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SUPPRESSION OF VIBRATION OF A CLAMPED BEAM VIA PIEZOCERAMICS

SUMMARY

Contribution deals with the suppression of the resonance vibration of a clamped beam with sensors and actuators made of piezoceramics material. It will be discussed the problem of influencing of a response of vibration by higher frequencies so called control and observation spillover. The model of a beam with piesoceramics was designed in environment ANSYS. The properties of the model have been identified from the response on an unit step impuls (voltage). The control was realized by controller with PPF method. Simulation of control has been realized on the clampd beam in environment MATLAB. There was designed an experimental test bed for verification of the simulation results.

Keywords: *vibration control, smart material systems, suppression of vibration*

TŁUMIENIE DRGAŃ UMOCOWANEJ BELKI Z PIEZOELEKTRYKIEM

Artykuł dotyczy tłumienia drgań rezonansowych jednostronnie zamocowanej belki z czujnikiem i aktuatorem wykonamym z materiału piezoelektrycznego. Omówiono problem wpływu odpowiedzi w postaci drgań o częstotliwościach większych od częstotliwości Nyquista na sterowanie i sprzężenie zwrotne, nazywanego w literaturze angielskiej: control and observation spillover. Model belki z piezoelektrykiem został zaimplementowany w programie ANSYS. Właściwości modelu zostały zidentyfikowane na podstawie odpowiedzi układu na wymuszenie w postaci skoku jednostkowego (skoku napięcia). Sterowanie układem zostało zrealizowane przy wykorzystaniu regulatora z dodatnim sprzężeniem zwrotnym od przemieszczenia. Symulacja sterowania jednostronnie zamocowanej belki zrealizowana została w środowisku MATLAB. Zaprojektowano również eksperymentalne stanowisko badawcze do weryfikacji badań symulacyjnych.

Słowa kluczowe: *sterowanie drganiami, materiały inteligentne, tłumienie drgań*

1. INTRODUCTION

One of the predominant difficulties in the control theory of mechanical structure systems (MS) comes from the fact that mechanical structures are basically distributed parameter systems. This implies that such structures have a very high if not infinity number modes of vibration within and beyond-the bandwidth of the controller. Usually, there are also modes within the bandwidth that are not targeted for control. The presence of uncontrolled or unmodelled modes within the bandwidth of the closed loop system results in the well-known phenomenon of “spillover”. Spillover is the coupling of the control system to the “residual” dynamics, which occurs because sensors and actuators are not continuously spatially distributed. It has long been know that spillover can destabilize residual dynamics, especially at higher frequencies where the dynamics is least well modeled. A great amount of research has centered on developing techniques to manage these destabilizing influences.

In the area of structural vibration suppression, the technique with perhaps the greatest immunity from the destabilizing the effects of spillover is collocated direct velocity feedback, which, in the absence of actuator dynamics, is unconditionally stable. In the presence of actuator dynamics, however, instability may result if *a priori* precaution is not taken. It has been shown that the stability boundary of modes near the natural frequency of the actuators is critically dependent on the inherent natural damping in these modes a quantity not well known in most cases. In addition, the technique requires rate measurement a quantity that becomes vanishingly small at low frequency.

Another crucial problem which has received very little attention is the problem actuator dynamics. Balas [4] furnished a brief analysis to justify the negligence of fast actuator dynamics, the conclusion of which is questionable due to conflicting assumptions. Basically the bandwidth of practical actuators is finite while that of MS is very large if not infinite. Extreme care must be exercised so that control of low--frequency modes does not destabilize the intermediate or higher frequency modes.

The technique implemented in the work of Caughey and Goh [2], Positive Position Feedback (PPF), was originally suggested by as an alternative to collocated direct velocity feedback. Like velocity feedback, the method is not sensitive to spillover but in addition, it is not destabilized by finite actuator dynamics. PPF requires only generalized displacement measurements which make it amenable to a strain--based sensing approach. While PPF is not unconditionally stable, as will be seen later, the stability condition is non-dynamic and minimally restrictive. The objective of these experiments is to examine the feasibility of using Positive Position Feedback as a vibration suppression control strategy on

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a structure with low inherent stiffness, and to investigate the feasibility of using PZT materials as control actuators and strain sensors-simulating a PZT active-member.

2. COLLOCATED VELOCITY FEEDBACK CONTROL IGNORING ACTUATOR DYNAMICS

Mechanical structures are, in principle, continuous distributed systems described by partial differential equations (PDE). For practical consideration, however, it is common to model the system by finite element analysis (FEM), hence approximating the PDE by a ordinary differential equations

$$M\ddot{y} + B\dot{y} + Ky = F(t) \tag{2.1}$$

where y and F are n-dimensional vectors representing the system physical state and control force respectively. The system mass matrix $M(n{\times}n)$ (including the mass of the collocated sensors and actuators) is positive definite, while the natural passive-damping matrix $B(n{\times}n)$ and stiffness matrix K $(n{\times}n)$ are positive semi-definite for a real mechanical structure. Note that we have reduced the infinite dimension of the original system to n for accurate modelling (n should be fairly large).

