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# FUNDAMENTAL SOLUTION OF THE PROBLEM DESCRIBING SHIP MOTION IN WAVES

**Abstract.** The problem describing a ship motion in waves comprises the Laplace equation, boundary condition on wetted surface of the ship, condition on the free surface of the sea in the form of a differential equation, the radiation condition, and a condition at infinity. This problem can be transformed to a Fredholm equation of second kind, and then numerically solved using the boundary element method, if the fundamental solution of the problem is known. This paper presents the derivation of the fundamental solution. In physical interpretation, the fundamental solution represents the moving and pulsating source under free surface of the sea. The free surface elevation, generated by the source for different forward speed and frequency of pulsation, is presented in this paper.

**Keywords:** ship hydrodynamics, boundary value problem, free surface potential flows, fundamental solution.

**Mathematics Subject Classification:** 35J05, 35J25, 76B07, 76B20, 35R35.

## 1. INTRODUCTION

A ship moving in waves diffracts sea waves and generates waves radiating off the ship. Assuming that water is inviscid and incompressible, as well as that the flow is irrotational, the flow around the ship is described by the potential flow theory.

The diffraction and radiation potential velocity fields in the sea are determined by the following boundary-value problem [3]:

1. Laplace equation
$$\Delta\varphi(x) = 0, \quad x \in \mathbb{R}_3^- \setminus V, \tag{1}$$
where $V$ is a closed domain occupied by the ship, and $\mathbb{R}_3^- = \{x : x_3 < 0\}$,

2. The boundary conditions:
— on the free surface $x \in S_F = \{x : x_3 = 0\} \setminus V$

$$-\nu\varphi(x) - 2i\tau\frac{\partial\varphi(x)}{\partial x_1} + \frac{1}{k_0}\frac{\partial^2\varphi(x)}{\partial x_1^2} + \frac{\partial\varphi(x)}{\partial x_3} = 0, \tag{2}$$

where $\nu = \frac{\omega_E^2}{g}$, $\tau = \frac{\omega_E u_0}{g}$, $k_0 = \frac{g}{u_0^2}$, $\omega_E$ is the encounter frequency, $\omega_E = \omega(1 - \omega u_0/g\cos\beta)$, $\omega$ is the wave frequency, $u_0$ is the ship forward speed, and $g$ is the gravity acceleration;
— on the wetted surface of the ship:
- for diffraction potential

$$\frac{\partial\varphi_D(x)}{\partial\mathbf{n}} = -\frac{\partial\varphi_W(x)}{\partial\mathbf{n}}, \quad x \in S, \tag{3}$$

where $\varphi_W$ is the potential of the undisturbed waves on the free surface, determined by the function

$$\varphi_W(x) = i\frac{g}{\omega}e^{kx_3 - i\mathbf{k}\cdot\mathbf{x}},$$

$\mathbf{k} = \mathbf{k}(\cos\beta, \sin\beta, 0)$, $\beta$ is the angle between the wave vector $\mathbf{k}$ and axis $x_1$, $\mathbf{x} = (x_1, x_2, 0)$, $\omega$ is the wave frequency, $k = \omega^2/g$, $S$ is the wetted surface of the ship, and $\mathbf{n} = (n_1, n_2, n_3)$ is the normal vector, outward to the ship;
- for the radiation potential

$$\begin{aligned}\frac{\partial\varphi_{ri}^R(x)}{\partial\mathbf{n}} &= n_i, \\ \frac{\partial\varphi_{ri}^I(x)}{\partial\mathbf{n}} &= 0, \quad i = 1\ldots6, \quad x \in S,\end{aligned} \tag{4}$$

3. The conditions at infinity:
— for ship forward speed $u_0 = 0$:
1° radiation condition [2]:

$$\lim_{\rho\to\infty}\sqrt{\rho}\Big(\frac{\partial\varphi}{\partial\rho} + i\nu\varphi\Big) = 0, \tag{5}$$

2° condition at infinity on the free surface:

$$\lim_{\rho\to\infty}\sqrt{\rho}|\varphi| \le c, \quad x \in S_F = \{x : x_3 = 0\}, \tag{6}$$

where $\rho = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$, and $c$ is a constant;
— for ship forward speed $u_0 > 0$:
1° condition imposed on the wave system generated:

- for $\tau \le \frac{1}{4}$ $(\tau = \frac{\omega_E u_0}{g})$:
  one wave with forward phase velocity greater than the ship speed, escapes from the ship; one wave with forward phase velocity, lower than the ship speed, follows the ship; two waves with backward phase velocity move away from the ship;
- for $\tau > \frac{1}{4}$: no wave in front of the ship is generated; two waves with backward phase velocity move away from the ship;

(The formulation of the radiation conditions for ship with forward speed in strict mathematical form still poses a problem).

