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George Juraj STEIN*, Radúz ZAHORANSKÝ*

ACCOUNTING FOR DRY FRICTION INFLUENCE IN OSCILLATORY SYSTEMS EXPOSED TO RANDOM EXCITATION

SUMMARY

In analysis of translatory oscillatory systems the basis is a simple linear oscillator, whose mathematical treatment is well known. However, an important issue in many practical applications is the friction. This is specifically so for low intensity random kinematic excitation in form of acceleration, as is often the case in ground transportation. In the contribution two approaches to friction modelling are described and compared – the signum function approach and the physically correct stick-slip approach, together with the commonly used harmonic balance method of Den Hartog. The differences are highlighted, indicating that the physically correct stick-slip approach describes the reality better than the computationally simpler signum approach. These effects are dependent on the relation between the dry friction force value and isolated body driving force, described by Den Hartog's factor K. It will be shown, that depending on the K factor value it is possible to decide which simulation method furnishes better prediction of vibration mitigation properties of an oscillatory system with friction. Moreover, the stick-slip model is of a generic nature and can be widely used for systems modelling. Its application circumvents deeper knowledge of advanced methods of non-linear analysis and enables better exploitation of the commercial simulation software.

Keywords: *dry friction, acceleration transmissibility, stick-slip, signum, random excitation*

RACHUNEK WPŁYWU TARCIA SUCHEGO NA UKŁAD OSCYLATORA MECHANICZNEGO PODDANEGO WYMUSZENIU LOSOWEMU

W analizie układów oscylacyjnych ruchu drgającego do podstawowych układów należy prosty oscylator liniowy, którego model matematyczny jest dobrze znany. Jednakże istotną kwestią w wielu zastosowaniach praktycznych jest tarcie. Jest to ściśle związane z wymuszeniem kinematycznym o niskim nasileniu w postaci przyspieszeń, co jest częstym przypadkiem w transporcie lądowym. W artykule opisano i porównano dwa podejścia do modelowania tarcia – podejście z funkcją signum oraz podejście z fizycznie zmodyfikowanym efektem drgań ciernych łącznie z powszechnie stosowaną metodą kompensacji harmonicznej Den Hartoga. Główne różnice pokazują, że podejście z fizycznie zmodyfikowanym efektem drgań ciernych opisuje rzeczywistość lepiej niż obliczeniowo prostsze podejście z funkcją signum. Efekty te są zależne od relacji pomiędzy wartością siły tarcia suchego a siłą napędzającą przyłożoną do izolowanego ciała, opisaną przez Den Hartoga jako współczynnik K. Pokazano, że w zależności od wartości współczynnika K można podjąć decyzję co do tego, która z metod symulacji zapewnia lepszą predykcję złagodzonych warunków drgań układu oscylatora z tarciem. Poza tym model drgań ciernych ma typową naturę i może być szeroko używany w modelowaniu układów. Jego stosowanie pozwala pominąć głęboką wiedzę zaawansowanych metod analizy nieliniowej oraz pozwala na lepsze wykorzystanie komercyjnego oprogramowania symulacyjnego.

Słowa kluczowe: *tarcie suche, przełożenie przyspieszenia, drgania cierne, funkcja signum, wymuszenia losowe*

1. INTRODUCTION

The analysis of translatory oscillatory systems is an essential part of machine dynamics and the starting point of further studies in engineering vibrations. The essential form of such an oscillator is a combination of rigid mass m, linear mass-less spring with spring constant k_x and an idealised viscous damper with resistance proportional to relative velocity, described by damping constant b (Fig. 1). The mathematical treatment of such a system is well known. The external excitation can be either a time variable force $F(t)$ acting on the mass m or a kinematic excitation in form of absolute displacement $u(t)$ and its derivatives $\dot{u}(t)$ or $\ddot{u}(t)$, acting on the oscillatory system support, as is the case analysed here. The equation of motion is then

$$m\ddot{x} + k_x(x-u) + b(\dot{x}-\dot{u}) = 0 \tag{1}$$

which can be re-written using the time dependent relative displacement $x_r = x - u$ and its time derivatives $\dot{x}_r, \ddot{x}_r$ as

$$m\ddot{x}_r + k_x x_r + b\dot{x}_r = -m\ddot{u} \tag{2}$$

However, for many real mechanical oscillatory systems this description is too simple:

– Structural constraints limit the free travel i.e. stroke (relative displacement x_r) of the oscillating mass.
– Description of vibratory energy dissipation by a linear, relative velocity dependent damper is too simple.

