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CHARACTERISTICS OF COMPLEX SEPARATION SCHEMES AND AN ERROR OF SEPARATION PRODUCTS OUTPUT DETERMINATION

The separation characteristic of the scheme is determined by separation characteristics of each operation, their number, and sequence of connection.

Let's assume that before mixture the quantity of particles with separating sign X was P_{i-1} , and after mixture it was P_i (Fig. 1), and was equal to the sum of quantities of particles before mixture and entered into the given operation with circulating loading, i.e.

$$P_i = P_{i-1} + \sum_{j=1}^m P_j$$

where j is a serial number of circulating loading.

The circulation is formed by consecutive connection of devices, i.e.

$$P_j = P_i \prod_{i=i+1}^{n_i} E_i$$

Then

$$P_i = P_{i-1} + P_i \sum_{j=1}^m \prod_{i=i+1}^{n_i} E_i$$

Whence

$$P_{i-1} = P_i \left(1 - \sum_{j=1}^m \prod_{i=i+1}^{n_i} E_i \right)$$

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Thus, the mixture operation separation characteristic is

$$P_{CMi} = \frac{P_i}{P_{i-1}} = 1 / (1 - \sum_{j=1}^m \prod_{i=i+1}^{n_i} E_i)$$

Let's apply the received formulas on examples.

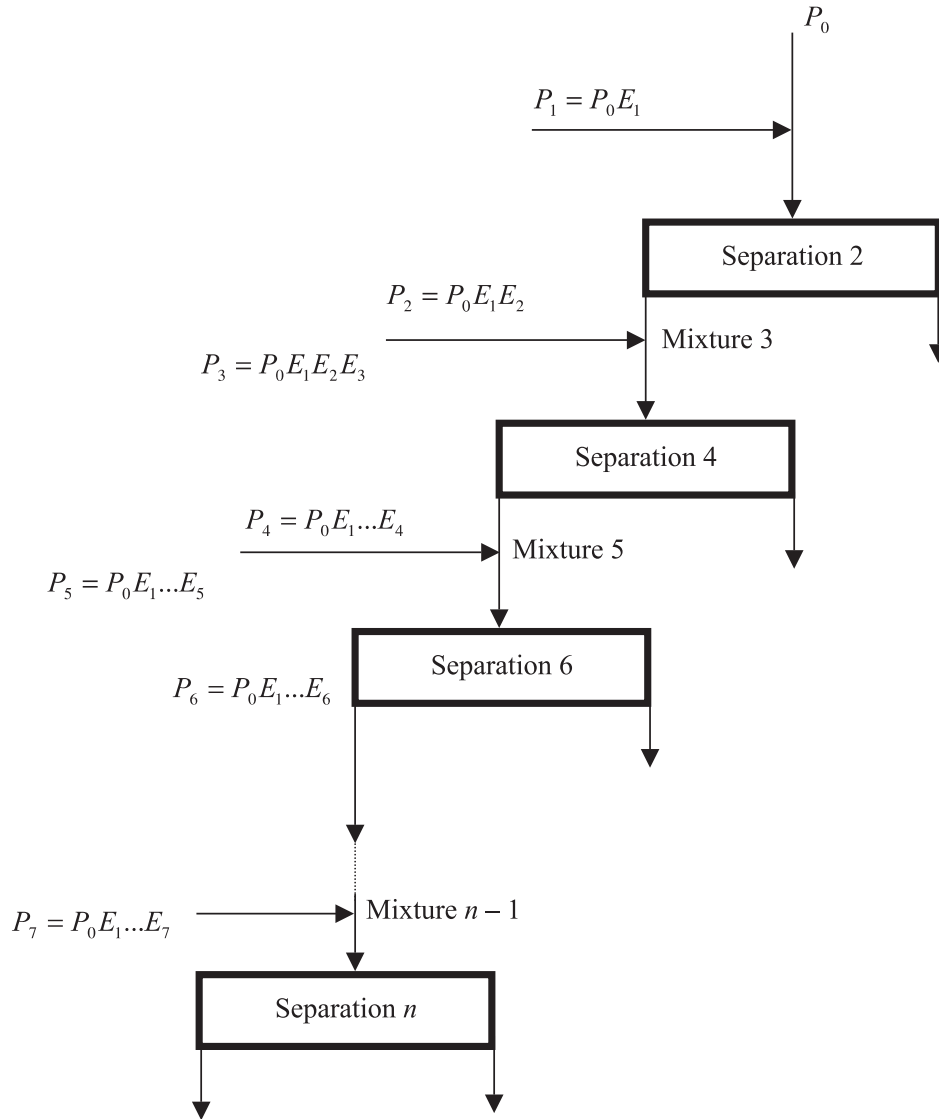


Fig. 1. Consecutive connection of separation and mixture operations

We assume there is a technological block for separation (Fig. 2).