Now we define the sensor/actuator (S/A) location matrix L (dimension $n_A{\times}n$), where n_A is the number of sensor/actuator pairs available. Sometimes L is written as the set $L = \{i_1, i_2, ..., i_{N_A}\}$ where the $i, j = 1, \ldots, n_A$ represent the ordered elements to which a sensor/actuator pair is attached. Assuming for the moment that the measurement is accurate for all frequencies and actuator dynamics is negligible, the control force $F(t)$ for direct velocity feedback [3] is simply given by

$$F(t) = -L^T G L\dot{y} \tag{2.2}$$

where G (dimension $n_A{\times}n_A$) is the gain matrix to be designed. It is easy to see that if G is positive definite, then the system is globally stable. Note that though spillover still exists, it will not destabilize the system. Furthermore, with n_A pairs of sensor/actuator available, we can design G such that the closed-loop damping of the first n_A modes can be prescribed approximately.

For purpose of illustration, consider a clamped beam of n elements with n_A pairs of sensor/actuator. Suppose we would like the first mode to the closed-loop damping ratio of approximately ζ_n and the other controlled modes to have approximately the same damping as the first. It can be shown that the closed loop damping of all the modes are dominated by the diagonal elements (γ_i) of the modal gain matrix G_m, where

$$G_m = \Phi^T L^T G L \Phi \tag{2.3}$$

and Φ is the modal matrix which simultaneously diagonalizes M and K, such that:

$$\begin{aligned} &\Phi^T M \Phi = 1 \\ &\Phi^T K \Phi = \Omega = \mathrm{diag}\,(\Omega_i^2) \\ &\Phi^T B \Phi = \Delta \end{aligned} \tag{2.4}$$

Since the structural natural damping B is small, but not clearly known, we shall assume that all modes possess the same damping ratio ζ_i, such that $\delta_i = 2\,\Omega_i\,\zeta_i$.

3. STABILITY PROBLEMS CAUSED BY THE INCLUSION OF ACTUATOR DYNAMICS

The previous assumption that actuator dynamics are negligible is never justified in practice, but this is unfortunately ignored by most researchers in the field of mechanical structures control. Balas [4] considered the control of a reduced system by deliberately ignoring all the fast modes. It was claimed that for such a reduced system, the negligence of actuator dynamics is justifiable and will not cause instability, if the actuator dynamics is sufficiently fast. This claim is questionable since fast modes certainly do exist whether we would like to model them as such or not. A flexible structure has essentially infinite bandwidth while the actuators, no matter how fast, have finite bandwidth. Thus the problem of finite actuator dynamics interaction is a crucial one which deserves extremely careful consideration.

Assuming that each actuator has identical second order dynamics, the velocity feedback control system with actuator dynamics is represented by:

$$M\ddot{y} + B\dot{y} + Ky = -L^T G x \tag{3.1}$$

$$\ddot{x} + 2\zeta_a \Omega_a \dot{x} + \Omega_a^2 (x - L\dot{y}) = 0 \tag{3.2}$$

where x is an n_A-dimensional actuator state vector, ζ_a and Ω_a the damping ratio and the natural frequency of the actuators respectively. It is convenient to analyze the system in modal space, so we introduce the following transformation:

$$\begin{aligned} y &= \Phi q \\ u &= L\Phi z \end{aligned} \tag{3.3}$$

where q and ζ are n-dimensional modal state vectors. (3.1) and (3.2) are transformed to:

$$\ddot{q} + \Delta\dot{q} + \Omega\, q = -\Phi^T L^T G L \Phi\, z \tag{3.4}$$

$$\ddot{z} + 2\zeta_a \Omega_a \dot{z} + \Omega_a^2 (z - \dot{q}) = 0 \tag{3.5}$$

The following paradoxical proposition is analogous to that of Balas [4] but is considered in the context of collocated velocity feedback control.

Proposition

If $\Omega_a >> \max \Omega_i$, $i = 1, 2, \ldots, n$, then the coupled system (3.4) and (3.5) is stable.

3.1. Positive position feedback control

In the previously published papers it has been shown (see for example Baz [3] and Inman [10]), that several features of PPF make it attractive MS environment. Features of PPF are best demonstrated by considering the scalar case.

