It is positive in operation mode I and negative in mode III. In these modes the absolute value of angle φ_1 is the higher, the smaller is the value of parameter $\dfrac{\alpha_o}{\omega_o}$.
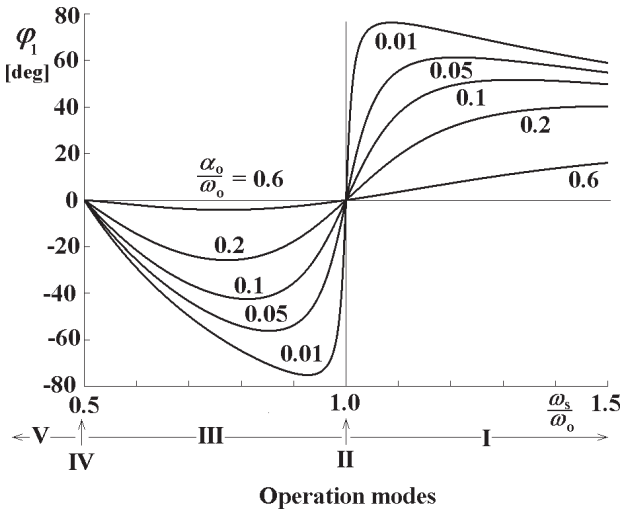

Fig. 12. Plots of angle $\varphi_1 = f\left(\dfrac{\omega_s}{\omega_o}\right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

To determine the maximum value of voltage U_{cm} across capacitor C_s two cases should be distinguished. At the inverter operation in mode I the maximum value of voltage across the capacitor occurs at $\alpha_1 = \dfrac{\omega_s}{\omega_o}\varphi_1$ (Fig. 4) while in each mode from II to V the voltage reaches its maximum at the instant corresponding to $\alpha_1 = \dfrac{\omega_s}{\omega_o}(\pi + \varphi_1)$ (Equ. 5) and Figs 5–8). Introducing the above expressions for α_1 into Equ. (7) and taking into account the fact that the magnitude of U_{cmw} is the same in modes IV and V, the following formulas describing U_{cmw} can be determined

$$U_{cmw} = \begin{cases} 1 + I_{mw4}\, e^{-\frac{\alpha_o}{\omega_o}\pi} & \text{at} \quad \dfrac{\omega_s}{\omega_o} \le 0.5 \quad \text{(modes IV i V)} \\[2ex] 1 + I_{mw}\, e^{-\frac{\alpha_o}{\omega_o}(\pi + \varphi_1)} & \text{at} \quad 0.5 < \dfrac{\omega_s}{\omega_o} \le 1 \quad \text{(modes II i III)} \\[2ex] -1 + I_{mw}\, e^{-\frac{\alpha_o}{\omega_o}\varphi_1} & \text{at} \quad \dfrac{\omega_s}{\omega_o} > 1 \quad \text{(mode I)} \end{cases} \quad (20)$$

where the value of I_{mw4} denoting the magnitude of I_{mw} in mode IV can be computed basing on (19) and (18).

The maximum voltage U_{cm} across capacitor C_s reaches highest values at $\dfrac{\omega_s}{\omega_o} = 1$ and small values of parameter $\dfrac{\alpha_o}{\omega_o}$ (Fig. 13). Voltage U_{cm} is much smaller at a high damping factor and in that case it depends to a much lesser degree on the operating frequency than at a low damping factor.
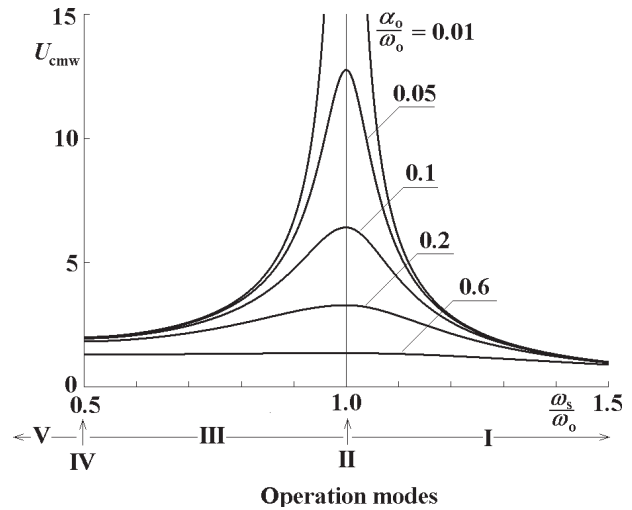

Fig. 13. Maximum voltage $U_{cmw} = f\left(\dfrac{\omega_s}{\omega_o}\right)$ across capacitor C_s for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

It results from Figures 10, 11 and 13 that highest magnitudes of current and voltages occur at a small damping factor of the circuit and the operating frequency equal to the frequency of free oscillations of the resonant circuit. In view of advantageous switching conditions, the operation at $\omega_s = \omega_o$ is preferred in case of transistor inverters. However, in view of high currents and voltages occurring in this operation mode, special care should be taken to avoid failure of the semiconductor devices used.

