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MODELING WITH SEISMIC RAY TRACING IN INHOMOGENEOUS GEOLOGICAL FORMATION

Modelowanie sejsmiczne ośrodków niejednorodnych metodą „punkt strzałowy – punkt odbioru”

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Abstract: Seismic ray tracing is an useful tool to solve the complicated problem of wave propagation. It makes possible obtaining essential information about wave propagation in geological media without necessity of computing the full wave field. In this paper the theoretical principle of seismic ray tracing has been presented. Moreover few typical models of inhomogeneous geological media have been taking into account. Ray tracing methods have been divided into several categories and shooting method has been particularly described. Using results of modeling realized with the SEISMIC UN*X system, advantages and possibilities of the practical application in seismic tomography have been shown.

Key words: seismic ray tracing, caustic points, shadow zone, seismic modeling

Treść: Śledzenie promienia sejsmicznego jest narzędziem do rozwiązywania złożonego zagadnienia propagacji fal. Pozwala uzyskać istotne informacje o rozchodzeniu się fal sejsmicznych w ośrodku geologicznym bez konieczności wyznaczania pełnego pola falowego. W artykule przedstawiono podstawy teoretyczne śledzenia promieni sejsmicznych w kilku wzorcowych modelach niejednorodnych ośrodków geologicznych. Szczególną uwagę zwrócono na modele ośrodków o stałym i o zmiennym gradiencie prędkości w warstwach. Szczegółowo opisano metodę „punkt strzałowy – punkt odbioru”. Wykorzystując wyniki modelowania wykonanego za pomocą systemu SEISMIC UN*X, pokazano jego zalety oraz możliwości wykorzystania zarówno w sejsmice głębokiej, jak i płytkiej.

Słowa kluczowe: śledzenie promienia sejsmicznego, kaustyki, strefa cienia, modelowania sejsmiczne

INTRODUCTION

Seismic recognition of shallow geological formations plays an important role in solving problems of prospecting and exploring shallow deposits, locating various underground cavities and holes near the Earth surface for environment protection tasks, it is also useful for hydrogeology. Modeling is a very important part of seismic survey, both in deep and shallow measurements, so in the paper some results of seismic modeling with SEISMIC UN*X programs are presented.

Modeling is a simulation of seismic waves propagation in a medium of known characteristics. It is the kernel of the inversion procedure, in which experimental data gathered at a site are compared to simulated data until a satisfactory match is reached. Modeling is also useful because it offers the possibility of simulating the test response, affording an evaluation of the sensitivity, the resolution and the investigation depth. The aim of modeling is to obtain synthetic seismograms, which could be recorded for established complex formation models. The useful tools for modeling and seismic data processing is SEISMIC UN*X program (SU) – a free software package. SU was developed and is regularly updated by the Center for Wave Phenomena at the Colorado School of Mines, USA. The package consists of the number of modules realizing different tasks, among other things some programs for seismic modeling.

In the SU package the following methods are proposed for generating synthetic seismic data:

- ray tracing method (TRISEIS),
- finite difference method (SUEA2DF, SUFMOD2),
- Kirchhoff method (SUSYNVXZ, SUSYNVXZSC).

There are two steps to the seismic modeling task. The first step is the construction of background wave speed profiles, which may consist of uniformly sampled arrays of floating point numbers. The second step is the construction of the synthetic wave information which propagates in that wave speed profile.

In this paper there is wide description of the ray tracing method. This approach is comfortable for the sake of the short calculation time. It gives possibility to construct simple geological models, for example the multilayered formation with increasing velocity in layers or settling tank. It is possible to use SU in more complicated models also, but it’s time consuming process.

Seismic rays going through different sort of heterogenities in geological media keep the same laws as light rays. It is important and very useful to apply for seismics the principle originally formulated for optics.