Another important issue is the dry friction, ever-present in any mechanical systems. The friction element may be introduced intentionally, to act as a means of vibratory energy dissipation, or, more likely, is an unwanted consequence of the system design or construction. In such oscillatory

* Institute of Materials and Machine Mechanics, Slovak Academy of Sciences, Bratislava, Slovakia, stein@savba.sk

systems the influence of friction cannot be neglected [1, 2, 16]. The simple linear SDOF oscillatory system may be an oversimplification of reality and its analysis may lead to erroneous conclusions. This is specially so for random kinematic excitation, as is often the case in ground transportation. In this class of problems the variable of interest is the vibratory acceleration $\ddot{x}(t)$ of mass *m* rather than the displacement variable. Hence it is worthwhile to analyse oscillatory systems in which both a linear damper and a friction element (described by a general friction force F_f) are present, as depicted schematically in Figure 1, from the point of view of acceleration transmissibility.

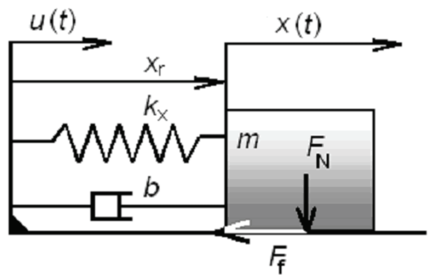

Fig. 1. Schematic layout of analysed horizontal oscillatory system with friction influence

Then the above equations of motion are slightly modified to become:

$$m\ddot{x} + k_x(x-u) + b(\dot{x}-\dot{u}) + F_C \operatorname{sgn}(\dot{x}-\dot{u}) = 0 \qquad (3)$$

$$\text{or: } m\ddot{x}_r + k_x x_r + b\dot{x}_r + F_f = -m\ddot{u} \qquad (4)$$

The general damping force F_d (or mixed damping force in Den Hartog's notation [4]) is described by expression

$$F_d = b(\dot{x}-\dot{u}) + F_C \operatorname{sgn}(\dot{x}-\dot{u}) = b\dot{x}_r + F_f \qquad (5)$$

From a mathematical point of view such an oscillatory system belongs to the class of non-conservative, non-linear systems with discontinuous type of non-linearity (dry friction, impacts, free-play, etc.) The discontinuous non-smooth non-linearity causes a time dependent change in the system dynamics [16, 17].

In analysing an oscillatory system with both viscous damper and dry friction the primary question concerns the relative contribution of both dissipative terms to the total vibratory energy dissipation and hence the influence of both terms on typical system response characteristics, e.g. transfer function, time response, etc.

2. DRY FRICTION MODELS

The first comprehensive scientific work on dry friction is attributed to Coulomb in 1785, however already the genius Leonardo da Vinci around 1500 was occupied by dry friction research [6]. Despite many years of research, the mathematical description of this phenomenon is not yet fully developed [1, 2, 6]. The phenomenon is not always reproducible, as its extent depends on surface state, lubrication, asperities, temperature, magnitude of normal force, relative velocity, etc. [1–3, 6, 15]. Various approaches to this problem are presented in the relevant literature, here only the simplest one, the so called static one, will be discussed a little.

Static friction models are based on the relation of the friction force F_f to the relative velocity v_r between the sliding surfaces in a phenomenological way. Basic approaches are described below, according to the notation of Figure 2.

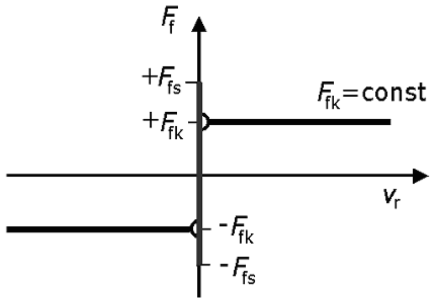

Fig. 2. Friction force F_f course as function of relative velocity v_r

The Coulomb type friction characteristics F_C, which may be mathematically described by the relay characteristics [1, 14, 15]

$$F_f = F_C \operatorname{sgn}(v_r),\ F_C \equiv F_{fk} = \mu_k F_N \qquad (6)$$

where:

F_f – friction force course,
F_C – Coulomb friction force,
v_r – relative velocity between the sliding surfaces.

This model involves a proportional relationship between the Coulomb friction force F_C, sometimes denoted as kinetic friction force F_{fk}, and the normal loading force F_N [3, 15], which is usually assumed to be constant. The proportionality constant μ_k is the dimension-less kinetic friction coefficient. The kinetic friction force F_{fk}, is independent of v_r; however for $v_r = 0$ it cannot be determined, i.e. the force F_f can have any value in interval $(-F_{fk}, +F_{fk})$.

The signum function $\operatorname{sgn}(v_r)$ is often [10, 14, 17, 19] mathematically described as

$$\operatorname{sgn}(v_r) = \begin{cases} +1 & \text{for } v_r > 0 \\ -1 & \text{for } v_r < 0 \end{cases} \qquad (7)$$

Different authors define different function values for the argument value $v_r = 0$ [9, 11]. Note also that the signum function, as defined by expression (7) has no limit for $v_r = 0$ and is therefore not differentiable for $v_r = 0$, and hence is not a smooth continuous function.