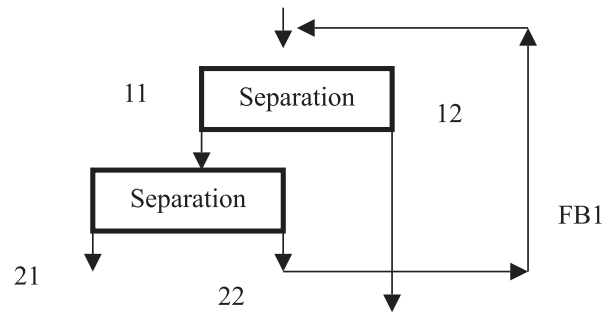


Fig. 2. The scheme of a technological separation block

Let's make the separation characteristic of this connection. According to laws of consecutive connection

$$P_{\beta} = P_1 P_{OC1} P_2$$

For the feedback technological connection the characteristic is

$$P_{OC1} = \frac{1}{1 - P_1 P_2^1}$$

Making identical transformations, we shall receive the required formula:

$$P_{\beta} = \frac{P_1 P_2}{1 - P_1 P_2^1} = \frac{P_1 P_2}{1 - P_1(1 - P_2)} = \frac{P_1 P_2}{1 - P_1 + P_1 P_2}$$

Let's complicate the separation devices connection (Fig. 3).

The sequence of separation characteristics for such connection is as follows

$$P_{\beta} = P_{FB2} P_1 P_{FB1} P_2 P_4$$

and the feedback characteristic is

$$P_{FB2} = \frac{1}{1 - P_2 P_3^1}$$

$$P_{FB1} = \frac{1}{1 - P_1 P_{FB2} P_2^1}$$

Making identical transformations we shall receive:

$$P_{\beta} = \frac{P_2 P_4 P_5}{1 - P_2 P_4^1 - P_4 P_5^1}$$

We shall not carry out the further transformations, as they are trivial.

The connection separation characteristic turns out rather complex, but there are identical transformations resulting in an obvious kind expression.

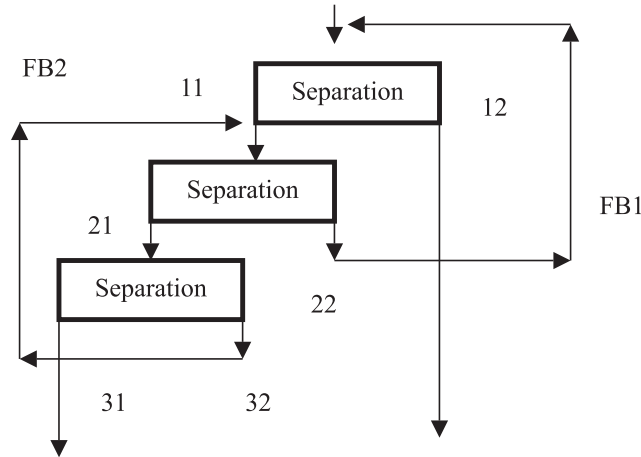


Fig. 3. The scheme of a complicated technological separation block

However, for the circuit shown in Figure 4 the identical transformations become so bulky that the probability of a mistake comes near to one. Therefore for such scheme it is necessary to make the equations of fractions balance for each separation reception:

$$\begin{aligned}
 p_{11} &= P_1(p_0 + p_{22} + p_{41}); & p_{12} &= P_1^1(p_0 + p_{22} + p_{41}); \\
 p_{21} &= P_2(p_{11} + p_{32}); & p_{22} &= P_2^1(p_{11} + p_{32}); \\
 p_{32} &= P_2^1 p_{21}; & p_{31} &= P_3 p_{21}; \\
 p_{41} &= P_4 p_{12}; & p_{42} &= P_4^1 p_{12}; \\
 p_{OC2} &= p_{22} + p_{41}
 \end{aligned}
 \tag{1}$$

To determine outputs of all products it is necessary to solve a system of equations (1). As it has a high order, the solution is carried out numerically, for example, by a simple iterations method. Numerical solutions demand special formal transformation to obtain an algorithmic form.

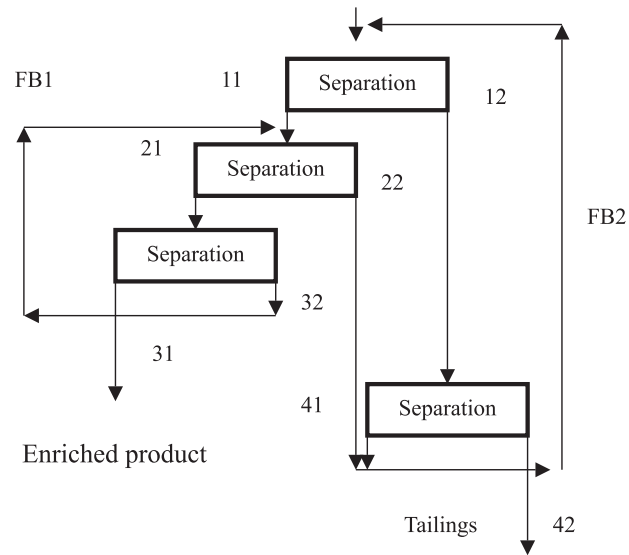


Fig. 4. The scheme of a technological separation block with deep feedback

For a system of linear equations it looks like:

$$\bar{X}^{(i)} = A\bar{X}^{(i-1)} + \bar{B} \quad (2)$$

where \bar{B} is a vector of free terms; A is a matrix of factors; \bar{X} is a vector of required roots approximations.