Seismic ray tracing is an useful tool to solve the complicated problem of wave propagation. It makes possible obtaining essential information about wave propagation in geological media without necessity of computing the full wave field. It causes that ray tracing is useful for accurate and effective determination of seismic ray paths and travel times of rays from source to receiver in established geological media model. Seismic ray tracing allows generating the synthetic seismograms similar to the field seismograms recorded in field stations. It composes part of seismic tomography velocity modeling. It helps us for example within oil prospecting as well as in seismology to locate earthquake hypocenter (a focus point).

We are fortunately in digital era of the powerful computers working up many new wave propagation modeling methods, especially in seismics, providing more accurate results at short time and researching more complicated geological media models.

A SHORT OUTLINE OF SEISMIC WAVE PROPAGATION IN INHOMOGENOUS MEDIA

The phenomena of wave propagation in inhomogeneous media can be described applying ray theory, in which wave propagation is described by rays or wave fronts (Scales 1995, Mendecki 1997, Shearer 1999).

Generally, two approaches to a problem are working: a solution of a ray equation and a solution of an eikonal equation (Scales 1995, Mendecki 1997). The ray equation describes a ray path in geological media and the eikonal equation determines seismic wave propagation. It shows how ray paths are changing depending on distribution of wave velocity in geological media. In practice, both eikonal equation and ray equation are applied in tomography, where ray paths must be known, and to migration, where we only need to known travel times. However, ray equation is more important in seismic tomography, while eikonal equation is more popular in seismic migration.

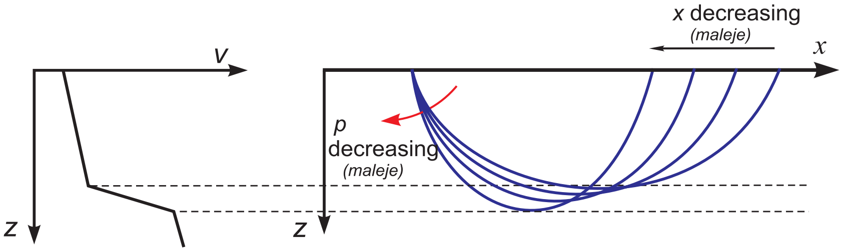

Fig. 1. Seismic ray paths in formation with variable velocity gradient in layers

Fig. 1. Trajektorie promieni dla modelu ośrodka o zmiennym gradiencie prędkości, $p = dT/dx$ – parametr promienia

Ray paths have various forms in different media thus few complex inhomogeneous geological standard models can be considered (Fig. 1):

- laterally homogeneous model where a ray travels downward through a series of layers, in condition as a lower layer is faster than an upper layer above (Shearer 1999);
- formation with continuous velocity gradients where velocity increases with depth; in this model rays are circle arcs;
- formation with variable velocity gradient in layers.

The transition from prograde to retrograde and back to prograde generates a triplication in the travel time curve. The endpoints on the triplication are termed caustics. In this case passing from weak to very sudden rising velocity and return to a very weak rising velocity is observed generating triplication of travel time curve and rays inversion. Bending points at the curve are called caustics (Fig. 2).

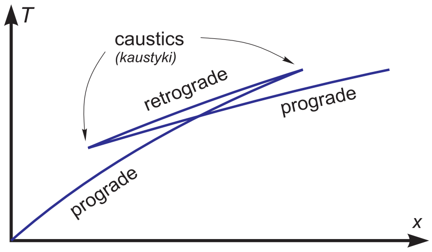

Fig. 2. A triplication in a travel time curve resulting from a steep velocity increase

Fig. 2. Wykres czasu przebiegu dla przypadku szybkiego wzrostu prędkości

Generally velocity increases with depth. However, occasionally at real geological media we can encounter the case where velocity decreases with depth, creating so called a low velocity zone. Within the negative gradient velocity at the top of low velocity zone, the rays are bent downward. Note that no rays originating at the surface can turn within the low velocity zone itself. Rays with horizontal slownesses corresponding to values within the low velocity zone turn above the zone. These rays then pass through the low velocity zone and turn in the region below the low velocity zone in which velocities are once again higher than any velocities in the overlying formation. A presence of a low velocity zone creates a gap, termed a shadow zone in the curve. The absence of rays turning the low velocity zone often makes the velocity structure within low velocity zones difficult to determine. A related phenomenon arises when rays originate within a low velocity zone. In this case, some rays are trapped within the low velocity zone and are forever curving back in toward the velocity minimum. The low velocity zone acts as wave guide and, in case of attenuation, seismic energy can propagate very long distances (Fig. 3).