The voltages across the switches are square waves of magnitude U_d. Hence, one of the advantages of the inverter is the low voltage across the switches, which is equal to supply voltage.

5. CONDUCTION TIMES OF SWITCHES AND DIODES

Conduction time t_T of each switch $S_1 - S_4$ in mode I $\left(\dfrac{\omega_s}{\omega_o} > 1\right)$ of the inverter operation corresponds to the interval $(\alpha_1 \div \pi)$ in Figure 4 and equals

$$t_T = \frac{T_s}{2} - \frac{\varphi_1}{\omega_o} \qquad (21)$$

whereas in modes II – V $\left(\dfrac{\omega_s}{\omega_o} \leq 1 \right)$ time t_T corresponds to the interval $(0 \div \alpha_1)$ in Figures 5–8 and it yields

$$t_T = \frac{\pi + \varphi_1}{\omega_o} \qquad (22)$$

In modes I – III $\left(\dfrac{\omega_s}{\omega_o} > 0.5 \right)$ the inverter current is continuous and conduction time t_D of diodes equals

$$t_D = \frac{T_s}{2} - t_T \qquad (23)$$

whilst in modes IV and V $\left(\dfrac{\omega_s}{\omega_o} \leq 0.5 \right)$ time t_D equals conduction time t_T of the switches

$$t_D = t_T = \frac{T_o}{2} \qquad (24)$$

Introducing relative values of conduction times of switches and diodes:

$$t_{Tw} = \frac{t_T}{T_s}; \quad t_{Dw} = \frac{t_D}{T_s} \qquad (25)$$

gives:

$$t_{Tw} = \begin{cases} \dfrac{1}{2} \dfrac{\omega_s}{\omega_o} \left(1 + \dfrac{\varphi_1}{\pi} \right) & \text{at} \quad \dfrac{\omega_s}{\omega_o} \leq 1 \quad \text{(modes II – V)} \\[3mm] \dfrac{1}{2} \left(1 - \dfrac{\omega_s}{\omega_o} \dfrac{\varphi_1}{\pi} \right) & \text{at} \quad \dfrac{\omega_s}{\omega_o} > 1 \quad \text{(mode I)} \end{cases} \qquad (26)$$

$$t_{Dw} = \begin{cases} \dfrac{1}{2} \dfrac{\omega_s}{\omega_o} & \text{at} \quad \dfrac{\omega_s}{\omega_o} \leq 0.5 \quad \text{(modes IV, V)} \\[3mm] \dfrac{1}{2} \left[1 - \dfrac{\omega_s}{\omega_o} \left(1 + \dfrac{\varphi_1}{\pi} \right) \right] & \text{at} \quad 0.5 < \dfrac{\omega_s}{\omega_o} \leq 1 \text{ (modes III, IV)} \\[3mm] \dfrac{1}{2} \dfrac{\omega_s}{\omega_o} \dfrac{\varphi_1}{\pi} & \text{at} \quad \dfrac{\omega_s}{\omega_o} > 1 \quad \text{(mode I)} \end{cases} \qquad (27)$$

Relative time t_{Tw} of conduction of switches is shown in Figure 14 and relative time t_{Dw} of conduction of diodes – in Figure 15. In mode II of the inverter operation the switches conduct during half-cycle $\dfrac{T_s}{2}$ and the diodes do not conduct at all. In modes I and III the difference between the conduction time of the switches and the conduction time of the diodes depends strongly on the damping factor of the circuit – it is the bigger, the bigger the damping factor.

By way of example, at $\dfrac{\omega_s}{\omega_o} = 0.95$ (mode III) relative conduction times equal respectively $t_{Tw} = 0.280$ and $t_{Dw} = 0.220$ at $\dfrac{\alpha_o}{\omega_o} = 0.01$ compared with $t_{Tw} = 0.471$ and $t_{Dw} = 0.029$ at $\dfrac{\alpha_o}{\omega_o} = 0.6$. In operation mode IV each element conducts during ¼ of cycle $T_s = 2T_o$, and in mode V conduction time of each element is also the same and equals less than ¼ of cycle T_s.