The matrix of factors A will be as follows

| | | | | | | | |
|----------|----------|----------|----------|--|----------|----------|--|
| | | | P_1 | | | P_1 | |
| | | | P_{21} | | | P_{21} | |
| P_2 | | | | | P_2 | | |
| P_{21} | | | | | P_{21} | | |
| | P_3 | | | | | | |
| | P_{31} | | | | | | |
| | | P_4 | | | | | |
| | | P_{41} | | | | | |

and the vector of free terms is $\bar{X} = (P_1 X_0, P_1^1 X_0, 0, 0, 0, 0, 0, 0)$.

For the analysis of the separation scheme, separation characteristics of all devices are set. As the separation is binary, each device has two separation characteristics. As the result the scheme shown in Figure 4 is characterized by eight separation characteristics. They are

represented as discrete set values, for example by six values. Thus, the initial data on the separation scheme will be as a matrix of 6×8 size.

The calculation of the separation circuit can be done in the following way. The first column of the separation characteristics matrix is chosen and the system of the equations (2) is solved, as a result we shall receive the vector of the first fraction of particles separation. Then the second column of the separation characteristics matrix is chosen, and again the system of the equations is solved. The vector of the second fraction of particles separation is received. The solution is made as many times, as many columns are in the separation characteristics matrix. As a result we shall receive a new matrix of 6×8 size.

To determine the i -th product output, it is necessary to sum the elements of the corresponding i -th solution matrix row. To determine quality indices, it is necessary to multiply row elements and the fraction quality parameters and to sum the received products.

So for the circuit in Figure 5 the solution is obtained as the matrix

| | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.002 | 0.21 | 0.003 | 0.34 | 0.002 | 0.22 | 0.001 | 0.1 |
| 0.049 | 0.28 | 0.05 | 0.297 | 0.03 | 0.195 | 0.016 | 0.09 |
| 0.135 | 0.31 | 0.1 | 0.24 | 0.07 | 0.16 | 0.033 | 0.07 |
| 0.2 | 0.136 | 0.21 | 0.14 | 0.14 | 0.09 | 0.066 | 0.044 |
| 0.32 | 0.08 | 0.28 | 0.07 | 0.184 | 0.04 | 0.088 | 0.022 |
| 0.5 | 0.03 | 0.33 | 0.02 | 0.216 | 0.1 | 0.1 | 0.006 |
| | | | | 0.642 | | | 0.362 |

The fifth product is the enriched product, and the eighth one is depleted. The sum of outputs of these products is equal to one, as it is received as a result of calculations.

The technological balance of concentration processes is connected with the determination of output parameters of each technological point. When the technology has deep recycles, such calculations become inconvenient and are connected with iterative procedures to select the required parameters resulting in material balance. The purpose of the given argumentations is to find formal algorithm, which would allow making calculations by the computer.

Let's assume that we have some separation circuit (Fig. 5).

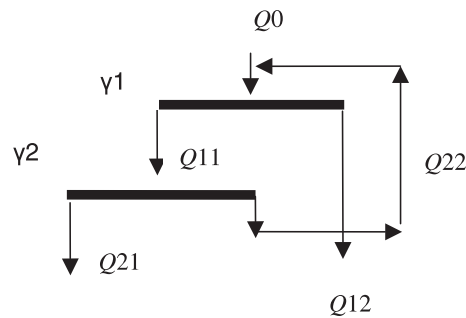


Fig. 5. The separation circuit with the recycle

Let's write down a system of balance equations for the given circuit:

$$Q_{11} = \gamma_1(Q_0 + Q_{22}); Q_{12} = \gamma_1^1(Q_0 + Q_{22});$$

$$Q_{21} = \gamma_2 Q_{11}; Q_{22} = \gamma_2^1 Q_{11}$$

The number of the equations is equal to the number of unknown values, therefore the solution is single-valued, and taking into account the low order of the system, it can be solved by a substitution. We shall determine an output of the enriched product:

$$Q_{21} = Q_0 \frac{\gamma_1 \gamma_2}{1 - \gamma_1 \gamma_2^1};$$

$$\gamma_{21} = \frac{Q_{21}}{Q_0} = \frac{\gamma_1 \gamma_2}{1 - \gamma_1 \gamma_2^1}$$

Thus, on the basis of known outputs on separate operations and known laws of products transformation in separation circuits, it is possible to determine the output of any product in relation to an initial feeding. Individual outputs are calculated when required by the

ratio of charges $\left(\gamma_{IJ} = \frac{Q_{IJ}}{Q_0} \right)$.