Scalar Case

The scalar system consists of two equations, one describing the structure, and one describing the compensator

System

$$\ddot{\xi} + 2\zeta\Omega_0\dot{\xi} + \Omega_0^2(\xi - \gamma\eta) = 0 \tag{3.6}$$

Compensator

$$\ddot{\eta} + 2\zeta_\phi\ \Omega_0\dot{\eta} + \Omega_f^2(\eta - \xi) = 0 \tag{3.7}$$

$\xi, \eta \in R,\ \ \gamma > 0,$

where:

- γ – scalar gain > 0,
- ε – modal coordinate,
- η – filter coordinate,
- Ω_0 and Ω_f – structural and filter natural frequencies, respectively,
- ζ and ζ_f – structural and filter damping ratios, respectively.

The compensator is composed of a second-order filter with the same form as the modal equation of (3.6), but with much higher damping ratio. The positive position terminology in the name PPF is derived from the fact that the position coordinate of (3.6) is positively fed to the filter, and the position coordinate of (3.7) is positively fed back to the structure.

Multivariate Case

A multivariate synthesis theory is based on the assumption that filters (such as equation (3.7)) can be tuned to individual modes that remain uncoupled to first order. If additional modes are included in the single-input-single-output (SISO) example above, the closed loop poles shift from the design locations. The uncoupled modes synthesis procedure is used to produce a specified damping ratio in the first mode, and equal settling times for the remaining modes. We assumed, that the poles and zeros are far enough apart to justify the uncoupled modes assumption. First mode damping ratios about of 0.30 are prescribed and the resulting damping ratios agree very closely. If actuator dynamics are present, a feedback circuit can be synthesized which cause to synchronize with the filter equation (3.7). This essentially removes the actuator dynamics from consideration. In fact, the PPF technique was itself motivated in response to the actuator dynamics problem to provide such a feature. The following necessary and sufficient condition results for stability (see for example [2]:

The combined system of (3.6) and (3.7) is stable if and only if $\gamma < 1$.

It is interesting to note that the stability condition does not depend on the damping in the structure. Instability occurs when the stiffness of the structure is made singular by the action of the control. We will see that a non-dynamic stability criterion is characteristic of PPF. Three cases are possible, depending on whether the damped frequency of the filter is greater than, equal to, or less than the damped frequency of the structure. It has been shown in [1], that the uncoupled modes assumption is violated if the poles and zeros are very close. Strictly speaking, if the real coordinate of the filter pole is of the same order as the spacing between structural pole and zero, the modes can be treated as uncoupled. In order to achieve large reductions in dynamic response, an alternative approach is taken: the pole associated with the PPF filter is designed to have a higher damped natural frequency than the structural pole. There are several approaches that can be taken at this point. For best performance in steady state response amplitude, which depends on the number of modes and the quantities ζ_i, Ω_i it is better to leave the filter pole farther in the left half plane than the structural pole. For this particular system we can adjust the actuator frequency such that the system frequency Ω_i lies just beneath the resonance peak in order to achieve maximum closed-loop damping while keeping γ well below unity thus ensuring stability.

For multivariate systems, however the actuator frequency cannot be simultaneously adjusted to suit more than one mode, hence the concept of tuning filters is introduced to overcome this difficulty.

Tuning filters are basically band-limited electronic filters with dynamics similar to those of the actuators, but with frequencies "tuned" to the controlled mode frequencies, in order to achieve maximum closed-loop damping. Assuming for the moment that actuator dynamics can be ignored (we shall show shortly that this is possible by appropriate arrangement) and n_A tuning filters are available to control n_A modes, then maximal damping effect can be realized if we set the peak corresponding to each of these tuning filters to lie right above the natural frequency of the corresponding controlled modes. In general, less than n_A (n_r say) sets of tuning filters can be used to control n_A modes. With the inclusion of these filters, however the complexity of the overall system is increased and symmetry is not preserved.

Multivariate Stability Criterion for PPF

The general collocated local control implementation of Positive Position Feedback can be written in matrix form as:

$$\begin{aligned} &\ddot{\xi} + B\,\dot{\xi} + \Omega\xi = a_1 C^T G\eta = 0 \\ &\ddot{\eta} + B_f\dot{\eta} + \Omega_f\eta = a_2\Omega_f C\xi \end{aligned} \tag{3.8}$$

where:

- G – gain diagonal matrix $n_f \times n_f$,
- C – participation matrix $n_f \times n_p$,
- Ω_f – spectral diagonal matrix of filters $n_f \times n_f$,
- Ω – spectral diagonal matrix of systems $n_p \times n_p$,
- B_f – modal damping matrix of filters $n_f \times n_f$,
- B – damping matrix of the system $n_p \times n_p$.

Theorem 1

System (3.8) is asymptotically stable if $\Omega - a_1a_2C^TGC > 0$ i.e. positive definite.

The stability criterion does not depend on the inherent damping in the structure.

