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A NEW VIBROACOUSTIC METHOD FOR SHAFT CRACK DETECTION

SUMMARY

Various excitation techniques have been developed to extract information on dynamic state of rotating machinery. This paper will address the effectiveness of selected excitations and analysed signals towards health monitoring of rotating machines from the shaft crack point of view. The computer simulation study is based on the uncracked and cracked Jeffcott rotor models and the efficiency of the selected approaches is examined. As the excitation we have considered the rotor unbalance, additional harmonic forces, and input conditions. The combined resonances as diagnostic indicators of the cracked shaft are good seen in the total frequency spectrum in the case when we consider the difference between the spectra of cracked and uncracked shaft. We have also introduced a new model of the rotor which allows us to use different method of signal processing for the crack detection and the evaluation of its deep. So more, in such model we can take into account the number of excitation planes and directions of the applied forces, the choice of the analysed signals (e.g. transient, steady-state).

Keywords: *health monitoring, crack, excitation techniques, Jeffcott rotor*

NOWA WIBROAKUSTYCZNA METODA DO WYKRYWANIA PĘKNIĘĆ WAŁU

Pękanie wału prowadzi do asymetrii jego sztywności. Według modelu pękania Mayesa sztywność zmienia się w sposób kosinusoidalny. Tego typu wymuszenie parametryczne prowadzi do kombinowanych rezonansów. Większość autorów bada zachowanie się dynamiczne wirnika w otoczeniu takiego rezonansu. W niniejszym artykule do badań diagnostycznych przyjęto model Jeffcotta dynamiki wirnika. Wyprowadzono równania ruchu uszkodzonego i nieuszkodzonego wału. Analizę drgań tego nieliniowego modelu przeprowadzono z wykorzystaniem metody małego parametru. Zaproponowano podejście transmitancyjne do modelowania uszkodzenia. Przeanalizowano możliwość pobudzania drgań maszyny z wykorzystywaniem różnych typów sygnałów. Z wielu możliwych rozwiązań prowadzących do uzyskania informacji diagnostycznej wybrano do dalszych badań model reprezentowany przez funkcję odpowiedzi częstotliwościowej. Różnica pomiędzy dynamiką uszkodzonego i nieuszkodzonego wirnika została użyta do uwypuklenia pików rezonansów kombinowanych w widmie drganiowym. Na bazie tych wymuszeń określono symptomy, z wykorzystaniem których można budować wskaźniki diagnostyczne. Rozważania analityczne poparto badaniami symulacyjnymi.

Słowa kluczowe: *pękanie, wirnik Jeffcotta, monitorowanie stanu technicznego, techniki wzbudzenia*

NOMENCLATURE

- $\mathbf{A}_{s0}$ – amplitude of transient vibrations
- $\mathbf{A}_{un0}$ – amplitude of unbalance forced vibrations
- $\mathbf{A}_{ex0}$ – amplitude of externally forced vibrations
- $\mathbf{A}_0$ – extended state matrix
- $\mathbf{A}$ – state matrix
- $\mathbf{B}$ – matrix of placement of the exciters
- $\mathbf{C}$ – damping matrix
- e – eccentricity of the rotor mass centre
- F_{ex} – external force
- F_{un} – unbalance force
- F_g – gravity force
- F_x – excitation force in direction of axis x
- $\mathbf{K}$ – stiffness matrix
- $\mathbf{K}_o$ – stiffness matrix of anisotropic rotor
- $\widetilde{\mathbf{K}}$ – stiffness of cracked shaft
- ΔK – shaft stiffness matrix in inertial coordinates
- k_o – stiffness coefficient of uncracked symmetric shaft
- k_α, k_β – stiffness of the cracked shaft with open crak
- m – mass of rotor
- $\mathbf{M}$ – mass matrix
- $\mathbf{q}$ – vector of transient vibrations
- $\mathbf{q}_{st}$ – steady state rotor displacement
- $\mathbf{q}_{ex}$ – vector of forced vibrations
- $\mathbf{T}$ – transformation matrix
- $\mathbf{u}(t)$ – time variables of the exciting forces
- u_x, u_y – additional variables
- x – displacement of mass centre in direction of axis x
- $\mathbf{x}_{res}(t)$ – residuum of the power series
- $\mathbf{x}(t)$ – state vector
- y – displacement of mass centre in direction of axis y
- ε – small parameter
- ξ – damping coefficient
- ϕ – angle between the crack and the rotor response
- ω_n – natural frequency
- ω_s – resonance frequency
- Ω – rotor angular speed