2° condition at infinity on the free surface

$$\lim_{\rho\to\infty} \sqrt{\rho}|\varphi| \le c, \quad x \in S_F = \{x : x_3 = 0\},$$

where $\rho = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$, and $c$ is constant,

4. The condition at the sea bottom:

$$\lim_{x_3\to-\infty} \frac{\partial\varphi(x)}{\partial x_3} = 0. \tag{7}$$

It is assumed that the solution of the problem has the following form of a single potential layer [1]:

$$\phi(x) = \int_S \mu(y)E(x,y)dS_y - \frac{u_0^2}{g}\int_l \mu(y)E(x,y)n_1^2 dl_y, \tag{8}$$

where $\mu$ is the complex function describing the source density, $l = \partial S$, and $\mathbf{n} = (n_1, n_2, n_3)$ is the normal vector, outward to the ship. Function $E(x,y)$ occurring in formula (8) is the fundamental solution to the above problem, satisfying the Laplace equation and boundary conditions, except the condition on the wetted ship surface. In the physical interpretation, the fundamental solution represents the velocity potential generated by pulsating source of unit strength and translating with forward speed under the free surface.

The following form of function $E$ occurring in formula (8) is assumed [3]:

$$E(x,y) = -\frac{1}{4\pi}\left[\frac{1}{|x-y|} - \frac{1}{|x-z|} + G(x,z)\right], \quad x, y \in \mathbb{R}_3^-, \quad x \ne y, \tag{9}$$

where $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$, $z = (y_1, y_2, -y_3)$,

$$|x-y| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2},$$
$$|x-z| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 + y_3)^2}, \tag{10}$$

$\omega_E$ is the encounter pulsation frequency, and $G$ is a harmonic function which is to be determined from Laplace equation (1) and the equation on the free surface (2).

Assuming the solution of problem (1)–(8) in the form of a single potential layer transforms the differential problem to the following Fredholm integral equation of second kind

$$\frac{1}{2}\mu(x) + \int_S \mu(y)\frac{\partial}{\partial \mathbf{n_y}}E(x,y)ds_y - \frac{u_0^2}{g}\int_\iota \mu(y)\frac{\partial}{\partial \mathbf{n_y}}E(x,y)n_1^2(y)dl_y = u_n(x), \qquad x\varepsilon S, \tag{11}$$

where $u_n$ is the boundary condition on wetted surface (3) or (4).

## 2. FUNDAMENTAL SOLUTION OBTAINED BY KHASKIND

Substituting $E$ to Laplace equation (1) and to condition (2) on the free surface, and applying the Fourier transform and the reverse Fourier transform, the function $G$ is obtained in an integral form. The fundamental solution obtained in such a way formally satisfies equations (1) and (2), but it cannot be used to determine the flow around the ship numerically, as the integrand is a singular function and the integration is over an infinite domain. Therefore, a different approach is applied in the next chapter to construct a function $G$ convenient for numerical computations.

Khaskind [2] applied artificial viscosity to remove the singularities in the integrand and obtained the following form of function $G$:

$$G(x,z,\omega_E,u_0) = \\ = -\frac{\nu}{\pi}\int_{-\pi}^{\pi}\int_0^\infty \frac{\lambda e^{\lambda(x_3-z_3)+i\lambda\chi}}{\tau^2\lambda^2\cos^2\vartheta - 2\tau\nu\lambda(1-i\beta)\cos\vartheta - \nu\lambda + \nu^2(1-2i\beta)}d\lambda d\vartheta, \tag{12}$$

satisfying the following condition on the free surface

$$-\nu(1-2i\beta)G(x,z) - 2i\tau(1-i\beta)\frac{\partial G(x,z)}{\partial x_1} + \frac{1}{k_0}\frac{\partial^2 G(x,z)}{\partial x_1^2} + \frac{\partial G(x,z)}{\partial x_3} = \\ = 2\frac{\partial}{\partial x_3}\frac{1}{|x-z|}, \tag{13}$$

where $\nu = \frac{\omega_E^2}{g}$, $\tau = \frac{\omega_E u_0}{g}$, $k_0 = \frac{g}{u_0^2}$, $\omega_E$ is the encounter frequency, $u_0$ is the forward speed of the ship, $\beta = \mu_1/2\omega_E$, and $\mu_1$ is the artificial viscosity.