In reality a larger force is needed to start the sliding motion, i.e. for overcoming the adhesion at zero relative velocity a larger force is required than when the two surfaces are continuously sliding over each other [3, 15, 16]. The static friction force F_{fs} at $v_r = 0$ has to be compared to a limit force F_L, external to the dry friction interface

$$\text{if } |F_L| \le F_{fs} \Rightarrow v_r = 0 \qquad (8)$$

The limit force F_L is obtained by analysing the force balance across the interface between the sliding surfaces and has to be compared to the static friction force value F_{fs}. If condition (8) is met the system is at standstill in the so-called stick state, indicated in Figure 2 by the vertical line segment. If at a certain time instant the static friction force F_{fs} is overcomed by the external force, the oscillatory systems starts to move abruptly and the relative velocity v_r attains some non-zero value, as described in more detail in [1, 2]. From this instant Eq. (6) is valid until v_r eventually decreases to zero and the system stops again for a certain time interval until the static friction force is overcome again. This start-slide-stop movement (stick-slip movement) leads to non-unique solution of equations describing the motion and poses mathematical difficulties [10, 11, 14, 15]. In analogy, the static friction coefficient μ_s is defined as $\mu_s = F_{fs}/F_N$, and $\mu_s > \mu_k$, because $F_{fs} > F_{fk}$.

The Stribeck's effect is observed in some cases of well-lubricated surfaces [1, 2, 23] when the sliding friction force is dependent on relative velocity v_r. It exhibits a certain minimum at a relative velocity known as the Stribeck's velocity v_S and then increases with higher velocities. It is described by a velocity dependent function $f(v_r)$:

$$\begin{aligned} &F_f = f(v_r), && \text{if } v_r \ne 0 \\ &F_f = F_L, && \text{if } v_r = 0 \text{ and } F_L < |F_{fs}| \\ &F_f = F_{fs}\,\mathrm{sgn}(F_L), && \text{if } v_r = 0 \text{ and } F_L \ge |F_{fs}| \end{aligned} \qquad (9)$$

In some simulation approaches the Stribeck's effect is modelled using a Gaussian distribution function [20] to account for the discontinuous natural dynamics of the change of state change at the start of the slipping motion (step transition $F_{fs} \to F_{fk}$ or $\mu_s \to \mu_k$). It is argued, that the restraining (adhesive) force is a composition of all the micro actions across the interface of contacting surfaces and their asperities [1, 2, 15, 16, 17]. Obviously these actions take place consecutively and not abruptly [1, 2, 23]. The Gaussian model introduced by Eq. (10) is a reasonable continuous approximation to this state change [20]

$$F_f = \left| F_{fk} + \left(F_{fs} - F_{fk}\right) \exp\left\{ -\left(\frac{v_r}{v_S}\right)^2 \right\} \right| \mathrm{sgn}(v_r) \qquad (10)$$

for $v_r \ne 0$

Further analysis will deal with an oscillatory system assuming two ways of dissipating vibratory energy by a linear viscous damping term and by a dry friction term – Eq. (5). Four different cases may arise, depending on the respective proportion of each dissipating term in Eq. (5) to the general damping force F_d, as illustrated in more detail in Figure 3, after [18].

Another issue is the type of excitation:

- harmonic (monoharmonic) with constant excitation frequency and amplitude;
- harmonic with amplitude and frequency varying in a narrow interval, as would be, for example, the case of many rotating machines;
- high intensity random excitation with some pronounced frequency bands where most of the vibration energy is concentrated;
- low intensity random excitation with one or two pronounced frequency bands where the vibratory energy is concentrated, as is often the case in transport industries;
- other possible types of excitation, e. g. parametric oscillations, self excited oscillations, etc.; however their treatment would go far beyond this article.

The most common approximate analytical approaches to the first excitation type are either to use the harmonic balance method, described first fully by Den Hartog in 1931 [4], or to solve the particular differential equations in respective time intervals [10, 12, 13]. This is rather laborious. Here the simulation approach will be followed, assuming excitation by vibratory acceleration $\ddot{u}(t)$, to arrive at solutions that are both viable from an engineering point of view, and realisable by commercially available simulation software, e.g. MATLAB/Simulink®. As will be indicated below the harmonic balance method is not a viable approach if low intensity random excitation is assumed.