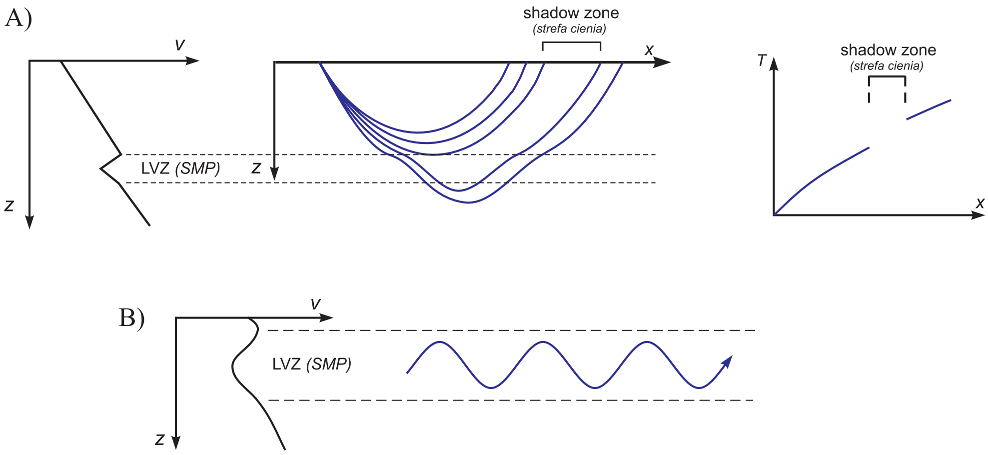

Fig. 3. Seismic ray paths and a travel time curve in the low velocity zone (LVZ) (A). A wave guide in the low velocity zone (B)

Fig. 3. Trajektorie promieni i krzywa czasu przebiegu powstałe w strefie małych prędkości (SMP) (A). Fala uwięziona powstała w strefie małych prędkości (B)

METHODS OF SEISMIC RAY TRACING

All ray tracing procedures can be summarized as kinematic ray tracing or dynamic ray tracing. Kinematic ray tracing includes ray paths and travel time determination, while dynamic ray tracing additionally estimates amplitudes and phase shift. Usually, kinematic ray tracing precedes dynamic ray tracing, but some methods like for example wave front construction methods estimate both kinematic and dynamic parameters simultaneously. Using the output of dynamic ray tracing, it is possible to compute synthetic seismograms provided that the source function is known.

Among several ray tracing methods shooting methods are simple and relatively frequently applied. Two-point seismic ray tracing exists in computing all ray paths, which propagate from source back to receivers for an arbitrary complex medium. Shooting methods are a basis of two-point ray tracing solutions (Scales 1995). The aim of a shooting method is to compute a ray going out from a source. The simplest solution to the boundary value problems for rays is to shoot a ray and see where it ends up. If it is not close enough to the desired receiver location one should make a correction of a take-off angle of the ray and shoot again. This way will be continued until the ray ends up sufficiently close to the receiver. The method of correcting a take-off angle of subsequent rays relies on finding an adjust angle θ, for which

$$\frac{dz}{dx} = f(x, z),$$

equation with an initial value problem $z(a) = z_a$ and $z'(a) = \theta$ is a solution to a boundary value problem. Thus, a first solution the initial value problem depending implicitly on angle θ, $z = z(x, \theta)$ is considered and next boundary value problem is reduced to finding roots of the function $z(b, \theta) - z_b$ with well-known methods, like for example the Runge–Kutty's or Newton's methods.