Equation (13), containing the viscosity coefficient $\beta = \mu_1/2\omega_E$, results from equation (2) after including the artificial viscosity $\mu_1$ and substituting fundamental solution (9) to this equation [2]. The viscosity is assumed equal to zero after obtaining a satisfactory form of function $G$. Function (12) also satisfies Laplace equation (1).

The singular points of integrand (12) satisfy the equation

$$\tau^2\lambda^2\cos^2\vartheta - 2\tau\nu\lambda(1-i\beta)\cos\vartheta - \nu\lambda + \nu^2(1-2i\beta) = 0. \tag{14}$$

The roots of the equation are the following complex numbers:

$$\overline{\lambda}_1 = \nu \frac{1 + 2\tau(1 - 2i\beta)\cos\vartheta + \sqrt{1 + 4\tau\cos\vartheta - 4\tau i\beta\cos\vartheta - 4\tau^2\beta^2\cos^2\vartheta}}{2\tau^2\cos^2\vartheta},$$
$$\overline{\lambda}_2 = \nu \frac{1 + 2\tau(1 - 2i\beta)\cos\vartheta - \sqrt{1 + 4\tau\cos\vartheta - 4\tau i\beta\cos\vartheta - 4\tau^2\beta^2\cos^2\vartheta}}{2\tau^2\cos^2\vartheta}. \tag{15}$$

Denoting the above fomulae for $\beta = 0$ by

$$\lambda_1 = \nu \frac{1 + 2\tau\cos\vartheta + \sqrt{1 + 4\tau\cos\vartheta}}{2\tau^2\cos^2\vartheta},$$
$$\lambda_2 = \nu \frac{1 + 2\tau\cos\vartheta - \sqrt{1 + 4\tau\cos\vartheta}}{2\tau^2\cos^2\vartheta}, \tag{16}$$

the following form of the roots

$$\overline{\lambda}_1 = \lambda_1 - i\,\frac{\mu_1}{2u\cos\vartheta}\frac{\sqrt{1 + 4\tau\cos\vartheta} + 1}{\sqrt{1 + 4\tau\cos\vartheta}} + O(\mu_1^2),$$
$$\overline{\lambda}_2 = \lambda_2 - i\,\frac{\mu_1}{2u\cos\vartheta}\frac{\sqrt{1 + 4\tau\cos\vartheta} - 1}{\sqrt{1 + 4\tau\cos\vartheta}} + O(\mu_1^2). \tag{17}$$

is obtained for small $\mu_1 = 2\beta\omega_E$.

Numbers $\lambda_1$ and $\lambda_2$ are:

— real for $|\vartheta| < \pi - \vartheta_0$,
— complex for $\vartheta \in [-\pi, \pi + \vartheta_0) \cup (\pi - \vartheta_0, \pi]$,

where

$$\vartheta_0 = \begin{cases} 0 & \text{dla} \quad |\tau| \le \frac{1}{4} \\ \arccos\frac{1}{4|\tau|} & \text{dla} \quad \tau > \frac{1}{4}. \end{cases} \tag{18}$$

The formulae imply that for $\vartheta$ rendering $\lambda_1$ and $\lambda_2$ real:

— the sign of the imaginary part of $\overline{\lambda}_1$ depends on $\cos\vartheta$, and
— the sign of the imaginary part of $\overline{\lambda}_2$ depends on the sign of $\tau$, but does not depend on $\vartheta$, as

$$\frac{\sqrt{1 + 4\tau\cos\vartheta} - 1}{\cos\vartheta} = \frac{4\tau}{\sqrt{1 + 4\tau\cos\vartheta} + 1}.$$

Further transformations of (12) by Khaskind [2], using roots $\lambda_1$ and $\lambda_2$, resulted in very complicated functions in the form of double integrals (one over an infinite domain) with integrands rapidly oscillating.