Nowadays, the mathematical approach is developed allowing calculation of estimating accuracy of a product output for a rather simple technology of mineral wealth enrichment [1]. Such estimation is carried out with the help of partial differentiation of the product output function. The more complicated is the circuit, the more complicated is the operation of differentiation. Therefore it is necessary to have a method allowing to execute the mentioned estimation of accuracy for the circuit of any complexity.

So, if to calculate outputs at known parameters of the product preparation, and then to give increments to these parameters and again make calculations, it is possible to receive an estimation of output derivatives on corresponding parameters that serves as a measure of accuracy of interested parameters estimation.

Separation characteristics $P(\alpha)$ and functions of particles distribution on fractions $F(\alpha)$ can be received experimentally. For this reason, each fraction definition error one can consider as known:

$$\sigma_{p_{\alpha_1}} \quad \sigma_{p_{\alpha_2}} \quad , \dots \dots \sigma_{p_{\alpha_n}} \quad \sigma_{F_{\alpha_1}} \quad \sigma_{F_{\alpha_2}} \quad , \dots \dots \sigma_{F_{\alpha_n}}$$

The complex circuit product output will be calculated on the basis of the repeated solution of system of the linear equations of products balance. At the beginning, we assume that in the result the certain value γ_1 is obtained.

Knowing it, we give an increment to the function $P(\alpha)$. For example, we shift it upwards or downwards, and again calculate an output. Now it will be γ_2 .

For the error definition it is necessary for functions $F(\alpha)$ and $P(\alpha)$ to have derivatives on all the range α .

In this case the differential of the function will be:

$$\Delta P(\alpha_i) = P_1(\alpha_i) - P_2(\alpha_i),$$

$$\Delta \Delta F(\alpha_i) = \Delta F_1(\alpha_i) - \Delta F_2(\alpha_i)$$

Then the differential of the argument is:

$$\Delta \gamma_P(\alpha_i) = P_1(\alpha_i) \Delta F(\alpha_i) - P_2(\alpha_i) \Delta F(\alpha_i),$$

$$\Delta \gamma_F(\alpha_i) = P_1(\alpha_i) \Delta F_1(\alpha_i) - P_1(\alpha_i) \Delta F_2(\alpha_i)$$

The function derivatives can be written down as follows:

$$\frac{\partial \gamma_P}{\partial P} = \frac{\Delta \gamma_P(\alpha_i)}{\Delta P(\alpha_i)},$$

$$\frac{\partial \gamma_F}{\partial F} = \frac{\Delta \gamma_F(\alpha_i)}{\Delta \Delta F(\alpha_i)}$$

Then the error from each variable change will be formed from errors of each product fraction definition, as functions $F(\alpha)$ and $P(\alpha)$ are integral:

$$\sigma_{\gamma_P}^2 = \sum_{i=1}^n \left(\frac{\Delta \gamma_P(\alpha_i)}{\Delta P(\alpha_i)} \sigma_{P\alpha_i} \right)^2;$$

$$\sigma_{\gamma_F}^2 = \sum_{i=1}^n \left(\frac{\Delta \gamma_F(\alpha_i)}{\Delta F(\alpha_i)} \sigma_{F\alpha_i} \right)^2$$

As a result, the general error of the numerical output estimation can be written by the following expression:

$$\sigma_{\gamma}^2 = \sigma_{\gamma_P}^2 + \sigma_{\gamma_F}^2$$

As an example we shall give the calculations of the output error for the separation circuit shown in Figure 6.

1) We determine the product output of the complex circuit on the basis of the repeated solution of the system of linear equations of products balance. As a result we have received the certain value γ_1 .

Knowing it, we shift the function $P(\alpha)$ downwards and again calculate the output γ_2 .

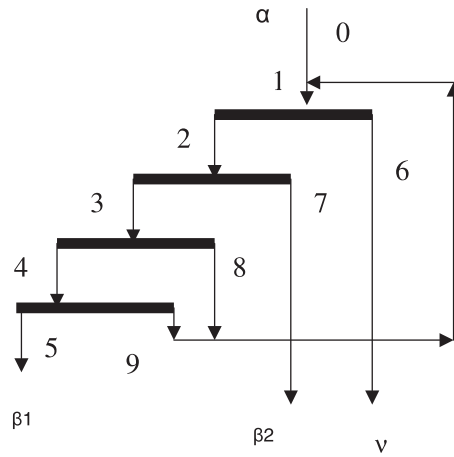


Fig. 6. The technological separation block

2) Using the given equations we have received their solutions, which are registered in Table 1.

$$\Delta P(\alpha_i) = P_1(\alpha_i) - P_2(\alpha_i),$$

$$\Delta \Delta F(\alpha_i) = \Delta F_1(\alpha_i) - \Delta F_2(\alpha_i)$$

TABLE 1

| α | 0 | 0.125 | 0.375 | 0.625 | 0.875 | 1 |
|---------------------------|------|-------|-------|-------|-------|------|
| $\Delta P(\alpha)$ | 0.02 | 0.02 | 0.06 | 0.03 | 0.03 | 0.05 |
| $\Delta \Delta F(\alpha)$ | 0.08 | 0.01 | 0.02 | 0.01 | 0 | 0.04 |