For ray paths and travel time calculating in inhomogeneous media there is necessity of using a special calculation grid at the model. Two types of calculation grids are applied in shooting method. In case of simple geological model rectangular grid is used, while in case of more complicated irregular geological media, triangular grid is the best (Czekaj, Idus & Bogacz 2000). Application of this grid is based on the triangulation method in which a model is divided into triangles. Irregular regions are divided into regular sub regions (cells) with constant velocity or constant velocity gradient and ray paths bend on cells boundaries according to Snell's law. An advantage of triangles grid is good matching of regions with different physical properties by triangles and possibility (if necessary) local grid condensation in order to minimize solution errors. The Delanuay triangulation method is one of many algorithms. Delaunay triangulation is based on a division of a model to triangles where each circle described on three neighboring points doesn't contain any different points.

PROGRAMS PACKET FOR SEISMIC RAYS MODELING

Development of the computer systems contributes to arise many systems and programs for seismic modeling. Among others the following can be mentioned: SEISMIC UN*X (Colorado School of Mines, USA), ANRAY (Geophysical Institute, Academy of Sciences of the Czech Republic), REFLEXW (Sandmeier Scientific Software) and GeoGraphix (Landmark Graphics Co., USA), as very friendly for using. Due to short time and good calculation accuracy for the academic needs (i.e. for example to seismic ray tracing) the most indicated is SEISMIC UN*X system, SU. It is on-line at the Center for Wave Phenomena in Colorado School of Mines web site. It was elaborated for seismic interpretation using UN*X computers. SEISMIC UN*X is composed of a big number of modules realizing diverse functions. Programs for ray tracing and synthetic seismograms generating are also included. Effective using of SU packet requires preparing scripts calling suitable programs sequence for preparation data (Stockwell 1998).

In the paper SU was used to seismic ray tracing in 2D geological formations using shooting method with Delanuay triangulation. Some options offered by SU packet enable:

- drafting 2D geological models,
- calculating and visualization of seismic ray paths for given initial angles rays from the source,
- calculating and printing synthetic seismograms.

SU programs require assignment values of input parameters. 2D model of geological formation is a basis for calculations. Boundaries of beds of different parameters are indicated by coordinates (x, z). To obtain smooth boundaries of beds (the first and last border should be located on the model boundary) program use 3D interpolation. Inside beds it's necessary to define values of density of formation and slowness of seismic wave. Source coordinates can be located at any of model points. Receivers can be distributed along established profiles. For better visualization of seismic ray trajectories it is possible to determine number of rays, minimal and maximal angles leaving source and step changing angles. Moreover it is possible to choose bended and reflected rays. These features make the SU programs very flexible and friendly for user. There are also disadvantages of SU programs. The most important of them is very scant description of program (short and not clear manual). SU programs are realized like usual UN*X system, what can be considered as advantage for UN*X system users, while it is less comfortable to the other systems users get used to interactive programs effects.

SU packet includes the following programs for seismic ray tracing: TRIMODEL, SXPLOT, TRIRAY, XGRAPH, GBBEAM, TRISEIS.

TRIMODEL is used for defining and for triangulation of 2D geological model. SXPLOT is used to generate model with triangle division. TRIRAY defines seismic ray parameters. XGRAPH generates seismic ray trajectories. GBBEAM determines parameters for synthetic seismograms. TRISEIS generates synthetic seismograms.

MODELING TRAJECTORY OF SEISMIC RAYS

Three seismic ray tracing models were used for calculating synthetic seismograms in different geological situations. The first example shows the model of bedded geological formation with constant velocity in each layer. The first layer of the 0.3 km thickness has 1.1 km/s velocity, the second layer of the same thickness has 1.3 km/s velocity and the third layer of 0.4 km thickness has 1.8 km/s velocity. The end-on spread has been applied and directed rays option (Fig. 4A, C) and reflected rays option from the second layer (Fig. 4B, D) has been chosen. For better visualization narrow angles range has been selected and a number of rays has been reduced only to one (Fig. 4A, C).

In modeling a sliding ray effect along the second boundary has been obtained in case of directed rays option (Fig. 4A, C) while, in case of reflected rays option (Fig. 4B, D) this effect has not been acquired despite preserving output angles range like presented in Fig. 4A, C). Seismic records on the seismogram have been presented for a full range of incidence angles and for the all receivers.