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1. INTRODUCTION

Vibroacustical signals of rotating machinery are used for the identification and diagnostics purposes. Such signals can be generated by phenomena connected with operational state of the machinery or can be excited by additional devices added to the machinery. The devices are typically the force generators. They should be able to generate forces in desired frequencies range called a frequency bandpass. Presently there are a few such devices like electrohydraulic or electromagnetic exciters, magnetic bearings, or piezo-electric actuators with sufficient bandpass for diagnostics or identification procedures.

The rotor in rotating machinery during its operations regime usually has a high kinetic energy so its break can result in a catastrophic failure and be dangerous for the environment. The failure starts from the shaft partial crack and crack increases during machinery life time. Detection of the crack in the early stages may save the rotor from irreparable damage. By monitoring of the type and severity of the crack it would be possible to extend the useful life of the cracked rotor without risking the catastrophic failure [1]. Many authors have looked for symptoms for detection of earlier stage of the shaft crack. All models of crack leads to nonlinear description of rotor dynamics because of asymmetric rigidity of the rotor [2] and because of the additional parametric excitations of the rotor vibrations [3]. For early stage crack the changes of shaft flexibility during are modeled by cosine functions [4] what lead to the parametric, external and combination resonances [5].

Particularly, combined resonances are good diagnostic indicators in the case of cracked rotor. Unfortunately, they have small amplitudes in comparison to the other resonances and the rotor answers to the natural excitations like unbalance. This paper focuses on the problems related to excitation signals and signal processing. We are looking for means how to intensify combined resonances signals in comparison to the other components of the vibration signals. It is appeared that the difference of the vibration signals of the cracked and uncracked shaft can be a good base for further signal processing to design the early stage detection and evaluation of the rotor crack.

2. MODELING OF EXTERNAL EXCITATION

The excitation signals generated by external devices can have different form. We can generate sinusoidal signals, chirp, impulse, pulse, step, or pseudo-white noise signals. The internal (operational) excitations usually have the form of periodic (sum of sinusoidal) excitations. So, we can gather the signals into two groups: trigonometric ones and pulse (train of pulses) ones. First group is usually used for steady state analysis while pulses are used for transient response analysis. According to our opinion the transient response is much richer in the useful information about systems than steady state response. It is confirmed by Kalman theorem about observability and controllability of the systems. To correctly excite the mechanical system we should avoid the location of exciters in the planes with the nodal points of system modes. The trigonometric signals we can join to the model of the system as it is shown below.

Consider mathematical model of simple unbalanced rotor:

$$\begin{aligned} &\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x = e\Omega^2\cos\Omega t+\frac{F_x(t)}{m} \\ &\ddot{y}+2\xi\omega_n\dot{y}+\omega_n^2 y = e\Omega^2\sin\Omega t \end{aligned} \tag{1}$$

with zero initial conditions excited by unbalance forces and by additional exciter generating force F_x only in the direction x. Such model can also be presented in the more homogeneous form:

$$\begin{aligned} &\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x-e\Omega^2 u_x = \frac{F_x(t)}{m} \\ &\ddot{y}+2\xi\omega_n\dot{y}+\omega_n^2 y-e\Omega^2 u_y = 0 \\ &\ddot{u}_x+\Omega^2 u_x = 0 \\ &\ddot{y}_x+\Omega^2 u_y = 0 \end{aligned} \tag{2}$$

with the following nonzero initial conditions: $u_x(0) = 1$, $\dot{u}_y(0)=\Omega$. The solution of the above equations in the state space representation can be found in the following form

$$\mathbf{x}_0(t)=e^{A_0 t}\mathbf{x}_0(0)+\int_0^t e^{A_0(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda \tag{3}$$

where we have:

$$\mathbf{A}_0=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\omega_n^2 & -2\xi\omega_n & 0 & 0 & e\Omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & e\Omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Omega^2 & 0 \end{bmatrix},$$

$$\mathbf{x}_0(t)=\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ u_x \\ \dot{u}_x \\ u_y \\ \dot{u}_y \end{bmatrix}, \qquad \mathbf{x}_0(0)=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \Omega \end{bmatrix} \tag{4}$$

$$\mathbf{Bu}(t)=[\,0 \quad F_x(t)/m \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,] \tag{4a}$$

Truly speaking, in the digital simulations two last equations from (2) should be one step ahead calculated to use its results in two first equations (2). In the case of linear time-invariant system the above approach is fully admissible, because it causes only the change of the initial conditions. For time depended systems (parametric excited systems) the above approach is also possible in the case of very short sampling time in comparison to the system time constants.