Functions in such forms cannot be used in the numerical computations, therefore, formula (12) will be transformed, using the above determined $\lambda_1$ and $\lambda_2$, to a feasible form easy for practical application. Considerations will be carried for $\tau > 0$ (the case $\tau < 0$ is analogous).

## 3. FUNDAMENTAL SOLUTION USEFUL FOR NUMERICAL COMPUTATIONS

Using the following decomposition of the integrand in (12) into simple fractions

$$\frac{1}{\tau^2\cos^2\vartheta(\lambda-\overline{\lambda}_1)(\lambda-\overline{\lambda}_2)}=\frac{1}{\nu p}\left(\frac{\overline{\lambda}_1}{\lambda-\overline{\lambda}_1}-\frac{\overline{\lambda}_2}{\lambda-\overline{\lambda}_2}\right),$$

where $p=\sqrt{1+4\tau\cos\vartheta-4\tau i\beta\cos\vartheta-4\tau^2\beta^2\cos^2\vartheta}$, function (13) can be written in the form

$$G=-\frac{\nu}{\pi}\int\limits_{-\pi}^{\pi}\frac{1}{\nu p}\int\limits_{0}^{\infty}\left(\frac{\overline{\lambda}_1}{\lambda-\overline{\lambda}_1}-\frac{\overline{\lambda}_2}{\lambda-\overline{\lambda}_2}\right)e^{\lambda(x_3-z_3)+i\ \lambda\chi}d\lambda d\vartheta. \tag{19}$$

In further transformations the following theorem on complex variable functions will be used [5]:

**Theorem 1.** *If the function*

$$F(z)=\int\limits_a^b\frac{\omega(t)}{t-z}dt$$

*is determined in the complex plane and* $\omega$ *is a smooth function, then*

$$F(\xi\pm 0)=\pm i\ \pi\omega(\xi)+P.V.\int\limits_b^a\frac{\omega(t)}{t-\xi}dt,\quad \xi\in(a,b)\subset\mathbb{R},$$

*where P.V. stands for the principal value of the integral.*

Taking the limits as $\overline{\lambda}_1\to\lambda_1$ and $\overline{\lambda}_2\to\lambda_2$ in (19), using (15) and the above theorem, the following form of function $G$ is obtained:

$$\begin{aligned}G(x,z;\omega_E,u_0)=\;&-\tfrac{1}{\pi}P.V.\int\limits_{-\pi+\vartheta}^{\pi-\vartheta_0}\tfrac{1}{\sqrt{1+4\tau\cos\vartheta}}\int\limits_0^\infty\tfrac{\lambda_1}{\lambda-\lambda_1}e^{\lambda(x_3-z_3)+i\lambda\chi}d\lambda d\vartheta+\\&+\tfrac{1}{\pi}P.V.\int\limits_{-\pi+\vartheta}^{\pi-\vartheta_0}\tfrac{1}{\sqrt{1+4\tau\cos\vartheta}}\int\limits_0^\infty\tfrac{\lambda_2}{\lambda-\lambda_2}e^{\lambda(x_3-z_3)+i\lambda\chi}d\lambda d\vartheta+\\&+i\int\limits_{-\pi+\vartheta}^{\pi-\vartheta_0}\tfrac{\lambda_1}{\sqrt{1+4\tau\cos\vartheta}}e^{\lambda_1(x_3-z_3)+i\lambda_1\chi}\mathrm{sign}(\cos\vartheta)d\vartheta-\\&-i\int\limits_{-\pi+\vartheta)}^{\pi-\vartheta_0}\tfrac{\lambda_2}{\sqrt{1+4\tau\cos\vartheta}}e^{\lambda_2(x_3-z_3)+i\lambda_2\chi}d\vartheta-\\&-\tfrac{\nu}{\pi\tau^2}\int\limits_{-\vartheta_0}^{\vartheta_0}\tfrac{1}{\cos^2\vartheta}\int\limits_0^\infty\tfrac{\lambda}{(\lambda-a)^2+b^2}e^{\lambda(x_3-z_3)-i\lambda\chi}d\lambda d\vartheta.\end{aligned}\tag{20}$$

where

$$a = \frac{1 + 2\tau\cos\vartheta}{2\tau^2\cos^2\vartheta}, \quad b = \frac{\sqrt{|1 + 4\tau\cos\vartheta|}}{2\tau^2\cos^2\vartheta}.$$