The second example shows increase of velocity in the layers (Fig. 5). At the first layer of the 0,4 km thickness, velocity increases from 0.2 km/s to 0.3 km/s, at the second layer, of the 0,1 km thickness velocity rises to 0.7 km/s, at the third layer of the 0.5 km thickness velocity increases to 0.8 km/s. The profile consisting of 80 steadily arranged receivers has been spreaded on 1.5 km length. For better visualization of seismic ray trajectories 4 rays have been chosen at narrow angles range.

As a result seismic ray inversion has been obtained and a distinct triplication of travel time curve at synthetic seismogram was observed. Unfortunately, it was not as distinctly visible as it can be expected on the basis of theoretical considering.

The third example shows model of geological formation with low velocity zone (Fig. 6). At the first layer of the 0.4 km thickness, velocity increases from 1.0 km/s to 1.5 km/s, at the second and third layer of the 0.14 km thickness, velocity first decreases to 0.5 km/s and then increases to 1.5 km/s. At the forth layer of the 0.33 km thickness, velocity increases to 2.0 km/s. The end-on spread has been applied and the profile consisting of 50 steadily arranged receivers has been spreaded on 4 km length. For better visualization 10 ray trajectories have been chosen at narrow angles range (Fig. 6A) and 1 ray trajectory has been selected in the model with low velocity zone located near the surface (Fig. 6B).

As a result it has been observed area, where rays are not coming called shadow zone. At the synthetic seismogram it can't be observed shadow zone because of average rays option using by program (Fig. 6A). In case of low velocity zone located near the surface wave guide has been generated (Fig. 6B).

The last figure (Fig. 7) shows model of the settling tank. In the first low velocity layer of the 0,02 km thickness velocity increases from 0.15 km/s to 0.2 km/s. The second one of the 0.2 km thickness has higher changeable velocity increasing from 2.0 km/s to 3.0 km/s. The profile consisting of 80 steadily arranged receivers has been spreaded on 1 km length.

The result of this sudden rising velocity is a seismic ray inversion and a distinct triplication of travel time curve at synthetic seismogram.