If we introduce the external excitation we write the diagnostic model in the following way:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad (5)$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

where for the single measurement of rotor displacement in x direction we have: $\mathbf{C} = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$. In the matrix **B** we take into account the placement of the exciters and elements and their other constant parameters while the vector u contains the time variables of the exciting forces. In the matrix **C** and **D** we take into account the placement of sensors and their other constant parameters.

3. MODEL OF THE CRACKED ROTOR DYNAMICS

We will take into account the Mayes model [6] of the cracked shaft. Jeffcott model with small crack have the form [3]

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{T}^T \widetilde{\mathbf{K}}(\phi, t)\mathbf{T}(\mathbf{q} + \mathbf{q}_{\mathbf{st}}) = \mathbf{F}_g + \mathbf{F}_{un} + \mathbf{F}_{ex} \qquad (6)$$

where:

$$\widetilde{\mathbf{K}}(\phi, t) = \begin{bmatrix} 0.5\,(k_0 + k_\alpha) + 0.5\,(k_0 - k_\alpha)\cos\phi & 0 \\ 0 & 0.5\,(k_0 + k_\beta) + 0.5\,(k_0 - k_\beta)\cos\phi \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix},$$

and **M**, **C**, $\mathbf{F}_g$, $\mathbf{F}_{un}$, $\mathbf{F}_{ex}$ are matrices of mass, damping, gravity force, unbalance force and external force, respectively. Above stiffness matrix **K** is designed in rotating coordinate system $0\alpha\beta$ and k_0 is stiffness coefficient of uncracked symmetric shaft, while k_α, k_β are stiffness of the cracked shaft with open crack, $\phi = \Omega t + \phi_0$ is the angle between the crack and the rotor response. After some calculations we obtain simple model in inertial coordinate systems

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + [(\mathbf{K}_0 - \Delta\mathbf{K}(t)](\mathbf{q} + \mathbf{q}_{\mathbf{st}}) = \mathbf{F}_g + \mathbf{F}_{un} + \mathbf{F}_{ex} \qquad (7)$$

or

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + [\mathbf{K}_0 - \Delta\mathbf{K}(t)]\mathbf{q} = \Delta\mathbf{K}(t)\mathbf{q}_{\mathbf{st}} + \mathbf{F}_{un} + \mathbf{F}_{ex} \qquad (8)$$

with:

$$\mathbf{K}_0 = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix}, \quad \Delta\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix},$$

where:

$$k_{11} = 0.5(k_{\Delta\alpha} + k_{\Delta\beta}) + 0.5(k_{\Delta\alpha} - k_{\Delta\beta})\cos 2\phi - 0.25[(3k_{\Delta\alpha} + k_{\Delta\beta})\cos\phi + (k_{\Delta\alpha} - k_{\Delta\beta})\cos 3\phi),$$

$$k_{22} = 0.5(k_{\Delta\alpha} + k_{\Delta\beta}) + 0.5(k_{\Delta\beta} - k_{\Delta\alpha})\cos 2\phi - 0.25[(3k_{\Delta\alpha} + k_{\Delta\beta})\cos\phi + (k_{\Delta\beta} - k_{\Delta\alpha})\cos 3\phi],$$

$$k_{12} = k_{21} = 0.25(k_\alpha + k_\beta)[\sin 2\phi - 0.5\,(\sin\phi + \sin 3\phi)],$$

$$k_{\Delta\alpha} = 0.5(k_0 - k_\alpha), \quad k_{\Delta\beta} = 0.5(k_0 - k_\beta).$$