We calculate an argument differential. The received values are registered in Table 2.

$$\Delta \gamma_P(\alpha_i) = P_1(\alpha_i) \Delta F(\alpha_i) - P_2(\alpha_i) \Delta F(\alpha_i),$$

$$\Delta \gamma_F(\alpha_i) = P_1(\alpha_i) \Delta F_1(\alpha_i) - P_1(\alpha_i) \Delta F_2(\alpha_i)$$

TABLE 2

| α | 0 | 0.125 | 0.375 | 0.625 | 0.875 | 1 |
|---------------------------|---------|--------|-------|--------|--------|--------|
| $\Delta \gamma_P(\alpha)$ | -0.0046 | 0.0065 | 0.023 | 0.0091 | 0.0036 | 0.0358 |
| $\Delta \gamma_F(\alpha)$ | -0.0096 | 0.0015 | 0.008 | 0.007 | 0 | 0.0328 |

Function derivatives are written down as follows:

$$\frac{\partial \gamma_P}{\partial P} = \frac{\Delta \gamma_P(\alpha_i)}{\Delta P(\alpha_i)},$$

$$\frac{\partial \gamma_F}{\partial F} = \frac{\Delta \gamma_F(\alpha_i)}{\Delta \Delta F(\alpha_i)}$$

The values received at their calculation are given in Table 3.

TABLE 3

| α | 0 | 0.125 | 0.375 | 0.625 | 0.875 | 1 |
|---|-------|-------|----------|----------|-------|-------|
| $\Delta \gamma_P(\alpha)/\Delta P$ | -0.23 | 0.325 | 0.383333 | 0.303333 | 0.12 | 0.716 |
| $\Delta \gamma_F(\alpha)/\Delta \Delta F$ | -0.12 | 0.15 | 0.4 | 0.7 | 0 | 0.82 |

The error from each variable change is calculated as follows:

$$\sigma_{\gamma_P}^2 = \sum_{i=1}^n \left(\frac{\Delta \gamma_P(\alpha_i)}{\Delta P(\alpha_i)} \sigma_{P\alpha_i} \right)^2;$$

$$\sigma_{\gamma_F}^2 = \sum_{i=1}^n \left(\frac{\Delta \gamma_P(\alpha_i)}{\Delta F(\alpha_i)} \sigma_{F\alpha_i} \right)^2$$

We have received the values of each variable and have written them in Table 4.

TABLE 4

| | | | | | | |
|-----------------------|----------|----------|----------|----------|----------|----------|
| $\sigma_{\gamma_P}^2$ | 0.000005 | 0.000011 | 0.000015 | 0.000009 | 0.000001 | 0.000051 |
| $\sigma_{\gamma_F}^2$ | 0.000001 | 0.000002 | 0.000016 | 0.000049 | 0.000000 | 0.000067 |

Now it is possible to calculate the general error of numerical output estimation:

$$\sigma_{\gamma}^2 = \sigma_{\gamma_P}^2 + \sigma_{\gamma_F}^2$$

$$\sigma_{\gamma}^2 = 0,000092 + 0,000136 = 0,000228$$

$$\sigma_{\gamma} = 1,5\%$$

This circuit is not too complex, and so we still can calculate an output error by a classical method with the purpose to check the offered numerical approach. Devices' separation characteristics are identical and are given in Table 5. There also the distribution function of joints of raw material going to separation is given.

TABLE 5

| α | 0 | 0.125 | 0.375 | 0.625 | 0.875 | 1 |
|------------|------|-------|-------|-------|-------|------|
| P | 0.12 | 0.15 | 0.4 | 0.7 | 0.78 | 0.82 |
| ΔF | 0.17 | 0.26 | 0.27 | 0.08 | 0.12 | 0.1 |

First it is necessary to determine outputs of all products. For this purpose the separation characteristics for each technology point are set.

The feedback 1 separation characteristic is

$$P_{FB1} = \frac{1}{1 - P_1 P_2 P_3^1}$$

The feedback 1 separation characteristic is

$$P_{FB2} = \frac{1}{1 - P_1 P_2 P_3 P_4^1}$$

For each circuit point the separation characteristic is formed by the scheme of multiplication of separation characteristics from the input up to a point.

$$\text{Point 1: } P_1 = \frac{1}{1 - P_1 P_2 P_3^1 - P_1 P_2 P_3 P_4^1 + P_1 P_2 P_3^1 P_1 P_2 P_3 P_4^1},$$