In order to enable numerical computations of function (20), the principal values (P.V.) of the integrals occurring in (20) are expanded into series. First, the following integrals are expanded:

$$J_1 = \int\limits_0^\infty \frac{1}{\beta - \lambda} e^{y\lambda} \cos x\lambda d\lambda \quad \text{and} \quad J_2 = \int\limits_0^\infty \frac{1}{\beta - \lambda} e^{y\lambda} \sin x\lambda d\lambda, \tag{21}$$

where $\beta, x, y \in \mathbb{R}$ and $y < 0$.
Let

$$J = J_1 - iJ_2 = \int\limits_0^\infty \frac{1}{\beta - \lambda} e^{\lambda(y - ix)} d\lambda. \tag{22}$$

Denoting $z = x + iy$, equation (22) can be written in the form

$$J = e^{-i\beta z} \int\limits_0^\infty \frac{1}{\beta - \lambda} e^{iz(\beta - \lambda)} d\lambda.$$

Substituting $\beta - \lambda = u$, and then $izu = t$, the following formula is obtained

$$J = J_1 - iJ_2 = e^{-i\beta z} \int\limits_{-\infty}^{iz\beta} \frac{e^t}{t} dt = \mathrm{Ei}(i\beta z), \tag{23}$$

where Ei is the exponential integral function defined as follows:

$$\mathrm{E}(z) = \int\limits_{-\infty}^{z} \frac{e^t}{t} dt, \tag{24}$$

The integral is taken over an arbitrary path $L$ in the complex plane cut along the positive real axis [4].

In order to expand the exponential integral function into a function series, equation (24) is written in the following form:

$$\mathrm{Ei}(z) = \int\limits_{-\infty}^{-1} \frac{e^t}{t} dt + \int\limits_{-1}^{0} \frac{e^t - 1}{t} dt + \int\limits_{0}^{z} \frac{e^t - 1}{t} dt + \int\limits_{-1}^{z} \frac{dt}{t}.$$

The sums of the two first integrals can be written (after substituting $t = -u^{-1}$ in the first, and $t = -u$ in the second one) in the form

$$\gamma = \int\limits_0^1 \frac{1 - e^{-u} - e^{-\frac{1}{u}}}{u} du.$$

This number is the Euler constant $\gamma = 0,57721\ldots$; thus,

$$\mathrm{Ei}(z) = \gamma + \ln(-z) + \int\limits_0^z \frac{e^t - 1}{t} dt,$$

where the main branch of the logarithm was assumed. The integrand on the right side of the last equality is an entire function, thus its integral is an entire function too, and can be expanded into series, convergent in the entire plane. Expanding the integrand first, and then integrating term-wise gives

$$\int\limits_0^z \frac{e^t - 1}{t} dt = \int\limits_0^z \sum_{n=1}^{\infty} \frac{t^{n-1}}{n!} = \sum_{n+1}^{\infty} \frac{z^n}{n!n}, \quad |z| < \infty.$$

Thus,

$$\mathrm{Ei}(z) = \gamma + \ln(-z) + \sum_{n+1}^{\infty} \frac{z^n}{n!n}. \tag{25}$$

This function is well defined for all for points $z$ of the plane cut along positive real axis. Taking $i\beta z = \beta(-y + ix) = \beta r e^{i\theta}$ instead of $z$ in (25), the following formula is obtained

$$\mathrm{Ei}(i\beta z) = \gamma + \ln(\beta r) + i[\theta - \text{ sign } (\mathrm{Im}\ i\beta z)\pi] + \sum_{n=1}^{\infty} \frac{(\beta r)^n e^{in\theta}}{n!n}. \tag{26}$$