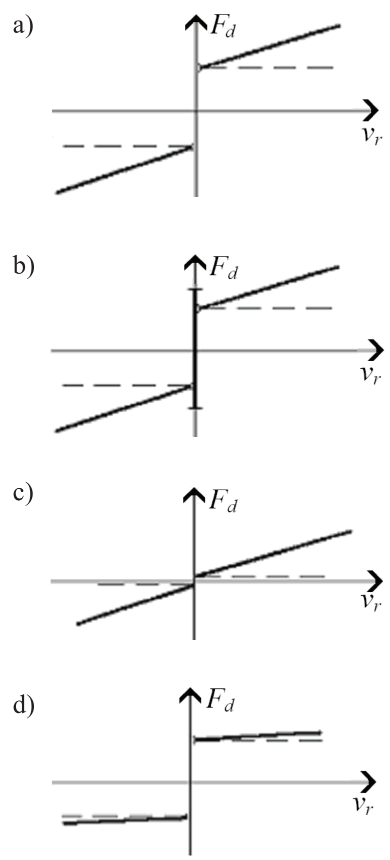

Fig. 3. Possible (a, b, c, d) dependences of the damping force F_d on relative velocity v_r

3. APPROACHES TO DRY FRICTION ANALYSIS AND MODELLING

3.1. Introduction

In employing static friction models for the analysis of oscillatory systems, essentially two general approaches are feasible:

(i) An approximate analytical one, based on the *harmonic balance method* approach.

(ii) A simulation one, employing contemporary simulation software, making use of conditioned switching between solutions in a time-scale that is short in comparison to the dominant period of the excitation signal [16, 17]. The simulation approach uses either the signum function approach, described by Eqs. (6) and (7) or a more physically sophisticated approach using the limit force analysis of Eqs. (8) and (6) and possibly also Eq. (10). The merits of both approaches have to be thoroughly assessed in the context of the specific case to be analysed.

One of the first rigorous attempts at computer simulation of the influence of friction on dynamic systems was made by Karnopp [8]. He considers the causality issues and introduces a region of small relative velocity Dv_r around zero, indicated by the vertical line segment in Figure 2. Outside of this region the Coulomb approach is valid, whereas within this region F_f is determined by other forces acting in the system in such a way that the F_f remains within the region, until a breakaway force (i.e. the static friction force) is exceeded. He illustrates the advantages of this approach on various examples using the bond graph approach and in this way designing a set of appropriate conditions.

3.2. The harmonic balance method

The harmonic balance method assumes a harmonic excitation by acceleration $\ddot{u}$ (with root mean square (RMS) value a_{0u}) or rather by absolute displacement u with amplitude u_0 and variable angular frequency ω_x. The method is fully explained in standard textbooks – e.g. [3, 7, 9, 19] for an oscillatory system without viscous damping, i.e. for $b = 0$. The equivalent damping coefficient b_e in vicinity of resonance of an equivalent linear oscillator is introduced, depending on the amplitude of relative displacement ζ_e

$$b_e = \frac{4F_{fk}}{\pi\,\zeta_e\omega_x} \tag{11}$$

If the assumed harmonic solution is resolved then in steady state

$$\zeta_e(\omega_x) = \frac{F_0}{k_x}\cdot\left[1-\left(\frac{\omega_x}{\omega_0}\right)^2\right]^{-1}\cdot\sqrt{1-\left(\frac{4F_{fk}}{\pi F_0}\right)^2} \tag{12a}$$

where $F_0 = -mu_0\omega_x^2 = -\sqrt{2}ma_{0u}$ is the amplitude of an equivalent excitation force and $\omega_0 = \sqrt{k_x/m}$ is the system natural frequency.

The formula can be expressed in a more transparent way

$$\left|\frac{\zeta_e(\omega_x)}{\sqrt{2}a_{0u}/\omega_0^2}\right| = \left|1-\left(\frac{\omega_x}{\omega_0}\right)^2\right|^{-1}\sqrt{1-K^2} \tag{12b}$$

with a non-dimensional factor K after [5]

$$K = \frac{4}{\pi}\frac{F_{fk}}{F_0} \tag{12c}$$

The positive factor K will subsequently be termed as Den Hartog's factor and (except for a multiplicative constant) it relates the kinetic friction force F_{fk} to the body driving force F_0. Specifically for a horizontal oscillatory system of Figure 1 with constant normal force $F_N = mg_n$ the Den Hartog's factor has the form (g_n is standard gravity acceleration) [7]

$$K = \frac{4}{\pi}\frac{\mu_k g_n}{\sqrt{2}a_{0u}} \tag{12d}$$

Eq. (12b) describes the modulus of the frequency response function (FRF) of the relative displacement for harmonic excitation with constant displacement amplitude in vicinity of resonance. However, expressions (12a) and (12b) are approximate and valid only for $K < 1$, i.e. for $F_0 > (4/\pi)F_{fk} \cong 1.273\,F_{fk}$, i.e. for base horizontal acceleration $a_{0u} > (2\sqrt{2}/\pi)g_n\mu_k$. In other words the driving force has to be sufficiently large in comparison to the kinetic friction force to permit the use of Eq. (12b).