4. SOLUTIONS OF EQUATIONS

To calculate the answer of above system we will use the method of small parameter ε. We join ε to the $\Delta\mathbf{K}$ and assume the following solution

$$\mathbf{q}(t) = \mathbf{q}_0(t) + \varepsilon\mathbf{q}_1(t) + \varepsilon^2\mathbf{q}_2(t) + \ldots \qquad (9)$$

and we introduce it to the nonlinear equations (8). We collect the elements of equations containing the small parameter ε in the same power into separate sets of equations:

$$\mathbf{M}\ddot{\mathbf{q}}_0 + \mathbf{C}\dot{\mathbf{q}}_0 + \mathbf{K}_0\mathbf{q}_0 = \mathbf{F}_{un} + \mathbf{F}_{ex} \qquad (10a)$$

$$\mathbf{M}\ddot{\mathbf{q}}_1 + \mathbf{C}\dot{\mathbf{q}}_1 + \mathbf{K}_0\mathbf{q}_1 = \Delta\mathbf{K}(t)\mathbf{q}_0 + \Delta\mathbf{K}(t)\mathbf{q}_{st} + \mathbf{F}_{un} + \mathbf{F}_{ex} \qquad (10b)$$

....

Let us try to find the solution of two first sets of equations. We see that the first set (10a) is equivalent to the equations (1) so its solution can be presented in the form of expression (3) or in more traditional way

$$\mathbf{q}_0(t) = e^{-\zeta\omega_n t}\mathbf{A}_0 \sin(\omega_s t + \varphi_{s0}) + \mathbf{A}_{un0}\sin(\Omega t + \varphi_{un0}) + \mathbf{A}_{ex0}\sin(\Omega t + \varphi_{ex0}) + \mathbf{q}_{ex}(t) \qquad (11)$$

For $\phi = \Omega t$ we have: $\Delta\mathbf{K}(t) = \sum_{m=0}^{3} \Delta\mathbf{K}_m \cos(m\Omega t + \varphi_m)$.

Since $\cos\alpha\sin\beta = 0.5\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$, and so on, one can use the trigonometric functions to find the excitation in the equation (10b)

$$\Delta\mathbf{K}(t)\mathbf{q}(t) = e^{-\xi\omega_n t}\sum_{n=0}^{3} A_{1n}\cos(|n\Omega \pm \omega_s| t + \varphi_n) + \sum_{m=0}^{2} A_{2m}\cos(m\Omega t + \varphi_m) + \mathbf{F}_3(t) \qquad (12)$$

Many researchers [3, 5] have indicated that only the excitations with frequencies $|m\Omega \pm \omega_s|$ give enough strong resonances which can be a base to diagnose the crack process.

In similar way to the equation (2) we join all excitations of equation (10b) (except excitation generated by external exciter) to the homogenous part of the state space model what lead to the state matrix $\mathbf{A}_1$. For example the excitation member $e^{-\xi\omega_n t}A_{1m}\cos(|m\Omega \pm \omega_s| t + \varphi_m)$ has the following representation (submatrix) in the matrix $\mathbf{A}_1$:

$$\begin{bmatrix} 0 & 1 \\ -|m\Omega \pm \omega_s|^2 & -\xi\omega_n \end{bmatrix}, \quad \omega_s = \omega_n\sqrt{1-\xi^2} \qquad (13)$$

In the state space the solution of equation (10b) has the form

$$\mathbf{x}_1(t) = e^{\mathbf{A}_1 t}\mathbf{x}_1(0) + \int_0^t e^{\mathbf{A}_1(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda \tag{14}$$

This way the second order final solution of nonlinear system is

$$\begin{aligned}\mathbf{x}(t) &= e^{\mathbf{A}_0 t}\mathbf{x}_0(0) + \int_0^t e^{\mathbf{A}_0(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda + \\ &+ \varepsilon\left(e^{\mathbf{A}_1 t}\mathbf{x}_1(0) + \int_0^t e^{\mathbf{A}_1(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda\right) + \mathbf{x}_{res}(t)\end{aligned} \tag{15}$$

where $\mathbf{x}_{res}(t)$ is a residuum of the power series containing the higher elements of the series, and small parameter $\varepsilon = 1$. Above solution provides many possibilities to extract information about the crack. In the paper we will analyze above solution of rotor vibration with external and without external excitation.