The denominator at all separation characteristics will be the same, therefore further we shall write down:

$$\text{point 2: } P_{11} = P_1 P_1,$$

$$\text{point 3: } P_{111} = P_1 P_2 P_1,$$

$$\text{point 4: } P_{1V} = P_1 P_2 P_3 P_1,$$

$$\text{point 5: } P_V = P_1 P_2 P_3 P_4 P_1,$$

$$\text{point 6: } P_{V1} = P_1^1 P_1,$$

$$\text{point 7: } P_{V11} = P_1 P_2^1 P_1,$$

$$\text{point 8: } P_{V111} = P_1 P_2 P_3^1 P_1,$$

$$\text{point 9: } P_{1X} = P_1 P_2 P_3 P_4^1 P_1$$

Let's calculate a denominator of separation characteristics (Table 6):

TABLE 6

| | | | | | | |
|-------------|--------|--------|--------|--------|--------|--------|
| α | 0 | 0.125 | 0.375 | 0.625 | 0.875 | 1 |
| P | 0.12 | 0.15 | 0.4 | 0.7 | 0.78 | 0.82 |
| Denominator | 0.9858 | 0.9781 | 0.8693 | 0.7652 | 0.7757 | 0.7917 |

Let's write down all separation characteristics calculated according to the above-stated expressions (Table 7).

TABLE 7

| | | | | | | | | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| α | P_1 | P_2 | P_3 | P_4 | P_5 | P_6 | P_7 | P_8 | P_9 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 1.0143 | 0.1217 | 0.0146 | 0.0017 | 0.0002 | 0.8926 | 0.1071 | 0.0128 | 0.0015 |
| 0.125 | 1.0224 | 0.1533 | 0.0230 | 0.0034 | 0.0005 | 0.8690 | 0.1303 | 0.0195 | 0.0029 |
| 0.375 | 1.1503 | 0.4601 | 0.1840 | 0.0736 | 0.0294 | 0.6902 | 0.2760 | 0.1104 | 0.0441 |
| 0.625 | 1.3068 | 0.9147 | 0.6403 | 0.4482 | 0.3137 | 0.3920 | 0.2744 | 0.1921 | 0.1344 |
| 0.875 | 1.2891 | 1.0055 | 0.7843 | 0.6117 | 0.4771 | 0.2836 | 0.2212 | 0.1725 | 0.1345 |
| 1 | 1.2630 | 1.0357 | 0.8492 | 0.6964 | 0.5710 | 0.2273 | 0.1864 | 0.1528 | 0.1253 |

We calculate the output and quality in each point of technology. Point 1 (Table 8).

TABLE 8

| | | | | | |
|----------|--------|------------|----------|----------|---------|
| α | P_1 | ΔF | 2 · 3 | 1 · 4 | 1 · 3 |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1.0144 | 0.17 | 0.172444 | 0 | 0 |
| 0.125 | 1.0224 | 0.26 | 0.265832 | 0.033229 | 0.0325 |
| 0.375 | 1.1504 | 0.27 | 0.3106 | 0.116475 | 0.10125 |
| 0.625 | 1.3068 | 0.08 | 0.104544 | 0.06534 | 0.05 |
| 0.875 | 1.2891 | 0.12 | 0.154694 | 0.135357 | 0.105 |
| 1 | 1.2631 | 0.1 | 0.126305 | 0.126305 | 0.1 |
| | | | 1.13 | 0.48 | 0.389 |

$$\alpha_m = 0.389 = 0.39,$$

$$\beta_1 = 0.42$$

Similarly we calculate for other points of technology. Therefore we have:

$$\gamma_2 = 0.45, \quad \beta_2 = 0.64,$$

$$\gamma_3 = 0.29, \quad \beta_3 = 0.76,$$

$$\gamma_4 = 0.2, \quad \beta_4 = 0.82,$$

$$\gamma_5 = 0.15, \quad \beta_5 = 0.85,$$

$$\gamma_6 = 0.65, \quad \beta_6 = 0.26,$$

$$\gamma_7 = 0.19, \quad \beta_7 = 0.45,$$

$$\gamma_8 = 0.09, \quad \beta_8 = 0.62,$$

$$\gamma_9 = 0.05, \quad \beta_9 = 0.72$$

Let's calculate partial outputs for each separation reception.

The first reception:

$$\gamma_1 = \frac{0.42 - 0.26}{0.64 - 0.26} = 0.42,$$

$$\gamma_1^1 = 1 - \gamma_1 = 0.58$$

The second reception:

$$\gamma_2 = \frac{0.64 - 0.45}{0.76 - 0.45} = 0.61,$$

$$\gamma_2^1 = 1 - \gamma_2 = 0.39$$

The third reception:

$$\gamma_3 = \frac{0.76 - 0.62}{0.82 - 0.62} = 0.7,$$

$$\gamma_3^1 = 1 - \gamma_3 = 0.3$$

The fourth reception:

$$\gamma_4 = \frac{0.82 - 0.72}{0.85 - 0.72} = 0.77,$$

$$\gamma_4^1 = 1 - \gamma_4 = 0.23$$

The function of the concentrate output is made similarly to the separation characteristics and looks like:

$$\gamma = \frac{\gamma_1 \gamma_2 \gamma_3 \gamma_4}{1 - \gamma_1 \gamma_2 (1 - \gamma_3) - \gamma_1 \gamma_2 \gamma_3 (1 - \gamma_4) + \gamma_1^2 \gamma_2^2 \gamma_3 (1 - \gamma_3) (1 - \gamma_4)}$$

Let's generate expressions for partial derivatives.