For $F_0 < F_{fk}$, or rather for $F_0 < F_{fs}$ no movement is possible as the driving force would not overcome the adhesion force. If $F_0 \in (F_{fk}, (4/\pi)F_{fk}) \approx (F_{fk}, 1.273\,F_{fk})$ the movement is not pure harmonic, but has one or more stops within one period [2, 4, 5] and *is not described* by the above approximate formula, as $K \geq 1$ and the square root term is not real. If the frequency of excitation ω_x approaches ω_0, the amplitude of oscillations at resonance will eventually grow beyond any limits [3, 19]. The system behaves as an undamped one with linearly increasing relative displacement amplitude x_r of the oscillations [3, 19] until the structural limit is reached.

The Den Hartog's approach cannot account for the stick-slip phenomenon, which is accounted for by a procedure illustrated in [12, 13] for specific cases under harmonic excitation. None is applicable when random excitation is assumed, as often occurs in practice. This is especially so, if the equivalent excitation force amplitude $F_0 = \sqrt{2}ma_{0u}$ would randomly fluctuate below F_{fk} and above $(4/\pi)F_{fk}$ and v_r would be low, i.e. if the friction force would be commeasurable with the driving force of the isolated body.

3.3. Use of the signum function

Use of the signum function for simulation of an oscillatory system with friction is an easy option that is facilitated by any simulation software. The describing equation of motion has the form (4), repeated here as Eq. (13)

$$\text{for } v_r \neq 0{:}\; m\ddot{x}_r + k_x x_r + b\dot{x}_r + F_f = -m\ddot{u} \tag{13}$$

For $v_r = 0$ the sgn function is set to zero [9, 11, 21] and so *analysis for $v_r = 0$ is completely omitted*. Sometimes the discontinuous signum function (7) is substituted by a "smoothed" continuous function, which approximates the sgn function with a required degree of accuracy [14, 15]

$$\operatorname{sgn}(v_r) \approx \frac{2}{\pi}\arctan(cv_r) \approx \tanh(cv_r) \approx \approx \operatorname{erf}(cv_r) \approx \frac{cv_r}{1+c|v_r|} \tag{14}$$

Constant c in each of the functions describes the numerical "match" between the sgn function and the respective continuous function used for approximation. The selection should be governed by following rules [14, 15]:

– if it were too small, approximation would differ too much from the non-smooth one;
– if it were too large the effort is too great and the approximation is not sufficiently smooth.

In [14] selected numerical values are analysed. It is demonstrated, that a value of $c \geq 10^3$ suffices to fulfil both conditions and the fit with analytical solution is within 1%. It is suggested that the last formula is better with regard to computational speed in attaining the same level of accuracy.

Any of the above approaches circumvents the problem of solving differential equation (13) with the discontinuous non-smooth signum function by introducing a continuous smooth function with arbitrary large derivative at zero crossing. The last formula of Eq. (14) was further used and a simple simulation program in MATLAB/Simulink® has been developed.

3.4. Use of the stick-slip approach

If the stick-slip phenomena are to be accounted for following approach has to be followed:

1. For $v_r \neq 0$ Eq. (13) is valid.
2. When the $v_r \neq 0$ to $v_r = 0$ transient occurs the movement stops and the force balance condition across the friction interface has to be tested by the following set of conditions:

(i) Slipping: $|v_r| > \varepsilon$ OR $|F_L| > F_{fs}$ (15a)

(ii) Sticking: $|v_r| < \varepsilon$ AND $|F_L| < F_{fs}$ (15b)

while: $$F_L = m\ddot{x} + k_x x_r \tag{16}$$

and ε is a sufficiently small number, representing the vicinity of zero.

Conditions (15) can be expanded further into a more subtle set of conditions, which formed the basis of the computation algorithm [24]. This approach follows that one of Karnopp [8], using $\varepsilon \sim Dv_r$. For smoothing the $v_r = 0$ to $|v_r| \neq 0$ transient the Gaussian approximation (10) may be used.

Further the operation for numerical evaluation of condition $v_r = 0$, or in numerical systems rather the condition $|v_r| < \varepsilon$, is generally called "variable zero-crossing operation" and is facilitated in standard simulation software by specific procedures (see e.g. [25]). The discriminating small quantity ε value has to be assessed independently. The main difficulty for numerical systems with equal time increments is the need for precise determination of the time instant, when the zero-crossing occurs, or when $|v_r| < \varepsilon$. This is very important when processing real-world data, commonly sampled at equal time increments. The standard stiff ordinary differential equations solvers use variable time increment and are not applicable, unless the sampled data set could be re-interpolated in the same way. Hence if measured data are used for comparison of simulation performance the standard stiff equations solvers must be supplemented by other means. The approach used here was to develop an ordinary differential equations solver with fixed time increment, which specifically caters for determining the $|v_r| < \varepsilon$ condition within the given fixed time increment Δt. Detailed analysis of the zero neighbourhood identification and selection of proper simulation time increment Δt is given elsewhere; e.g. in [22–24].