Now we will use the Laplace transformation to find the input-output relations (steady-state solution) in the case of SISO system (system with one exciter and one sensor)

$$\begin{aligned}y(s) &= \mathbf{C}_0(\mathbf{sI} - \mathbf{A}_0)^{-1}\mathbf{B}_0 u(s) + \\ &+ \varepsilon\mathbf{C}_1(\mathbf{sI} - \mathbf{A}_1)^{-1}\mathbf{B}_1 u(s) + \ldots\end{aligned} \tag{16}$$

The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, … have different dimensions. Therefore, the matrices $\mathbf{C}_0$, $\mathbf{C}_1$, …, and $\mathbf{B}_0$, $\mathbf{B}_1$, … have also different dimensions, but the difference is only in the number of null elements. We should underline the excitation in our model is generated only by external exciter while parametric excitations, unbalance weight and so on are inside the state matrices. Such approach is presented in Figure 1.

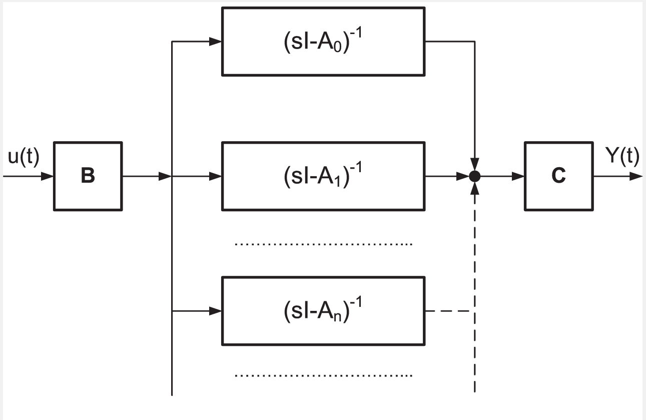

Fig. 1. Input-output model of the cracked rotor

5. DIAGNOSTIC METHOD

Models (15) and (16) give many possibilities to design the diagnostic indicator which on the early stage indicate the shaft crack and its depth. We can notice that widely exploited sinusoidal excitation $u(t) = A_u \cos(|m\Omega \pm \omega_s| t)$ (which excites one of the combined resonances) is only one of all possible excitations. For example we should take into account the following possibilities.

– SISO or MIMO systems. In the case of MIMO system we should use more than single sensor and/or exciter.
– Diagnostic system based on transient or steady state signals. In the case of transient signals we should carefully chose the initial conditions of considered signals.
– Signals can be analyzed in time or frequency domain.
– We can use different methods for signal processing.

It is evident that the second term in (15) is a residuum which can be used to the identification of the crack because it is approximately the difference between actual vibration measurement and dynamics of uncracked shaft

$$\begin{aligned}& e^{\mathbf{A}_1 t}\mathbf{x}_1(0) + \int_0^t e^{\mathbf{A}_1(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda \cong \\ &\cong \mathbf{x}(t) - e^{\mathbf{A}_0 t}\mathbf{x}_0(0) - \int_0^t e^{\mathbf{A}_0(t-\lambda)}\mathbf{Bu}(\lambda)d\lambda\end{aligned} \tag{17}$$

We can also define the residuum method by using of the steady state input-output relations for SISO system

$$y_1(s) = G_1(s)u(s) \cong y(s) - G_0(s)u(s) \tag{18}$$

where:

$$\begin{aligned}G_0(s) &= \mathbf{C}_0(\mathbf{sI} - \mathbf{A}_0)^{-1}\mathbf{B}_0 \\ G_1(s) &= \mathbf{C}_0(\mathbf{sI} - \mathbf{A}_1)^{-1}\mathbf{B}_1\end{aligned} \tag{19}$$

In the equation (17) there is a difference between solutions in the time domain. In such case we should properly choose the initial values of the signals what is rather difficult in the experimental approach. To avoid such problem we can use frequency domain as it is shown in equation (18). In the case of transient signals we can replace the time signals their autocorrelation functions. Autocorrelation also introduces the averaging operation and autocorrelation function starts from its maximal value.

6. COMPUTER SIMULATION

Before crack the rotor has isotropic parameters. The resonance has frequency 40 Hz and angular speed is 36 Hz. We assume 3% external damping and e = 0.1 mm mass displacement from geometric centre. It means we have used parameters from paper [7].