Stability by the inclusion of actuator dynamics

The overall system-filter-actuator dynamics can be expressed as:

System

$$M\ddot{y} + B\dot{y} + Ky = L^T Gx \tag{3.9}$$

Filters

$$I_{n_A}\left(\ddot{z}_i + 2\zeta_{f_i}\Omega_{f_i}\dot{z}_i + \Omega^2_{f_i} z_i\right) = \Omega_{f_i} G^{1/2}_{f_i} Ly \qquad i = 1, 2, \dots, n_f \tag{3.10}$$

Actuators

$$\ddot{x} + 2\zeta_a\Omega_a\dot{x} + \Omega_a^2 x = = \sum_{i=1}^{n_r} \Omega_{f_i} G^{T/2}_{f_i}\left(\ddot{z}_i + 2\zeta_a\Omega_a\dot{z}_i + \Omega_a^2 z_i\right) \tag{3.11}$$

For $\zeta_a\Omega_a$ sufficiently large and initial conditions sufficiently small, x converges rapidly to

$$\sum_{i=1}^{n_r} \Omega_i G^{T/2}_{f_i} z_i \tag{3.12}$$

and consequently, actuator dynamics fall out of the case completely. The overall system than reduces to the symmetrical form

$$M\ddot{y} + B\dot{y} + Ky = L^T \sum_{i=1}^{n_r} \Omega_i G^{T/2}_{f_i} z_i \tag{3.13}$$

$$I_{n_A}\left(\ddot{z} + 2\zeta_{f_i}\Omega_{f_i}\dot{z}_i + \Omega^2_{f_i} z_i\right) = \Omega_{f_i} G^{1/2} Ly \qquad i = 1, 2, \dots, n_f \tag{3.14}$$

Theorem 2

The overall system in (3.13) and (3.14) is stable if and only if the block matrix W is positive definite.

$$W = \begin{bmatrix} \Lambda & -H_1^T\Omega_{f_1} & . & . & . & -H_{n_r}^T\Omega_{f_{n_r}} \\ -H_1^T\Omega_{f_1} & \Omega^2_{f_1} I_{n_A} & & & & 0 \\ 0 & 0 & . & & & \\ & & & . & & \\ & & & & . & \\ -H_{n_r}^T\Omega_{f_{n_r}} & 0 & & & & \Omega^2_{n_r} I_{n_A} \end{bmatrix} \tag{3.15}$$

$$H_i = G_i^{1/2} L\Phi \tag{3.16}$$

$$W_i = H_i^{T/2} H_i^{1/2} = \Phi^T L^T G_i L\Phi \tag{3.17}$$

There remains the question of designing the feedback gain matrices G_{f_i} for the filters to achieve the desired closed-loop damping for the controlled modes. A specified level of damping ratio ζ_p is prescribed, for the first mode, and the control is such that all other controlled modes achieve approximately the same closed-loop damping as the first mode. This has the effect of causing all the controlled modes to decay at approximately the same rate. The necessary and sufficient condition in Theorem 2 is checked to ensure stability, and the scalar gains γ_i (the diagonal elements of their corresponding modal gain matrices) are also computed to indicate the relative stability margin. The filter parameters (i.e. Ω_f and ζ_f) are computed *a priori* such that only minimal gains are required to achieve the required damping. When a full complement of filters is available (i.e. $n_A = n_f$) closed-loop damping performance for the three cases can be ζ_p = 0.3; 0.4; 0.5. We can achieve closed-loop damping as high as ζ_p= 50% for the first mode without causing instability or other undesirable situations. The closed-loop damping ratio or the first-mode turns out to be within 6% of the actual prescribed value. This error is mainly attributed to the presence of off-diagonal coupling terms. As opposed to the result obtained using velocity feedback, the uncontrolled modes all result in higher than natural closed-loop damping.

If fewer filters than controlled modes are used, the technique still applies, thought the performance deteriorates.In practice we would assign one filter to tune each of the lower-order modes and share those remaining among the higher-order modes, as the lower modes are more critical. Note that position feedback is also much less sensitive to the uncertain natural damping of the structure than velocity feedback.

4. DESIGN OF THE EXPERIMENT

In this section we demonstrate the designing of scalar PPF control on experimental example of vibration suppression, where prismatic steel one side clamped beam with cross-section area of 40 × 2 mm is used as controlled structure. One 100 mm long piezopatch, product QP45W of Mide Technology Corporation, is attached on the surface of the beam and serves as self-sensing actuator.

Uniform Beam Test Structure

For purposes of validating the feasibility of PZT active-member control of a test structure consisting of a thin steel clamped cantilever beam was constructed. One PZT form the actuator which was glued on the top side of the clamp of the beam (because of the maximum of the stress energy at that location) to simulate the moment-producing effect of active part on a beam (Fig. 1). The PZT actuator with their poling geometries arranged such that a common voltage causes one to expand or contract, producing a bending moment on the cross section. The beam was clamped in a support fixture on table. The free end of the beam was driven by a permanent magnet shaker. This can produce a single frequency sine wave as an input to measure dynamic response and closed loop performance.