4. COMPARISON OF THE TWO DRY FRICTION FORCE SIMULATION APPROACHES

In many applications time courses of different variables are less important, while aggregate and statistical characteristics are preferred, e.g. maximum and minimum values, RMS, crest factors, power spectral density (PSD), amplitude distribution etc. Here the acceleration RMS values will be used, as explained above.

When the signum function method is used a high friction value in combination with low v_r results in numerical instability causing parasitic oscillations in the time interval where v_r course is crossing the zero value. The error appears in the response acceleration and not in the relative displacement. It can be explained in following way: the sign output of the signum function for the i-th step is determined by the value in the (i–1)-th step. In the vicinity of the zero crossing point the signum function forces the value for the next step to have the opposite sign and vice-versa. If the simulation interval Δt is too large, or the v_r change is too slow, false oscillations with period $2\Delta t$ occur, even if the real system would stop due to friction. In simulation studies this can be circumvented by setting a sufficiently small Δt at the expenses of the simulation time. In analysing real world sampled data, the time interval is set at the time of data acquisition, and later comparison by simulation means has to follow suit. Thus the choice of simulation interval Δt choice is to some extent limited. If the simulation interval is too large parasitic oscillations occur when using the signum approach. However these oscillations do not occur when using the physically correct stick-slip approach.

This is illustrated in Figure 4 based on simulation using measured input time histories and their comparison to output acceleration time histories measured under field experimental conditions with random input acceleration excitation $a_{0u} = 0.35$ ms^{-2}. Note false oscillations for $F_{fk} = 45$ N; whose consequence is a markedly different acceleration RMS value obtained by evaluating the output signal from the model using the stick-slip approach ($a_{0x} = 0.33$ ms^{-2}), compared with the signum approach, which gives $a_{0x} = 0.50\ \text{ms}^{-2}$. The difference is not marked for the lower $F_{fk} = 15$ N ($0.20\ \text{ms}^{-2}$ versus $0.21\ \text{ms}^{-2}$), where virtually no sticking occurs. This example illustrates the principal drawback of the signum method applied to oscillatory systems in which the driving force may fluctuate around the dry friction force value.

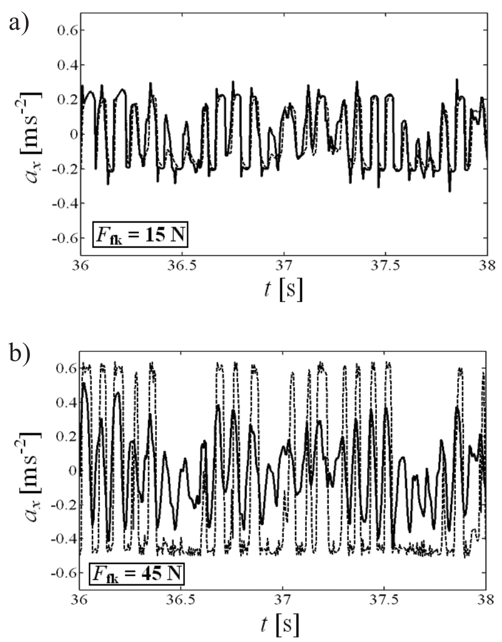

Fig. 4. Time histories of acceleration response for an oscillatory system with different dry friction force for random excitation: stick-slip model (——), signum model (- - - -)

It is of importance to decide whether for given conditions it is necessary to turn to the somehow complicated and more computationally demanding physically correct approach, or the signum approach suffices. As already indicated and described in more detail in [24] the decisive factor is the Den Hartog's factor K, relating the dry friction force F_f to the mass driving force F_0. In analogy to the damping ratio (relating the acting damping constant b to the critical one $b_c = 2\sqrt{mk_x}$) the den Hartog's factor can be also termed *the relative friction coefficient*.

Hence it is interesting to explore in more general way the influence of dry friction force magnitude on system behaviour. Due to system non-linearity classical description by frequency response function, assuming harmonic input and output variables (accelerations) is not applicable. Instead the acceleration transmissibility function in the x-direction T_{ax} is used, defined as: $T_{ax} = a_{0x}/a_{0u}$. Here a_{0x} and a_{0u} respectively are the RMS values of the respective acceleration time courses a_x and a_u. In this way the non-linear effects of the oscillatory system with dry-friction influence are accounted for. The oscillatory system has these parameters: $m = 88.4$ kg, $k_x = 11.5\ \text{kNm}^{-1}$, $b_c = 3017\ \text{Nsm}^{-1}$. Two systems are considered – a system without viscous damping and a system with damping ratio of 0.25, i.e. damping constant $504\ \text{Nsm}^{-1}$.