The principal value of integral (24) for $z = i\beta z$ is computed in the following way:

$$\begin{aligned}
P.V. \int\limits_{-\infty}^{i\beta z} \frac{e^t}{t} dt = & \lim_{\epsilon \to 0} \left( \int\limits_{-\infty}^{-i\epsilon z} + \int\limits_{i\epsilon z}^{i\beta z} \right) \frac{e^t}{t} dt = \\
= & \lim_{\epsilon \to 0} \left[ \int\limits_{-\infty}^{-i\epsilon z} \frac{e^t}{t} dt + \int\limits_{i\epsilon z}^{i\beta z} \frac{e^t - 1}{t} dt + \int\limits_{i\epsilon z}^{i\beta z} \frac{1}{u} du \right] = \\
= & \lim_{\epsilon \to 0} \left[ \gamma + \ln(i\epsilon z) + \sum_{n=1}^{\infty} \frac{(-i\epsilon z)^n}{nn!} + \sum_{n=1}^{\infty} \frac{(i\beta z)^n}{nn!} - \right. \\
& \left. - \sum_{n=1}^{\infty} \frac{(i\epsilon z)^n}{nn!} + \ln(i\beta z) - \ln(i\epsilon z) \right],
\end{aligned}$$

which gives

$$P.V. \int\limits_{-\infty}^{i\beta z} \frac{e^t}{t} dt = \lim_{\epsilon \to 0} \left[ \left( \gamma + \ln(\beta r) + \sum_{n=1}^{\infty} \frac{(\beta r)^n \cos\theta}{nn!} \right) + i \left( \sum_{n=1}^{\infty} \frac{(\beta r)^n \sin\theta}{nn!} + \theta \right) \right].$$

This formula can be written as

$$P.V. \int\limits_{-\infty}^{i\beta z} \frac{e^t}{t} dt = f(r, \theta) + ig(r, \theta), \tag{27}$$

where

$$f(r,\theta) = \gamma + \ln(\beta r) + \sum_{n=1}^{\infty} \frac{(\beta r)^n \cos n\theta}{n!n}, \quad r \neq 0,$$

$$g(r,\theta) = \theta + \sum_{n=1}^{\infty} \frac{(\beta r)^n \sin n\theta}{n!n}. \tag{28}$$

Taking into account (23), (27), and the equality $e^{-i\beta z} = e^{\beta y}(\cos\beta x - i\sin\beta x)$, the following formulae determining the principal values of integrals (21) are obtained

$$P.V.J_1 = e^{\beta y}[f\cos\beta x + g\sin\beta x],$$

$$P.V.J_2 = e^{\beta y}[f\sin\beta x - g\cos\beta x], \tag{29}$$

and thus

$$P.V.\int_0^{\infty} \frac{1}{\lambda-\beta} e^{\lambda(y+ix)} d\lambda = -P.V.(J_1 + iJ_2). \tag{30}$$

Substituting (30) into (20) the function $G$, being the main component of fundamental solution (9), takes the following form

$$\begin{aligned} G(x,z;\omega_E,u_0) = \quad & \frac{1}{\pi}\int_{-\pi+\vartheta}^{\pi-\vartheta_0} \frac{\lambda_1}{\sqrt{1+4\tau\cos\vartheta}} P.V.(J_1+iJ_2)d\vartheta - \\ & -\frac{1}{\pi}\int_{-\pi+\vartheta}^{\pi-\vartheta_0} \frac{\lambda_2}{\sqrt{1+4\tau\cos\vartheta}} P.V.(J_1+iJ_2)d\vartheta + \\ & +i\int_{-\pi+\vartheta}^{\pi-\vartheta_0} \frac{\lambda_1}{\sqrt{1+4\tau\cos\vartheta}} e^{\lambda_1[(x_3-z_3)+i\chi]}\mathrm{sign}(\cos\vartheta)d\vartheta - \\ & -i\int_{-\pi+\vartheta}^{\pi-\vartheta_0} \frac{\lambda_2}{\sqrt{1+4\tau\cos\vartheta}} e^{\lambda_2[(x_3-z_3)+i\chi]}d\vartheta - \\ & -\frac{\nu}{\pi\tau^2}\int_{-\vartheta_0}^{\vartheta_0} \frac{1}{\cos^2\vartheta}\int_0^{\infty} \frac{\lambda}{(\lambda-a)^2+b^2} e^{\lambda[(x_3-z_3)-i\chi]}d\lambda d\vartheta, \end{aligned} \tag{31}$$

where $\beta = \lambda_1$ in $J_1$ and $J_2$ in the first integral of formula (31) and $\beta = \lambda_2$ in $J_1$ and $J_2$ in the second one. The remaining symbols occurring in $J_1$ and $J_2$ have the following meaning:

$$r = \sqrt{(x_3-z_3)^2+\chi^2}, \quad \theta = \arctan\frac{\chi}{-(x_3-z_3)},$$

where $\chi = (x_1-z_1)\cos\vartheta + (x_2-z_2)\sin\vartheta$.

