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CABLE – MR DAMPER SYSTEM MOTION IN TRANSIENTS

SUMMARY

The paper addresses certain aspects concerning the interaction between a cable and a magnetorheological (MR) damper. An essential difference between the displacement of the point where the damper is attached and the displacements of other points remote from the damper has been observed during experiments. The results of experimental data imply non-steady state motion of the damper piston. Twice in one period of vibration, for a short time, the harmonic motion turns into the motion with almost constant velocity. After a few seconds, the damper can be blocked. During this time other points, remote from the damper, continue their harmonic motion. These phenomena have a significant meaning in cable – MR damper system motion and have to be taken into account while developing algorithms for MR damper control. A finite element model was formulated for the cable – MR damper system that describes the motion of the system with high precision. The influence of cable flexural stiffness in the neighbourhood of end clamps and damper handle were taken into account. Results of considerations are illustrated by numerical examples in which discussed effects are visible.

Keywords*: cable, MR damper, vibration damping*

CHARAKTERYSTYCZNE ZJAWISKA CHWILOWE WYSTĘPUJĄCE W RUCHU UKŁADU LINA–TŁUMIK MR

Przedmiotem artykułu jest wyjaśnienie pewnych zjawisk chwilowych występujących w ruchu układu lina–tłumik MR. Wyniki pomiarów wskazują na istotne, jakościowe różnice pomiędzy przemieszczeniem punktu liny, w którym zamocowany jest tłumik i przemieszczeniami pozostałych punktów liny. Ruch tłoka, który jest równocześnie punktem zamocowania tłumika, wykazuje charakterystyczne cechy. Dwukrotnie w czasie jednego okresu oscylacji, w krótkim przedziale czasu, oscylacyjny ruch punktu zamocowania tłumika zmienia się na ruch z prawie stałą prędkością. Po kilku sekundach ruch tłumika zostaje zablokowany. W tym czasie pozostałe punkty liny poruszają się ruchem harmonicznym o malejącej amplitudzie. Rozważane w artykule zjawiska mogą mieć istotne znaczenie przy projektowaniu układów redukcji drgań lin, z wykorzystaniem tłumików MR. Obliczenia przeprowadzono przy użyciu metody elementów skończonych. Uwzględniono wpływ sztywności giętnej liny, istotny zwłaszcza w bezpośrednim otoczeniu punktu zamocowania tłumika. Uzyskano dużą zgodność obliczeń z wynikami pomiarów.

Słowa kluczowe*: lina, tłumik MR, tłumienie drgań*

1. INTRODUCTION

Cables are very efficient in various types of supporting structures. Cables, however, have very low internal damping and therefore tend to develop dangerous vibrations and wave motion [10]. This motion takes a form of aeolian vibration or cable galloping. The types of motion differ in the way of energy transfer from wind to the cable. Besides, frequencies and amplitudes of vibrations are different.

Various countermeasures are used to protect cable against vibrations. For instance, in cable-stayed bridges magnetorheological (MR) fluid dampers are successfully employed [1, 2]. These dampers proved very efficient due to the fact that the MR damper force can be adjusted to the motion of cables. It is reported that MR dampers operate as passive dampers or various control algorithms are applied (i.e. they operate as controllable dampers). These control algorithms are still under research.

In order to simulate MR damper behaviour various physical and mathematical models were proposed, based on MR fluid research data [8]. Visco-plastic, visco-elastic-plastic and visco-elastic with hysteresis models were considered. Finally, the strongly nonlinear relationships between the force, velocity and control signal (voltage or current) were formulated. The fidelity of real MR damper behaviour is strictly associated with the underlying relationship.

The aim of this study is to explain the interaction between the cable and MR damper. The motivation was the analysis of experimental data obtained from the tests conducted in the specially designed experimental setup. The diagram of the setup is shown schematically in Figure 1. The results show an essential difference between the displacement of the point where the damper is attached and that of other points, remote from the damper (Fig. 5). After a few seconds of motion the damper can be blocked whereas other points continue their motion. These phenomena are very important and have to be taken into account in control algorithms developed for an MR damper.