Two different input excitations are further considered – one a wide-band random excitation and the other one stationary low-intensity excitation, measured in real operating conditions on the cabin floor of a lorry. Their power spectral densities (PSD) are depicted in Figure 5. Note two periodic components at 6.3 Hz and 12.6 Hz in the real signal in Figure 5b.

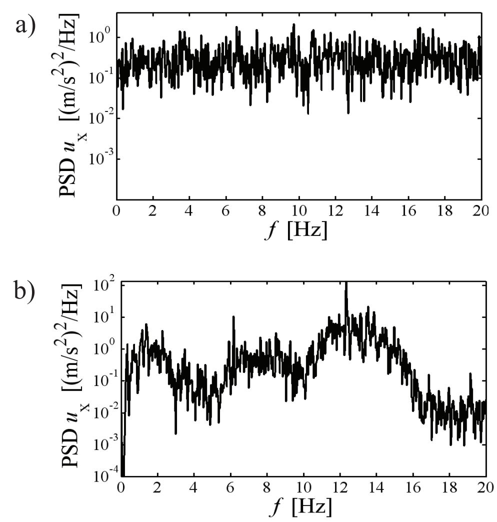

Fig. 5. Acceleration power spectral densities of a wide-band stationary random excitation (a) and low-intensity real-life excitation (b)

The transmissibility T_{ax} courses related to factor K are depicted in upper row of Figure 6: (a) for the wide-band random signal, (b) for the real excitation signal. In the lower row of Figure 6 the probability of sticking ρ for the stick-slip approach is depicted, defined as the time when the system is in the stick state in relation to total duration of simulation for given K (expressed in per-cent).

Note the large discrepancy between transmissibility T_{ax} calculated using the signum approach (dashed line in Figs. 6a and b) and by the physically correct model (solid line in Figs. 6a and b) for higher K factor values. This is due to parasitic numerical oscillations in the signum model, illustrated in Figure 4. From Figure 6 can be seen that the differences are significant when K factor reaches value of approx. 0.75, i.e. when ρ is larger than some 50%. As the K factor increases the stick state probability increases and the transmissibility T_{ax} asymptotically converges to unity, i.e. the oscillatory system becomes stuck and starts to move as a rigid body. Note there are still oscillations for $K \geq 1$, which are not accounted for by Eq. (12). From Figures 6a and b it can be concluded that both simulation approaches furnish same T_{ax} values for low friction influence, say for $K < 0.4$ to 0.45 when the sticking probability is less than 20–25%. Hence for K factor values smaller than, say, 0.40 the signum model suffices for description of oscillatory system behaviour, however for larger friction, if K is larger than, say, 0.75 the correct physical friction model use is inevitable.

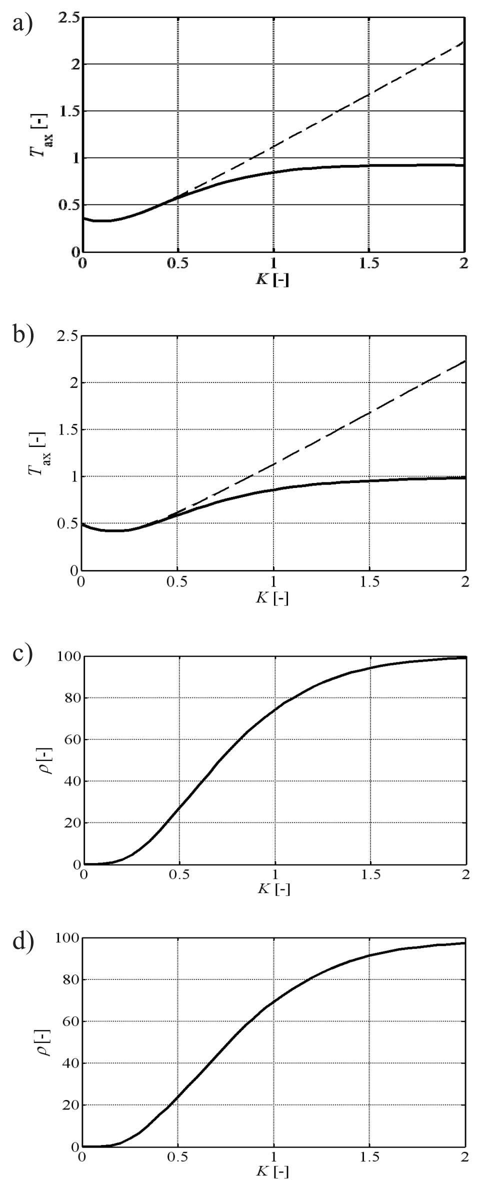

Fig. 6. Transmissibility T_{ax} dependence on factor K: a) random signal; b) real field-measured signal. Probability ρ dependence on factor K; c) random signal; d) real field-measured signal; (——) stick-slip approach, (– – –) signum approach