After 40% crack the rotor stiffness in crack direction is reduced by 9%, while in the perpendicular direction it is reduced by 4%.

We will start the computer simulations from the frequency solutions of the equations (1) and (2). The solutions of these equations are identical for the same parameters and are presented in Figure 2. It confirms that the all periodic excitations of the rotor resulting from its rotation we can express as the additional states of the model in the state space. Only the excitation generated by additional exciter will be considered as an external excitation denoted by $u(t)$. In the further simulations we assume if $u(s)$ is applied it excites rotor only in the direction of the axis x and has harmonic form. In all plots starting from Figure 2, the horizontal axis presents the frequency values of the spectrum in Hz while vertical axis gives values of the displacement amplitude in meters. Since we applied the FFT to the full solution of the motion equation (which is a sum of transient and steady-state signals) the carried out calculations have only quality character as it is explained in the Conclusions. Such approach is justified since we are looking for solutions in which the combined resonances can be detected in the spectrum solution.

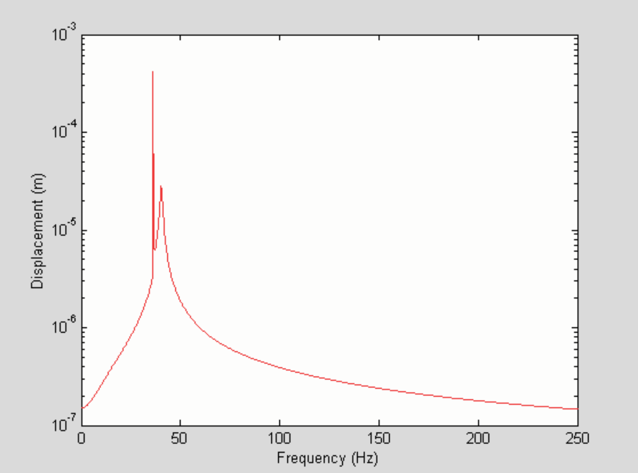

Fig. 2. Frequency solution of equations (1) or (2)

The frequency spectrum of the cracked rotor (crack reached 40% of the shaft radius) is shown in Figure 3. Since we have used FFT with high resolution (4096 samples) we can notice the combined resonances in the frequency spectrum. These resonances are seen for frequencies $(\omega_s - \Omega)$, $(\omega_s + \Omega)$, $(2\Omega - \omega_s)$ and in the vicinity of these frequencies what is connected with anisotropy of the cracked shaft stiffnes.

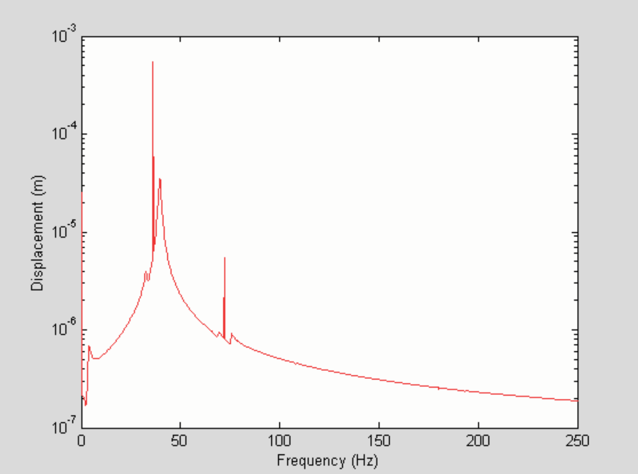

Fig. 3. Frequency spectrum of the cracked shaft without external excitation

Next we have excited the rotating rotor by the external harmonic force with amplitude 1 N and frequency 76 Hz. Such frequency equals the frequency of some combined resonance. The vibration spectrum of the uncracked rotor is shown in Figure 4 while the spectrum of cracked rotor is presented in Figure 5. We can see that vibration amplitude in the case of the cracked shaft increases not so much, so in the face of the measurement noise the crack could be undetected.