In the case of a thyristor inverter an adequately long diode conduction time should be assured, which is at the same time thyristor reverse recovery time.
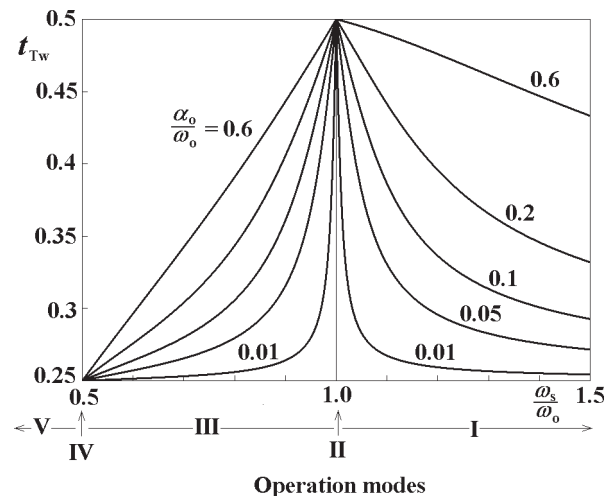

Fig. 14. Relative conduction time of switches $t_{Tw} = f\left(\dfrac{\omega_s}{\omega_o} \right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$
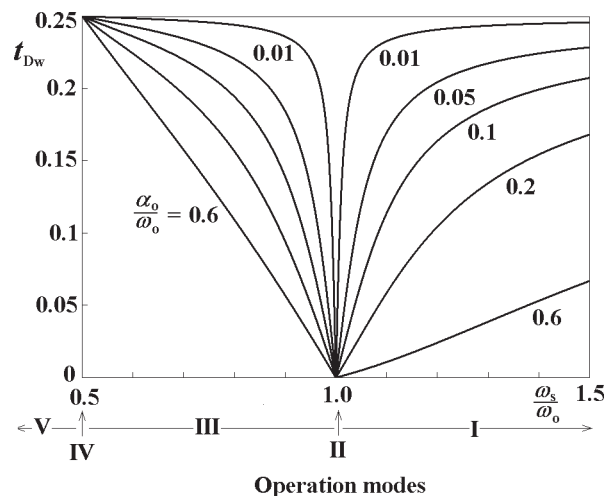

Fig. 15. Relative conduction time of diodes $t_{Dw} = f\left(\dfrac{\omega_s}{\omega_o} \right)$ for various values of parameter $\dfrac{\alpha_o}{\omega_o}$

6. POWER OF THE INVERTER

The power of the inverter can be determined basing on the relationship

$$P = \frac{2}{T_s}\int_o^{\frac{T_s}{2}} U_d \cdot i_d(t)dt = U_d I_s \qquad (28)$$

where I_s is mean value of inverter current i_d (Fig. 1), which was calculated in [1]. After introducing relative power defined as

$$P_w = \frac{P}{\dfrac{U_d^2}{\omega_o L_o}} \qquad (29)$$

and carrying out appropriate rearrangements one obtains the equation

$$P_w = \frac{2}{\pi}\frac{\omega_s}{\omega_o}\frac{U_{cpw}}{1+\left(\dfrac{\alpha_o}{\omega_o}\right)^2} \qquad (30)$$

valid for modes I – IV and the equation

$$P_w = 2\frac{\omega_s}{\omega_o}P_{w4} \qquad (31)$$

determining the relative power of the inverter in mode V, where P_{w4} denotes the relative inverter power in mode IV. Graphs of the relative inverter power are shown in Figure 16. For two characteristic operation modes II $\left(\dfrac{\omega_s}{\omega_o}=1\right)$ and IV $\left(\dfrac{\omega_s}{\omega_o}=0.5\right)$ relative powers $P_{w2}=4.046$ and

$P_{w4}=0.096$ have been determined assuming $\dfrac{\alpha_o}{\omega_o}=0.1$. The inverter power in mode IV equals only 2.37% power in mode II, whereas the inverter power in mode V is still less.

Thus, the power dissipated in the load can be varied over a wide range by changing the operating frequency f_s. This power control method is not advantageous in case of inverters using controllable power electronic elements operating at frequencies of the order of several dozens of kilohertz or more. In this case special attention should be paid to minimizing power loss. The inverter operation in mode II, IV or V, where all elements are turned ON and OFF at zero current, is the most advantageous in this respect. However, in view of inverter's small power in modes IV and V, the operation mode II is of fundamental importance. Power control can be achieved either by using a voltage-controlled supply source or special control methods, e.g. PDM [6], assuring zero-current switching.