If the mixed mode damping system would be compared to a SDOF oscillatory system without friction and with the same damping coefficient then, as seen from Figures 6a and b for low values of factor K, the course of T_{ax} would be below that one for the system without friction. The T_{ax} of a common SDOF system without friction would be a horizontal line at T_{ax} value for the mixed mode system for $K = 0$. This indicates that for low friction values the real system damps vibrations slightly better than the simplified SDOF system, i.e. small friction improves vibration mitigation properties. In contrary, for large K factor values there is a large discrepancy between the T_{ax} for the SDOF system without accounting for friction influence and the real system.

Further it is interesting to analyse in same way the same oscillatory system, however without viscous damping. The results of simulation are depicted in Figure 7. Again note the discrepancy between the T_{ax} values obtained by the signum approach and the stick-slip approach, which are occurring for essentially same K values, larger than 0.75, as described above. For low K values both approaches are identical and for decreased friction the T_{ax} approaches large values. Due to low friction the oscillatory system without viscous damping is not able to extract sufficient amount of vibratory energy transmitted throughout and the transmissibility T_{ax} increases beyond acceptable limits.

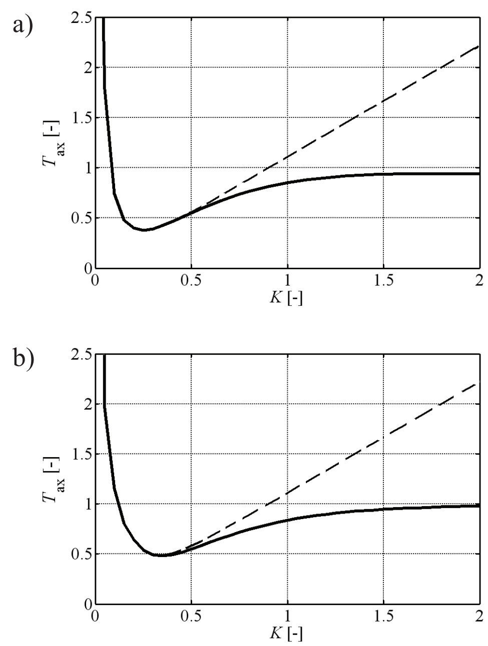

Fig. 7. Transmissibility T_{ax} dependence on factor K for a system without a viscous damper for wide-band stationary random excitation (a) and low-intensity real-life excitation (b): (——) stick-slip approach, (– – –) signum approach

5. CONCLUSION

The contribution deals with the analysis and simulation of a general single degree-of-freedom oscillatory system with vibratory energy dissipation by both an idealised linear viscous damper and a dry friction interface. For modelling the dry friction interface the phenomenological macro-slip approach is employed, described in mathematical form by the approximate harmonic balance approach, by the signum function approach and by the physically correct stick-slip approach assuming switching phenomena within a short time scale. The last two approaches are illustrated by simulation examples using as the input random excitation acceleration measured on a heavy lorry in real operating situation. The differences in the two approaches are highlighted, indicating that the physically correct stick-slip approach describes the reality better than the simpler signum approach. The signum approach is for higher friction force values prone to false numerical oscillations that completely distort the acceleration response signal. These effects are dependent on the relation between the friction force value, the isolated body driving force and the relative velocity between the sliding surfaces, best described by the Den Hartog's factor K (the relative friction coefficient).

It can be concluded that the simpler model, employing the signum function continuous approximation, is suitable for oscillatory systems with low inherent dry friction and high driving force, whereas for correct modelling of systems with higher friction and low driving force the limit force analysis approach is essential. A possible discrimination between using of either of these models is the Den Hartog's factor K. For lover K values (say, below 0.40) the signum model suffices, whereas for higher K values (say, above 0.75), when the probability of sticking rises above 50% of the simulation time physically correct model use is inevitable. These values have to be taken as a "rule of thumb" indication only.

The limit force analysis approach describes reality correctly from a physical point of view, including as it does also static friction. It is universally applicable to any oscillatory system with dry friction in a generic way, irrespective of the magnitude of the Den Hartog's factor. The SDOF oscillatory system including a physically correct dry friction model that has been developed and described here is of a generic nature and can be widely used for systems modelling in transport industries. Its application circumvents the deeper knowledge of advanced methods of non-linear systems analysis and enables more effective exploitation of simulation software for better understanding of the performance of oscillatory systems.

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