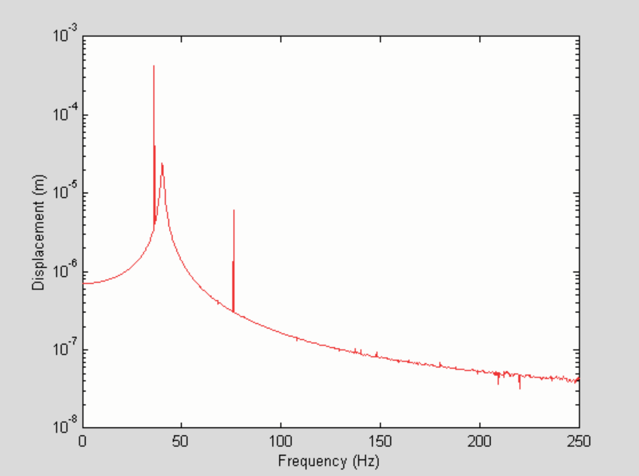

Fig. 4. Frequency spectrum of the unbalanced uncracked rotor vibrations excited with frequency $(\Omega + \omega_s = 76$ Hz)

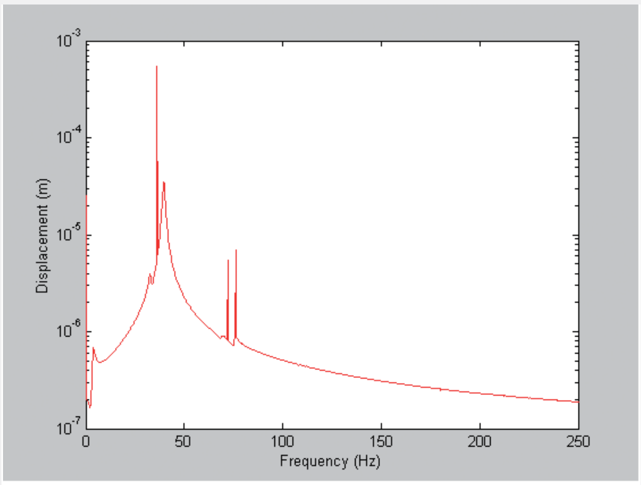

Fig. 5. Cracked rotor externally excited with frequency $(\Omega + \omega_s = 76$ Hz)

The better results are obtained in the case the rotor is excited with frequency 4 Hz (Fig. 6) which is the frequency of the other combined resonance. In this case the vibration amplitude increases over 10 dB. Such big change in the amplitude value is easy to detect. Additional adventure is the low value of exciting frequency. Such low frequency excitation can be generated by different kinds of exciters.

Much more interesting solution was obtained in the case of the above described method based on the investigation of residuum given by formulae (17) and (18). Let us start from the residuum for rotor without external excitation $(u(t) = 0)$.

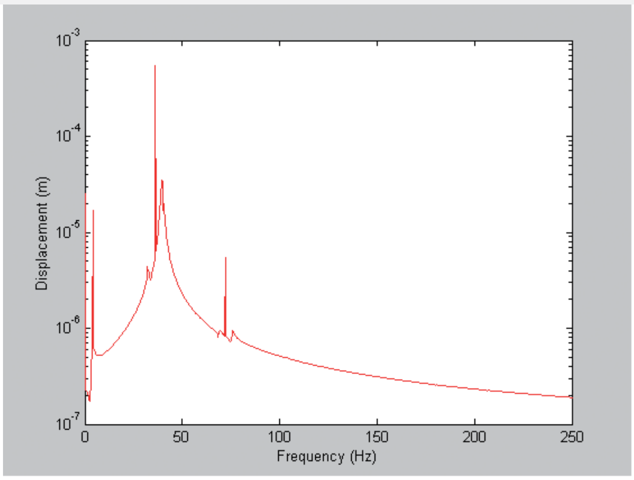

Fig. 6. Cracked rotor externally excited with frequency ($\Omega - \omega_s = 4$ Hz)

In this case the inertial conditions excite the rotor vibrations. The frequency spectrum of the difference between vibrations of cracked and uncracked rotor is shown in Figure 7. In contrary to Figure 3 the combined resonances are presently very well seen in the spectrum plot. Of course, the absolute values of resonance amplitudes have not increased but the amplitudes connected with other internal excitations have decreased as a result of spectrum difference so the relative values of combined resonance amplitudes have increased. Combined resonances in such spectrum are much simpler for the detection and evaluation. Some times it is possible to analyze the difference spectrum of do not externally excited rotor vibration to diagnose the shaft crack. It could be the simple and low cost diagnostic procedure in comparison for example with the method described in [5].