Function (31) enables one to perform the numerical computations which, in turn, enable the boundary value problem to be solved and the diffraction and radiation velocity fields to be determined.

The fundamental solution also makes possible the computation of the free surface elevation $\zeta$, generated by the pulsating source and translating under free sea surface, according to the following formula [3]:

$$\zeta = -\frac{1}{g}\mathrm{Re}(i\omega_E - u_0\frac{\partial}{\partial x_1})G, \quad x \in S_F = \{x : x_3 = 0\}. \tag{32}$$

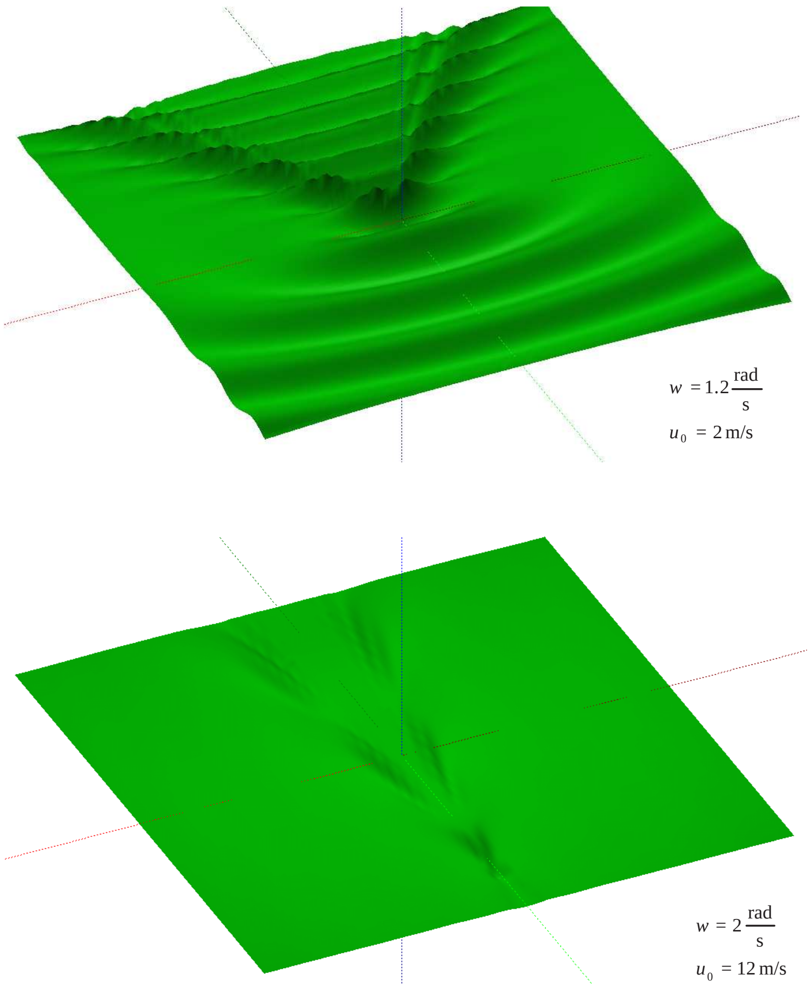


**Fig. 1.** Free surface generated by source $G(x,(0,0,-1);\omega_E,u_0)$, $x_1 \in [-50,50]$, $x_2 \in [-50,50]$, $x_3 = 0$

## 4. CONCLUSIONS

The problem describing ship motion in waves [3] can be solved in practice if the fundamental solution of the problem (1)—(8), determining the diffraction and radiation velocity fields, is determined in the form enabling numerical computations of its values. The paper presents the derivation of the fundamental solution from the one obtained by Khaskind [2] to the form supporting numerical computation.

The derivation is based on Theorem 1, on the exponential integral function and on the expansion of this integral into function series.

The fundamental solution in such form is used to determine the diffraction and radiation velocity fields around the ship [3]. In this paper, function given by (31) was used to compute the elevation of the sea free surface generated by a pulsating and translating source under the free surface (see Fig. 1).

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