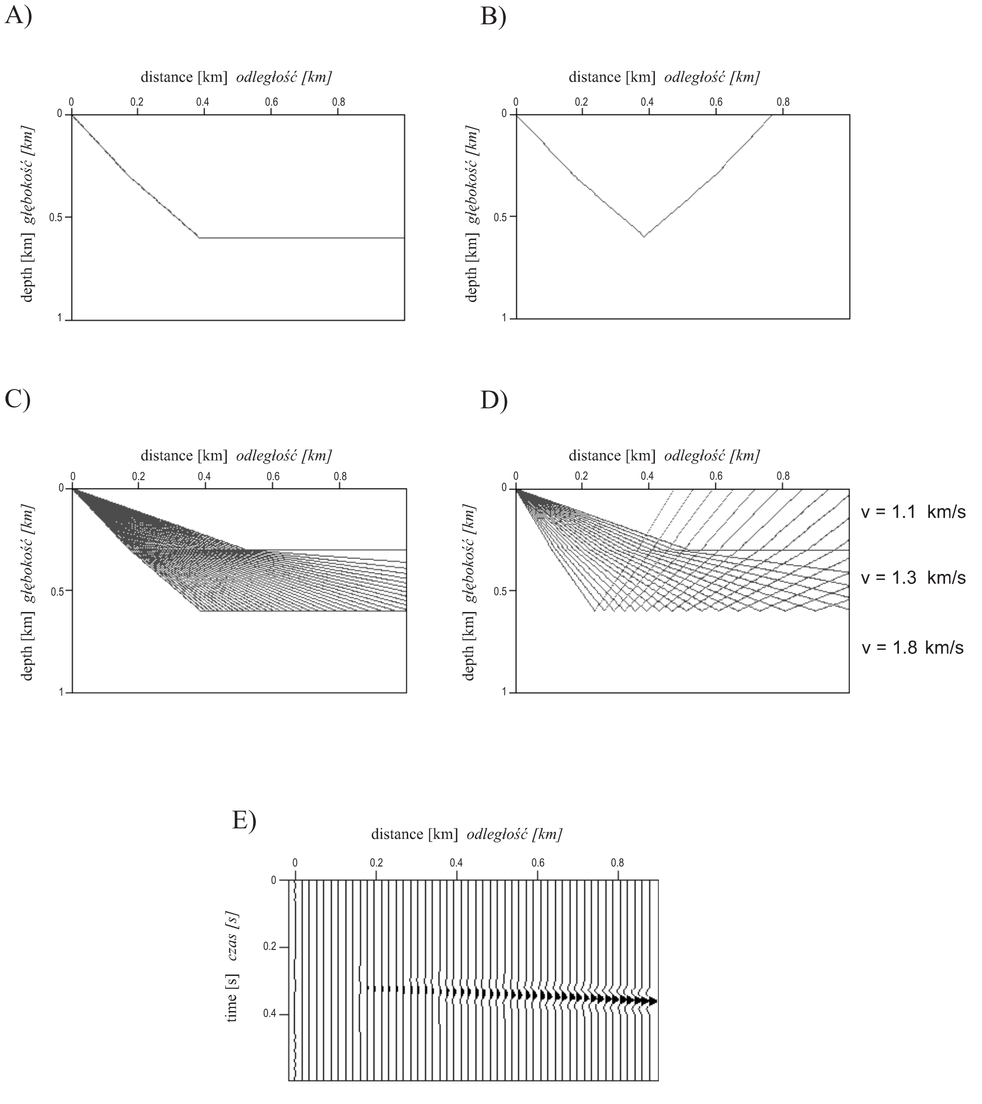

Fig. 4. Seismic ray paths in the model of bedded geological formation: A) for selected initial angle with directed rays option; B) for selected ray in reflected rays option; C) for defined initial angles range (directed rays); D) for defined initial angles range (reflected rays); E) synthetic seismogram for reflected rays

Fig. 4. Model ośrodka poziomo warstwowanego z trajektoriami promieni sejsmicznych: A) dla wybranego kąta wyjścia przy zadaniu opcji promieni bezpośrednich; B) dla wybranego kąta przy zadaniu opcji promieni odbitych; C) dla zadanego zakresu kątów wyjścia promieni (promienie bezpośrednie); D) dla zadanego zakresu kątów wyjścia promieni (promienie odbite); E) sejsmogram wykreślony dla przypadku promieni odbitych

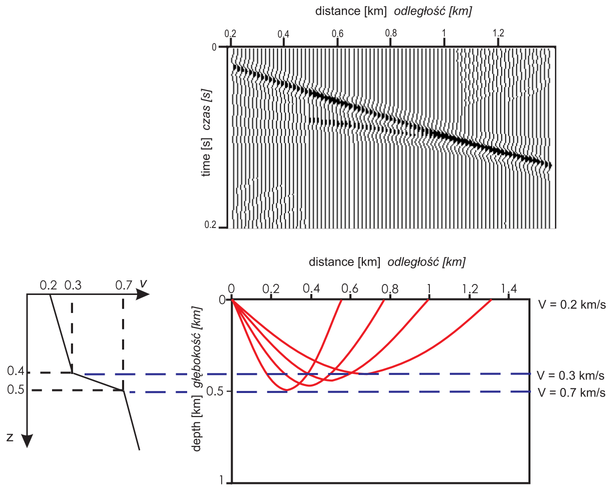

Fig. 5. Seismic ray paths and synthetic seismogram in the model of formation with variable velocity gradient in layers

Fig. 5. Model ośrodka z gwałtownym wzrostem prędkości z wykreślonymi trajektoriami promieni sejsmicznych i sejsmogramem