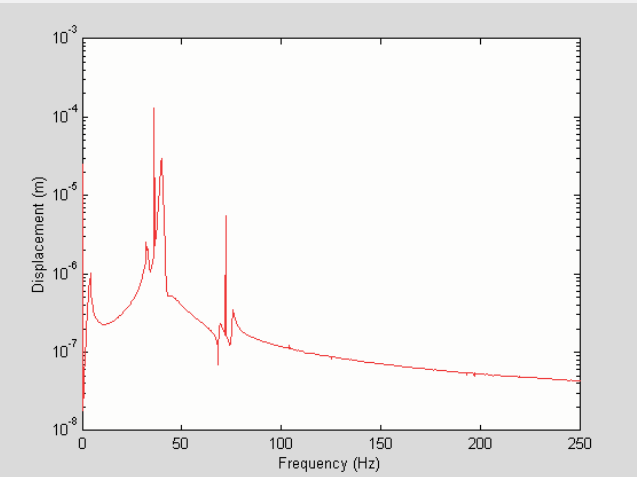

Fig. 7. Residuum spectrum as a difference between spectrum of cracked and uncracked rotor vibrations (according to equation (17))

Now we consider the case when the cracked and uncracked rotor is excited by 1 N force with frequency of combined resonance (4 Hz). The difference spectrum is presented in Figure 8. We can notice the amplitude change not only for this frequency but also for frequencies connected with other combined resonances. So we can design the diagnostic indicator on the combination of these amplitudes.

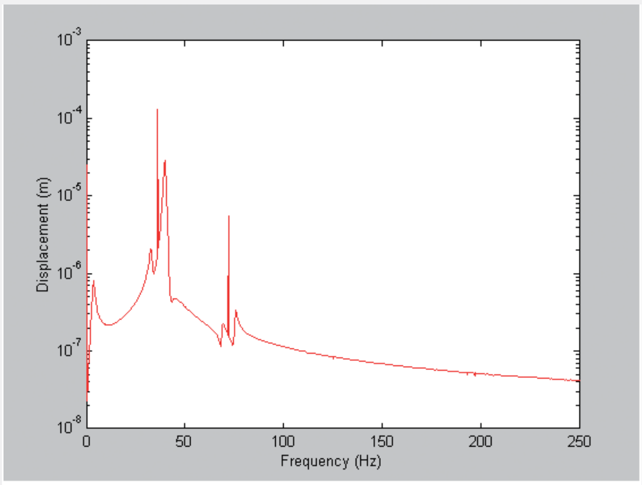

Fig. 8. Spectrum of vibration residuum for external excitation with frequency 4 Hz

7. CONCLUSION

A new vibroacustical method of shaft crack diagnostic is layout in the paper. By the introduction of the difference of the vibration signals of cracked and uncracked shaft we have obtained the relative increase in amplitudes of combined resonances which are closely related with the crack. It may in many cases allow us to eliminate the expensive external excitations or to apply the simple and cheap exciters.

It should be underlined the described investigations in the paper can be considered as a qualitative investigations. For example Figure 2 presents FFT of the transient and steady state process. The rotor vibrations are only excited by the initial conditions $\mathbf{x}_0(0)$. Since the Fourier transfer is the average procedure a big influence on the amplitude values has the speed of the transient process declining, the width of the measurement window, the choose of the collection start point of the measured samples. These factors have influence on the amplitude values but not on the relative relations between the values of the amplitudes. These relations are important for the crack detection realized with use of combined resonances. In all above simulations the starting point of sample collection is in the initial conditions, we have used the same sampling time and the same number of the samples.

In future work the authors will curry out the quantitative investigations to estimate the deep of the crack. As it was mentioned, it is possible to use known Impulse (Markov Parameter) method in the case of transient signals or Frequency Response Function method in the case of steady state signals to identify the model parameters. By identification procedure we can also estimate the advancement of the shaft crack. We will use the difference spectrum in all these procedures. Only difference of vibration signals for cracked and uncracked rotor increases the relative amplitudes of combined resonances in relation to the other spectrum components connected with other excitations.

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