7. CONCLUSIONS

Five various modes of the inverter operation can be distinguished in view of the shape of current and voltage waveforms in the inverter when using PFM power control. Each mode is determined by the value of ratio of operating frequency ω_s to frequency ω_o of resonant circuit free oscillations, where modes I, III and V correspond to the ratio $\dfrac{\omega_s}{\omega_o}$ from an appropriate range and either mode II and IV – to one particular value of $\dfrac{\omega_s}{\omega_o}$.

An inverter with controllable switches can operate at any mode, while an inverter with SCR thyristors can only operate in modes III – V, whereas in case of operation mode III an additional condition $\dfrac{\omega_s}{\omega_o}<\left(\dfrac{\omega_s}{\omega_o}\right)_{max}<1$ must be fulfilled.
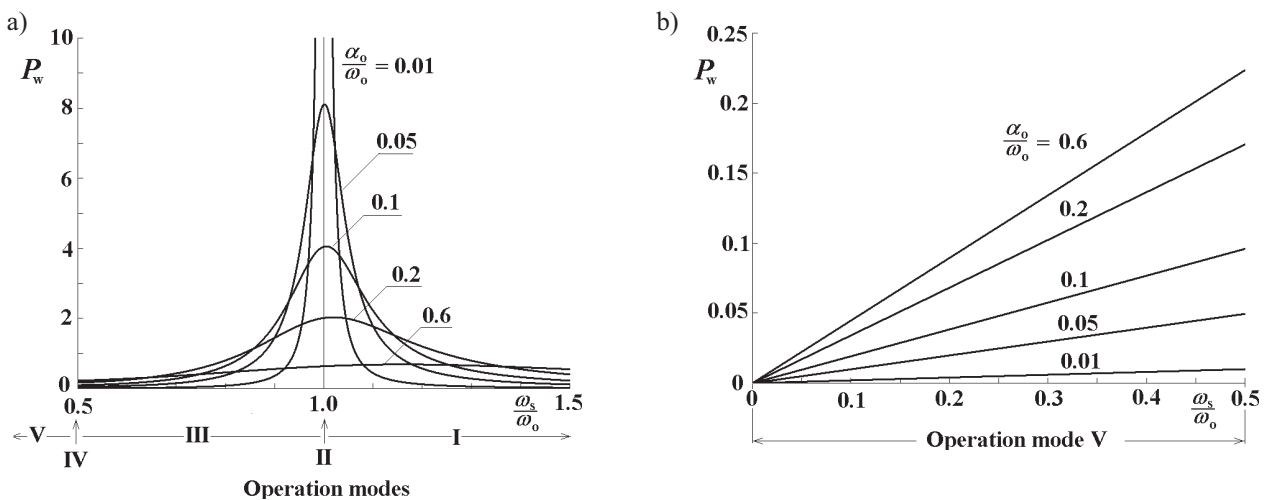

Fig. 16. Relative inverter power $P_w = f\left(\dfrac{\omega_s}{\omega_o}\right)$ for various values of parameter a) in modes I – IV; b) in mode V

This results from the necessity of assuring a sufficiently long reverse recovery time for outgoing thyristors.

The inverter parameters including current and voltage magnitudes, power dissipated in the load and conduction times of switches and diodes depend on ratios $\dfrac{\omega_s}{\omega_o}$ and $\dfrac{\alpha_o}{\omega_o}$. The range of inverter's parameters variations in terms of relative operating frequency $\dfrac{\omega_s}{\omega_o}$ decreases with increase in damping factor of the circuit. Maximum values of receiver current and voltage across the capacitor occur at operating frequency equal the frequency of the circuit free oscillations whereas the maximum power occur at operating angular frequency ω_s corresponding to resonant angular frequency $\omega_r = \dfrac{1}{\sqrt{L_o C_s}}$ of the circuit.

While operating the inverter in mode II or around this operating point ($\dfrac{\omega_s}{\omega_o}$ near 1), a special attention should be paid not to exceed permissible magnitudes of currents or voltages, especially in case of a low damping factor.

The voltage across the switches is low and, independently of operating frequency and damping factor, it is equal to supply voltage, which is one of the main advantages of the inverter.

The current through the load is distorted and its shape is the most similar to a sinusoidal one in mode II.

The operation of inverter is possible over a wide range of the operating frequency, where the power dissipated in the load can be controlled by varying that frequency. This way of power control can be used in inverters operating at low frequencies. In case of high frequency inverters special attention should be paid to minimizing switching loss. Zero current switching occurs in modes II, IV and V. However, due to low inverter power in modes IV and V, a practical importance has mode II with strictly determined operating frequency. In the case of high frequency inverters the power control can be achieved either by using a voltage controlled power source or by other control procedures different than PFM.

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