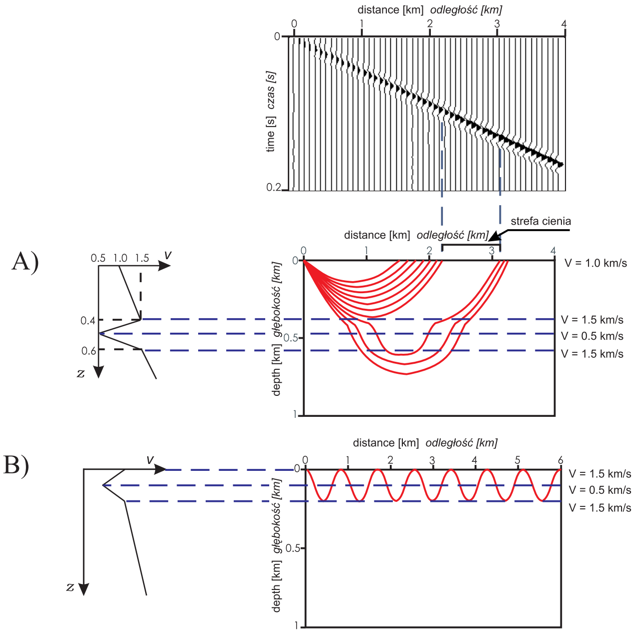

Fig. 6. Seismic ray paths and synthetic seismogram in the model of formation with low velocity zone (A). Wave guide in the model of formation with low velocity zone (B)

Fig. 6. Model ośrodka ze strefą małych prędkości: A) z wykreślonymi trajektoriami promieni sejsmicznych i sejsmogramem syntetycznym; B) z wykreśloną falą uwięzioną

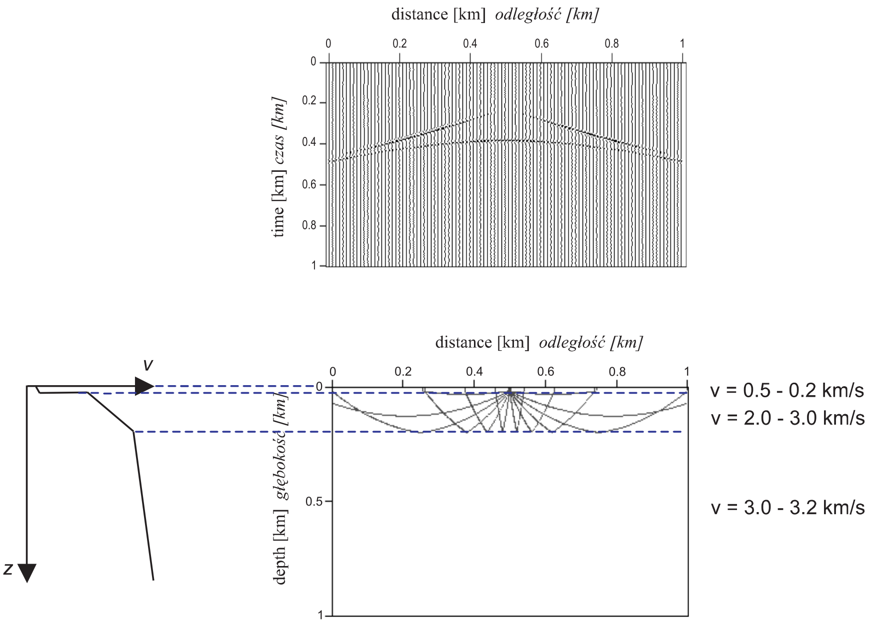

Fig. 7. Seismic ray paths and synthetic seismogram in the settling tank model

Fig. 7. Model osadnika z wykreślonymi trajektoriami promieni sejsmicznych i sejsmogramem

CONCLUSIONS

An analysis of acquired results allows formulating a series of conclusions:

- In the case of direct rays option given for stratified geological formation with constant velocity in each layer, increasing with depth, SU programs model the ray, which after reaching critical point slides itself along boundary of the layers (Fig. 4A, C). In the case of given reflected rays option, ray, which previously slides along boundary of the layers is reflected (Fig. 4B, D).
- In the case of formation model with sudden increase of velocity a seismic rays inversion has been observed. Moreover, SU gives a possibility of a travel time triplication on a generated curves.
- In the case of formation model with low velocity zone it has been observed area, where rays are not coming (shadow zone). SU programs besides given possibility to generate wave guide. At the seismogram generated by SU it can't be observed shadow zone because of average rays option using by program.