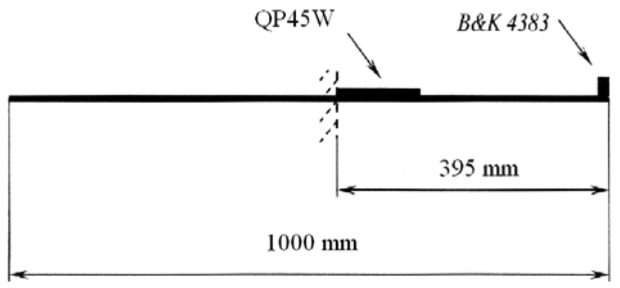

Fig. 1. Experimental one side clamped steel beam with self-sensing piezoactuator and acceleration sensor

Piezoceramics Actuators and Sensors

PZT material is an inherent electro-mechanical transducer. The electro-mechanical action of the sensor and actuator PZT thin sheet material used in the experiments (type QP 45W). The material is poled across the thickness, but the induced strain occurs along the length. The relationship between the applied electric field and the induced free strain for the actuator is given by

$$\varepsilon_3 = d_{31} E_f \tag{4.1}$$

where d_{31} is the PZT strain constant, or the transverse charge coefficient. The relationship between the applied mechanical stress and the induced electric field is given by

$$E_f = -g_{31}\sigma_3 \tag{4.2}$$

where g_{31} is the PZT voltage constant, or transverse voltage coefficient.

A mechanics analysis results in the following equation for the applied moment from the actuators

$$M_{ya} = F_{xa} z_a = K_{ya} u_a \Rightarrow K_{ya} = -\frac{E_a b_a d_{31} z_a}{1-v} \tag{4.3}$$

where:

M_{ya} – applied moment which is proportional to PZT transverse charge coefficient d_{31},
E_a – Young's modulus of the PZT material,
F_{xa} – force which originates from the voltage between electrodes,
z_a – distance from neutral axis,
K_{ya} – constant of the actuator,
v – Poisson constant.

This equation does not account for the compliance of the adhesive use to attach PZT which results in a slightly lower effective moment. The actuator is assumed to apply a constant magnitude moment across the composite beam cross section everywhere along the length of the actuator.

The sensor ceramics respond to the applied stress due to bending strain along their length. Assuming small strains, the sensor voltage can be approximated by

$$u_s = -\frac{K_{ya}}{C_p}(\varphi_y(x_2) - \varphi_y(x_1)) \tag{4.4}$$

where:

u_s – sensing voltage,
C_p – capacitance of PZT measured for constant strain,
φ_y – the slope of the beam about axis y.

The capacitance of the glued PZT can be given by

$$C_p = \left(\varepsilon_3^\sigma - \frac{2d_{31}^2 E_a}{1-v}\right)\frac{A}{t} \tag{4.5}$$

where:

ε_3^σ – PZT permitivity for constant mechanical stress,
A – cross section of electrode,
t – distance between electrodes.

In our case a self-sensing actuator was used, which makes both sensing and actuating.

Partial Differential Equation of Motion

Figure 1 shows the geometry of a cantilever beam acted on by PZT actuator spanning from station x_1 to station x_2. The actuator applies a constant moment of magnitude M along its length. The governing partial differential equation is given by

$$m(x)\frac{\partial^2 y}{\partial x^2} = -\left(\frac{\partial^2}{\partial x^2}\left(E(x)I(x)\frac{\partial^2 y}{\partial x^2}\right)\right) \tag{4.6}$$

By using of standard modal expansion we receive modal equations, and by multiplying modal equations by the mode shape, and integrating over the domain and making use of the fact th $\Phi_i(x)$ at the actuators do not span to the ends of the beam and after some manipulations follows

$$\ddot{\xi}_i(t) + \Omega_i^2 \xi_i(t) = a_i V_a [\Phi_i(x_2) - \Phi_i(x_1)] \tag{4.7}$$

We see that the actuators couple into the modes through the difference in the slopes of the mass-normalized mode shapes at the ends of the actuators.

5. EXPERIMENT AND SIMULATION RESULTS

The one Mode control experiments will be discussed.

The open loop and closed loop free decay of Mode 1 is shown in Figures 2, 3 and 4. The settling time has been reduced from about 20 seconds to about two seconds. It is a substantial reduction.

The result of application of PPF control method for suppression of the steady-state vibration is on Figure 5. The clamped beam has been excited at the free end by magnetic shaker up to the first resonance (at about 10 Hz). From Figure 5 we can see, that suppression of vibration has very good efficiency.

The resulting frequency response functions for the region of Modes 1 and 2 are shown in Figures 6 and 7. The line of the frequency function on Figure 7 indicates that, that the damping of Mode 1 has been increased (a wider half-power bandwidth) and the peak amplitude has been decreased from the open loop peak of 25 dB to about 18 dB of closed one.