According to the calculated partial outputs we shall write down the expressions:

$$\gamma_\beta = \frac{0.329 \gamma_1}{1 - 0.281 \gamma_1 + 0.018 \gamma_1^2},$$

$$\gamma_\beta = \frac{0.179 \gamma_4}{0.923 - 0.193(1 - \gamma_4)},$$

$$\gamma_\beta = \frac{0.226 \gamma_2}{1 - 0.194 \gamma_2 + 0.0085 \gamma_2^2},$$

$$\gamma_\beta = \frac{0.197 \gamma_3}{0.759 + 0.182 \gamma_3}$$

Let's calculate derivatives on corresponding outputs.

$$\frac{\partial \gamma}{\partial \gamma_1} = \frac{0.329(1 - 0.281 \gamma_1 + 0.018 \gamma_1^2) - 0.329 \gamma_1 (-0.281 + 0.036 \gamma_1)}{(1 - 0.281 \gamma_1 + 0.018 \gamma_1^2)^2} = 0.418,$$

$$\frac{\partial \gamma}{\partial \gamma_2} = \frac{0.226(1 - 0.194 \gamma_2 + 0.0085 \gamma_2^2) - 0.226 \gamma_2 (-0.194 + 0.017 \gamma_2)}{(1 - 0.194 \gamma_2 + 0.0085 \gamma_2^2)^2} = 0.29,$$

$$\frac{\partial \gamma}{\partial \gamma_3} = \frac{0.197(0.759 + 0.182 \gamma_3) - 0.182 \cdot 0.197 \gamma_3}{(0.759 + 0.182 \gamma_3)^2} = 0.19,$$

$$\frac{\partial \gamma}{\partial \gamma_4} = \frac{0.179(0.923 - 0.193(1 - \gamma_4)) - 0.179 \gamma_4 \cdot 0.193}{(0.923 - 0.193(1 - \gamma_4))^2} = 0.169$$

Let's further calculate partial derivatives from partial outputs $\gamma = \frac{\alpha - v}{\beta - v}$ on corresponding variables.

$$\frac{\partial \gamma_1}{\partial \alpha} = \frac{\partial \gamma}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \alpha} = 0.418 \frac{1}{0.38} = 1.1,$$

$$\frac{\partial \gamma_2}{\partial \alpha} = \frac{\partial \gamma}{\partial \gamma_2} \frac{\partial \gamma_2}{\partial \alpha} = 0.29 \frac{1}{0.31} = 0.93,$$

$$\frac{\partial \gamma_3}{\partial \alpha} = \frac{\partial \gamma}{\partial \gamma_3} \frac{\partial \gamma_3}{\partial \alpha} = 0.19 \frac{1}{0.2} = 0.95,$$

$$\frac{\partial \gamma_4}{\partial \alpha} = \frac{\partial \gamma}{\partial \gamma_4} \frac{\partial \gamma_4}{\partial \alpha} = 0.169 \frac{1}{0.13} = 1.3,$$

$$\gamma_1 = \frac{0,42 - 0,26}{\beta - 0,26}$$

$$\frac{\partial \gamma_1}{\partial \beta} = \frac{\partial \gamma}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \beta} = 0.418 \frac{-0.16}{(\beta - 0.26)^2} = -0.46.$$

Let's assume $\sigma_\alpha = \sigma_\beta = \sigma_v$. To determine their numerical values, which will be the same as for numerical determination of errors; let's do the next actions (Fig. 7).

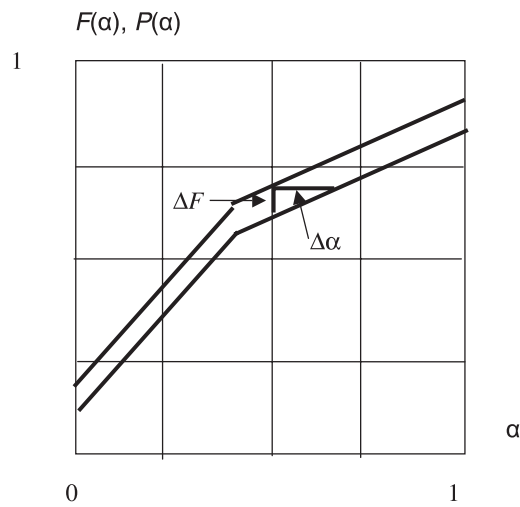


Fig. 7. The circuit of reception of output function arguments' increments

According to functions' increments we can determine an argument increment $\Delta\alpha_i$ for each accepted discrete value of the function using the plot. Then we can find the average value of such increment on all a range of α change. It will be the required $\overline{\Delta\alpha}$. In this case it we have