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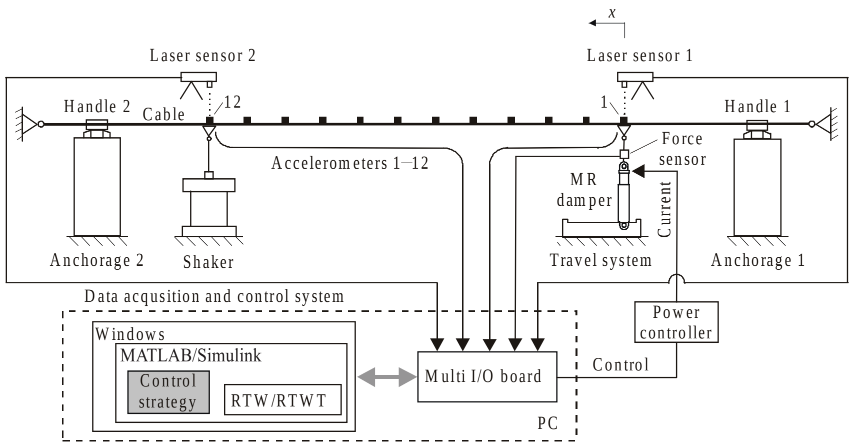

Fig. 1. Diagram of the experimental setup with data acquisition system

2. ENERGY DISSIPATION IN CABLE-DAMPER SYSTEM

Cables belong to the class of spatially distributed systems. Such system is often controlled by means of point-wise sensors and actuators [7]. In this case the controller must guarantee a certain level of performance, both with respect to space-coordinates and time.

Since the damper acts as an energy absorbing element, the overall energy of the system should decrease and the vibrations should eventually die out. But the energy dissipation is controlled by means of a point-wise damper. Improper location of the damper may cause that the cable may vibrate and the damper will be motionless. This is possible when the damper is attached to the cable at the node of any natural mode. It is apparent that in this case the energy of the cable remains constant. Using the definitions of the stability, it can be said that the equilibrium position is stable but it is not asymptotically stable. It is well-known that nodes for cable are located at points $x_d = ml/n$, where l is the length of the cable and n, m are integers ($m < n$). Thus the coordinate x_d is a rational part of the length of the cable. In order to assure the energy dissipation for any initial conditions, the coordinate x_d has to be an irrational part of the length of the cable. In the further consideration we assume that the damper is attached in an appropriate position to damp each natural mode.

Let $F(t)$ represents the damper force acting on the cable. The following equation describes the displacement $w(x, t)$ of the cable points

$$\mu \frac{\partial^2 w}{\partial t^2} - T \frac{\partial^2 w}{\partial x^2} = F(t)\, \delta(x - x_d) \tag{1}$$

where μ is linear mass density of the cable (mass per unit length), T is the cable tension.

The overall energy of the cable can be expressed as

$$E = \frac{1}{2} \int_0^l \left(\mu \left(\frac{\partial w}{\partial t} \right)^2 + T \left(\frac{\partial w}{\partial x} \right)^2 \right) dx \tag{2}$$

Differentiating Eq. (2) with respect to time, substituting Eq. (1), and integrating by parts yields

$$\frac{dE}{dt} = F(t)\, \dot{w}(x_d, t) \tag{3}$$

As expected, the first derivative of energy is equal to the power dissipated by the damper force.

In a viscous damper, force $F(t)$ acting on the cable is proportional to the velocity with the negative sign and can be expressed as

$$F(t) = -\alpha\, \dot{w}(x_d, t) \tag{4}$$

Substituting Eg. (4) in Eq. (3), the first derivative of the cable energy can be written as

$$\frac{dE}{dt} = -\alpha\, \dot{w}^2 (x_d, t) \tag{5}$$

Since the velocity is different from zero, the energy decreases and tends to zero as $t \to \infty$. The motions of any two points have the same character of oscillatory motion with the exponentially decaying amplitude but they are not in the same phase. It is apparent that when viscous damper is used the displacements decrease point-wise. The equilibrium position $w(x, t) = 0$ is strongly, asymptotically stable according to Def. (1).

Definition 1

The position $w(x, t) = 0$ is defined to be strongly asymptotically stable if the displacement decays point-wise: $w(x, t) \to 0$ as $t \to \infty$, $\forall x \in \langle 0; l \rangle$ for all initial conditions.

The similar behaviour of the system is observed when the damper force has the bilinear profile, as shown in Figure 2.