Aboved mentioned observations allow to state, that obtained results of ray trajectories and travel time curves calculated for complicated geological formation models generated by SU programs are very accurate. It is result of the division of a formation model to small cells with very precise assigning of physical parameters, which minimize calculations errors. Relatively easy using and short calculating rays time in SU programs allow also application it in seismic tomography.

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Streszczenie

Problem rozpoznania budowy geologicznej strefy przypowierzchniowej odgrywa znaczącą rolę w rozwiązywaniu zagadnień związanych z poszukiwaniem i rozpoznaniem płytkich złóż, lokalizacją pustek występujących na niewielkich głębokościach, jak również w ochronie środowiska i hydrogeologii.

Istotnym elementem badań sejsmicznych są modelowania. Ich celem jest uzyskanie sejsmogramu, który zostałby zarejestrowany dla założonego modelu geologicznego ośrodka. Modelowania można wykorzystać zarówno na etapie projektowania pomiarów, interpretacji, jak i przetwarzania danych w celu przetestowania procedur przetwarzania. Modelowania są przydatnym narzędziem w głębokich badaniach sejsmicznych oraz w badaniach prowadzonych w celu rozpoznania strefy przypowierzchniowej.

Rozwój systemów komputerowych oraz wzrost mocy obliczeniowej komputerów przyczynił się do powstania wielu systemów oraz pakietów programów do przetwarzania i modelowania sejsmicznego. Wśród nich można wyróżnić pakiet programów systemu Seismic Unix (Colorado School of Mines, USA) służący do śledzenia promienia sejsmicznego, który z uwagi na krótki czas i dokładność obliczeń jest najbardziej wskazany na potrzeby akademickie. W artykule przedstawiono podstawy teoretyczne śledzenia promieni sejsmicznych w kilku wzorcowych modelach niejednorodnych ośrodków geologicznych. W celu przedstawienia możliwości wykorzystania pakietu SU w sejsmice wybrano kilka przykładowych modeli ośrodków sejsmicznych. Na figurze 1 wykreślono trajektorie promieni sejsmicznych dla modelu ośrodka o zmiennym gradiencie prędkości. W tym przypadku następ-

puje przejście od słabego wzrostu prędkości do bardzo gwałtownego i powrót do słabego wzrostu prędkości, co generuje tzw. potrojenie (ang. *triplication*) krzywej czasu przebiegu (Fig. 2). Figura 3 przedstawia trajektorie promieni, krzywą czasu przebiegu i falę uwięzioną, powstałe w strefie małych prędkości.

W przypadku modelu poziomo warstwowanego ośrodka geologicznego o stałych prędkościach w warstwach uzyskano efekt ślizgania promienia po drugiej granicy ośrodka przy zastosowaniu opcji promieni bezpośrednich (Fig. 4A, C), natomiast nie uzyskano tego efektu w przypadku opcji promieni odbitych (Fig. 4B, D). W przypadku modelu ośrodka ze zmiennym gradientem prędkości w warstwach (Fig. 5) uzyskano inwersję promieni sejsmicznych oraz wyraźny podział krzywej czasu przebiegu na sejsmogramie syntetycznym. W wyniku modelowania ośrodka ze strefą małych prędkości (Fig. 6) na obrazie promieniowym obserwuje się strefę, do której nie docierają promienie (strefę cienia). Ponadto system SU daje możliwość wygenerowania fali uwięzionej (Fig. 6B). Na sejsmogramie syntetycznym nie uzyskano strefy cienia, gdyż program dokonuje uśrednień promieni (Fig. 6A). Figura 7 przedstawia trajektorie promieni oraz sejsmogram syntetyczny wygenerowane dla modelu osadnika. Ostatni przykład pokazuje możliwość wykorzystania modelowań w sejsmice inżynierskiej.