$$\gamma_2 = \frac{0.64 - 0.45}{\beta - 0.45},$$

$$\overline{\Delta\alpha} = 0.02$$

$$\frac{\partial\gamma_2}{\partial\beta} = \frac{\partial\gamma}{\partial\gamma_2} \frac{\partial\gamma_2}{\partial\beta} = 0.29 \frac{-0.19}{(\beta - 0.45)^2} = -0.5734,$$

$$\gamma_3 = \frac{0.76 - 0.62}{\beta - 0.62},$$

$$\frac{\partial\gamma_3}{\partial\beta} = \frac{\partial\gamma}{\partial\gamma_3} \frac{\partial\gamma_3}{\partial\beta} = 0.19 \frac{-0.14}{(\beta - 0.62)^2} = -0.665,$$

$$\gamma_4 = \frac{0.82 - 0.72}{\beta - 0.72},$$

$$\gamma_1 = \frac{0.42 - v}{0.64 - v},$$

$$\gamma_2 = \frac{0.64 - v}{0.76 - v},$$

$$\frac{\partial\gamma_2}{\partial v} = \frac{\partial\gamma}{\partial\gamma_2} \frac{\partial\gamma_2}{\partial v} = 0.29 \frac{(0.76 - v)(-1) + (0.64 - v)}{(0.76 - v)^2} = -0.362,$$

$$\gamma_3 = \frac{0.76 - v}{0.82 - v},$$

$$\frac{\partial\gamma_3}{\partial v} = \frac{\partial\gamma}{\partial\gamma_3} \frac{\partial\gamma_3}{\partial v} = 0.19 \frac{(0.82 - v)(-1) + (0.76 - v)}{(0.82 - v)^2} = -0.285,$$

$$\gamma_4 = \frac{0.82 - v}{0.85 - v},$$

$$\frac{\partial\gamma_4}{\partial v} = \frac{\partial\gamma}{\partial\gamma_4} \frac{\partial\gamma_4}{\partial v} = 0.169 \frac{(0.85 - v)(-1) + (0.82 - v)}{(0.85 - v)^2} = -0.3,$$

$$\sigma_{\gamma_1}^2 = \left(\frac{\partial \gamma_1}{\partial \alpha} \cdot \sigma_\alpha \right)^2 + \left(\frac{\partial \gamma_1}{\partial \beta} \sigma_\beta \right)^2 + \left(\frac{\partial \gamma_1}{\partial v} \sigma_v \right)^2,$$

$$\sigma_{\gamma_1}^2 = (1.1 \cdot 0.02)^2 + (-0.46 \cdot 0.02)^2 + (-0.637 \cdot 0.02)^2 = 0,001,$$

$$\sigma_{\gamma_1} = 0.032,$$

$$\sigma_{\gamma_2}^2 = \left(\frac{\partial \gamma_2}{\partial \alpha} \cdot \sigma_\alpha \right)^2 + \left(\frac{\partial \gamma_2}{\partial \beta} \sigma_\beta \right)^2 + \left(\frac{\partial \gamma_2}{\partial v} \sigma_v \right)^2,$$

$$\sigma_{\gamma_2} = 0.03,$$

$$\sigma_{\gamma_3}^2 = \left(\frac{\partial \gamma_3}{\partial \alpha} \cdot \sigma_\alpha \right)^2 + \left(\frac{\partial \gamma_3}{\partial \beta} \sigma_\beta \right)^2 + \left(\frac{\partial \gamma_3}{\partial v} \sigma_v \right)^2,$$

$$\sigma_{\gamma_3} = 0.03,$$

$$\sigma_{\gamma_4}^2 = \left(\frac{\partial \gamma_4}{\partial \alpha} \cdot \sigma_\alpha \right)^2 + \left(\frac{\partial \gamma_4}{\partial \beta} \sigma_\beta \right)^2 + \left(\frac{\partial \gamma_4}{\partial v} \sigma_v \right)^2,$$

$$\sigma_{\gamma_4} = 0.04$$

The error function is:

$$\sigma_{\gamma}^2 = \left(\frac{\partial \gamma}{\partial \gamma_1} \sigma_{\gamma_1} \right)^2 + \left(\frac{\partial \gamma}{\partial \gamma_2} \sigma_{\gamma_2} \right)^2 + \left(\frac{\partial \gamma}{\partial \gamma_3} \sigma_{\gamma_3} \right)^2 + \left(\frac{\partial \gamma}{\partial \gamma_4} \sigma_{\gamma_4} \right)^2 = 0.018$$

Though the divergence is about 15%, we believe that the approach of the error's numerical analysis is comprehensible as opportunities of such analysis are practically unlimited.

Conclusion

The offered method of the output determination of the accuracy estimation for enrichment products can be applied for enterprise technological balance forming.

REFERENCES

- [1] *Kozin V.Z.*: Approbation and control of technological processes of enrichment. M., Nedra, 1985, 294