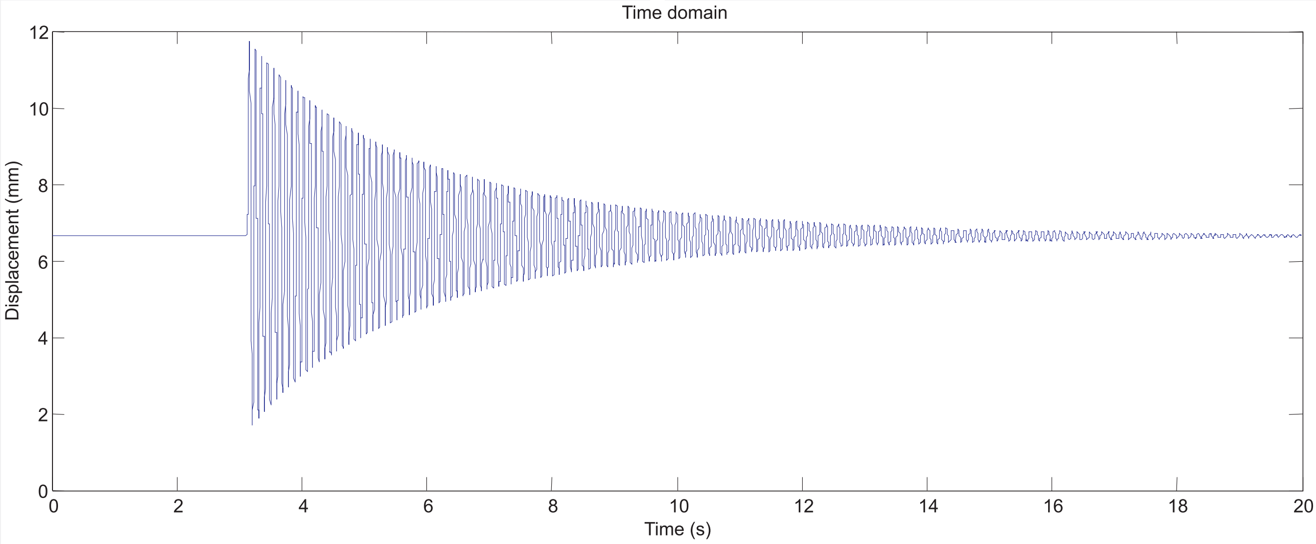

Fig. 2. Open loop free decay of Mode 1 – measured

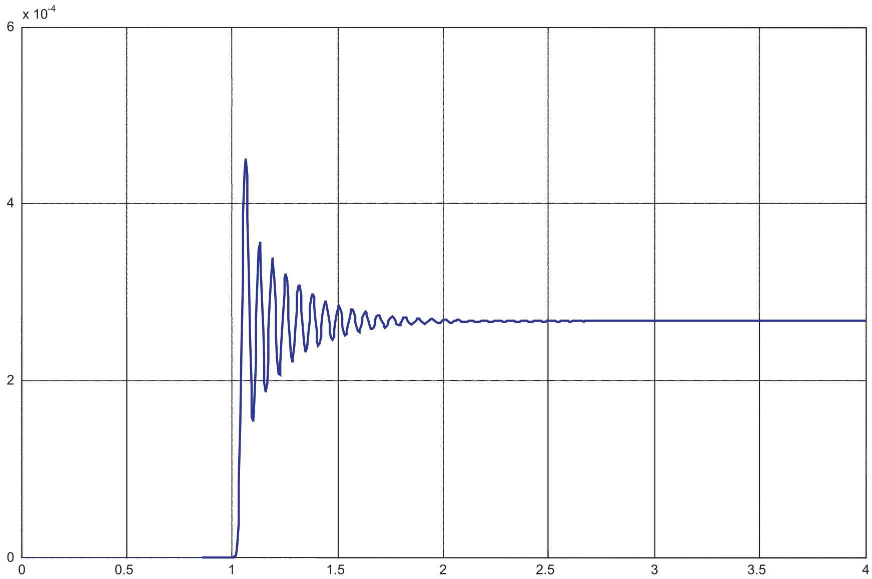

Fig. 3. Simulated closed loop free decay of Mode 1

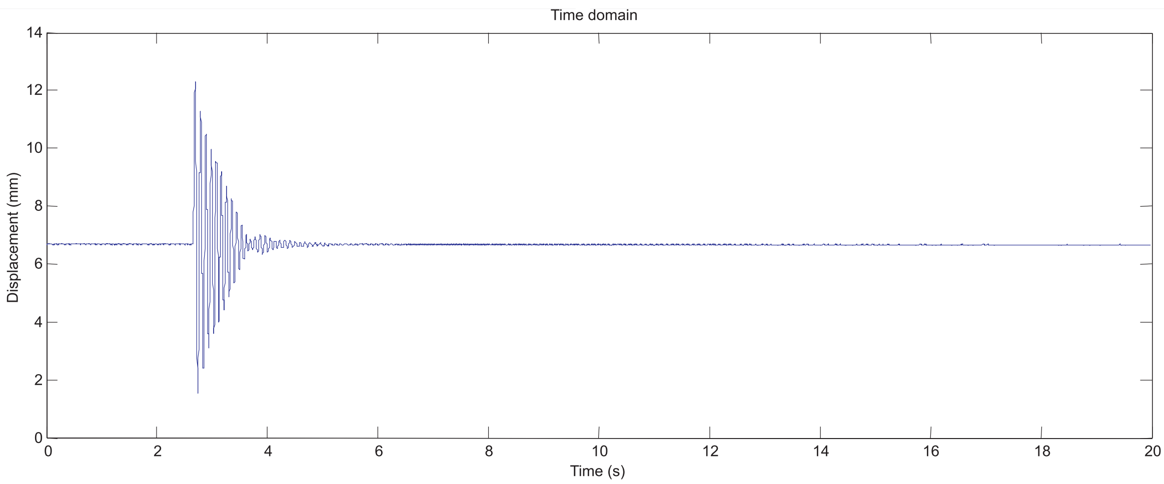

Fig. 4. Closed loop free decay of Mode 1 – measured

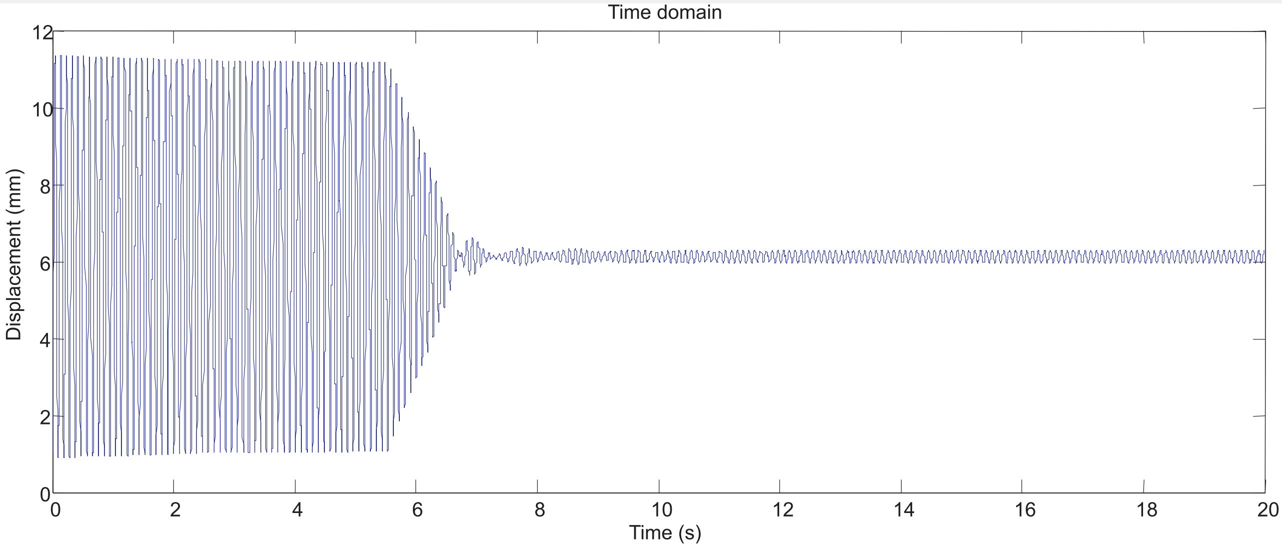

Fig. 5. Suppression of vibration of the forced vibration in resonance – measured

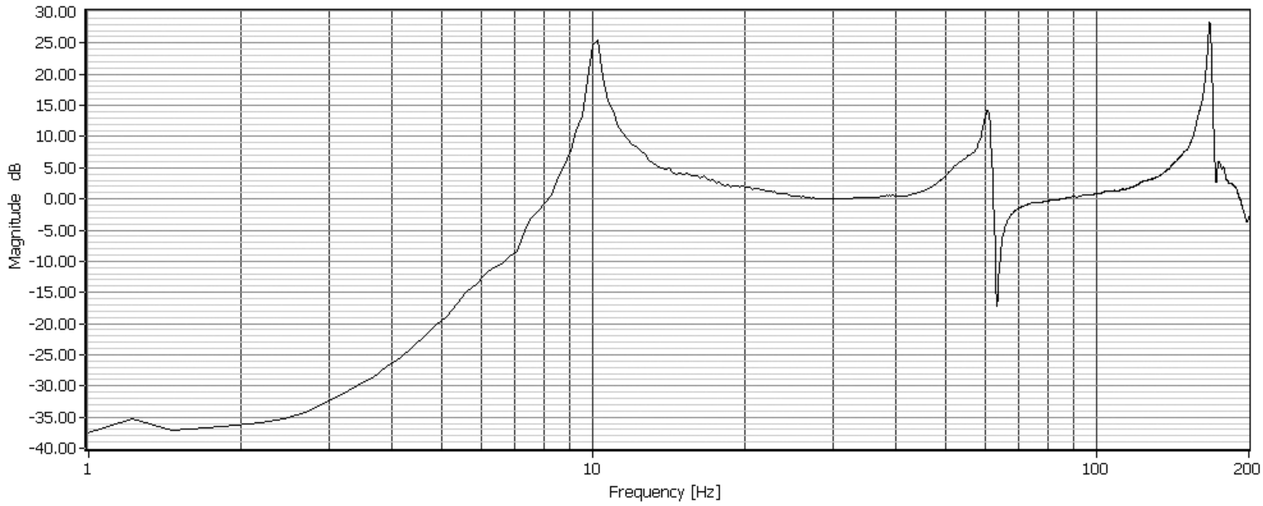

Fig. 6. Frequency response function for Mode 1 without control

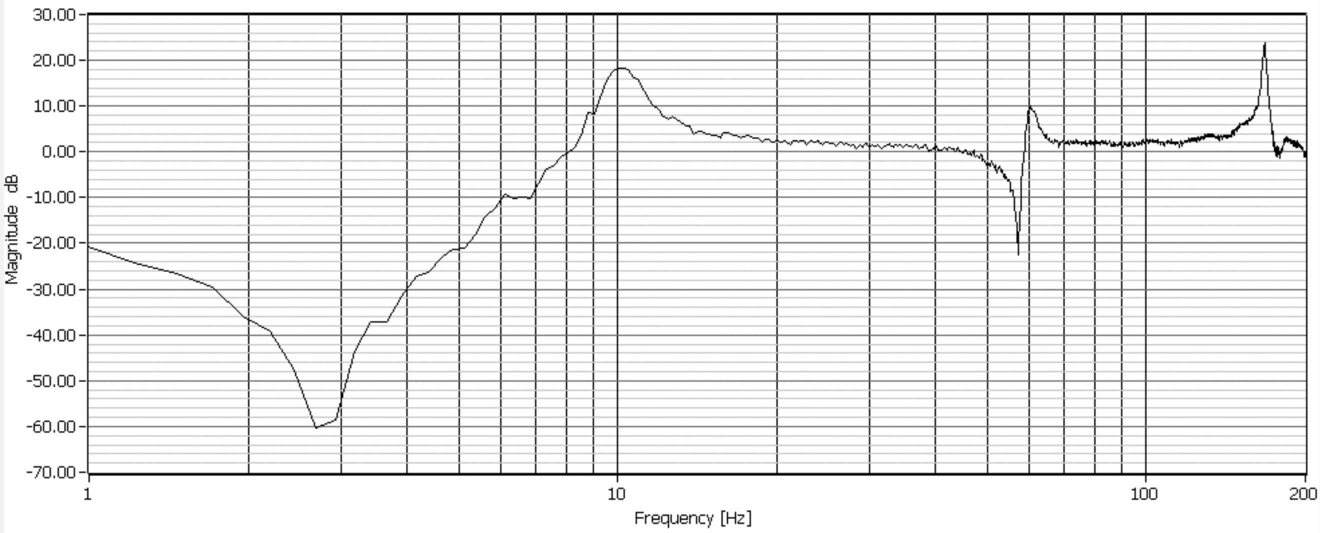

Fig. 7. Frequency response function for Mode 1 with control – measured

The solid line shows a reduction in response amplitude well below the open loop peak response as the damping from the Mode 1.

6. CONCLUSIONS

Table 1 summarizes the two mode control performance.

Table 1
Experimental natural frequencies and damping ratios of the clamped beam

Mode	Open loop values		Closed loop values	
	Natural frequencies [Hz]	Damping ratios [%]	Natural frequencies [Hz]	Damping ratios [%]
1	9.82	4.26	10.09	13.57
2	57.29	2.98	61.5	5.98

The open loop and closed loop free decay of Mode 1 is shown in Figures 2, 3 and 4. The settling time has been reduced from about 20 seconds to about two seconds, a substantial reduction.

A simple, necessary and sufficient stability criterion has been given for the case of local control PPF. This criterion is nondynamic, which accounts for the superior robust stability of the technique. The damping of Mode 1 has been increased from 4.26% to 13.57% (a wider half-power bandwidth). The frequency response function for the first two modes is shown in Figures 6 and 7.

From Figure 5 follows, that suppression of steady-state vibration, under resonance condition (magnetic shaker) indicates, that piezoceramics can be used for solving vibration suppression problem.

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