Unlike a viscous force, the Coulomb friction force existing as a component of damper force significantly changes the motion of the cable. For example, in Bingham model of MR damper the force is a sum of viscous damping force and frictional force as shown in Figure 3.

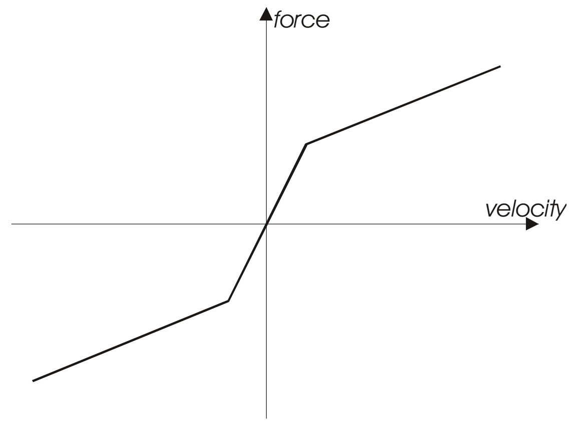

Fig. 2. Bilinear profile of the damper force

For sufficiently small cable amplitudes the resultant force exerted by the cable on the damper piston is less then the maximal value of friction component of the damper force. The damper is completely blocked. The velocity at the cable point where the damper is attached is equal to zero but the remaining points of the cable continue their motions. According to Eq. (3), the energy of the cable is not dispersed and remains constant. The equilibrium position of cable is not asymptotically stable but is stable according to Def. (2).

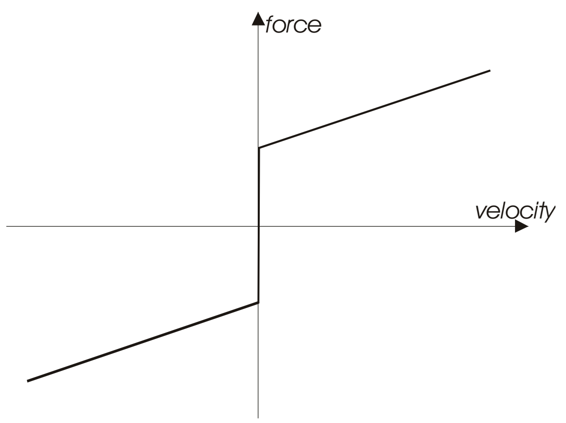

Fig. 3. Force *vs*. velocity for the Bingham model of MR damper

Definition 2

The position $w(x, t) = 0$ *is defined to be strongly stable, but not asymptotically stable if* $w(x, t)$ *is bounded* $\forall x \in \langle 0; l \rangle$ *and* $\forall t \in \langle 0; \infty)$ *for all initial conditions.*

Assuming the Bingham model of MR damper, the essential difference between the displacement of the point where the damper is attached and those of other cable points can be anticipated. When the cable approaches the turning position, the damper force can change direction to oposite one only after the damper piston comes to a halt. This is illustrated by the continuous line in Figure 4.

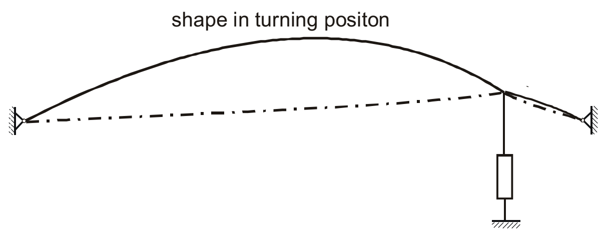

Fig. 4. Two positions of the cable limiting the possible shapes in which the damper piston is motionless

After changing the force direction, in order to set the piston in motion, the force between the cable and the damper has to reach the limit value that is equal to the maximal value of friction component of the damper force. It is possible only if the cable has moved from the turning position to the appropriate one (dash-dot line in Fig. 4). In this short interval of time the damper is clamped, the point where the damper is attached is motionless and the energy of the system is not dispersed.

3. SELECTED RESULTS OF EXPERIMENTS

Displacement of a point where the damper is attached is shown to be entirely different than that of a point located at some distance from the damper (Fig. 5). Twice per each period, the point where the damper is attached takes approximately the uniform motion for a short range of time (Fig. 5a). The velocity during these time intervals is almost constant. This type of motion is not observed at other points remote from the damper (Fig. 5b).

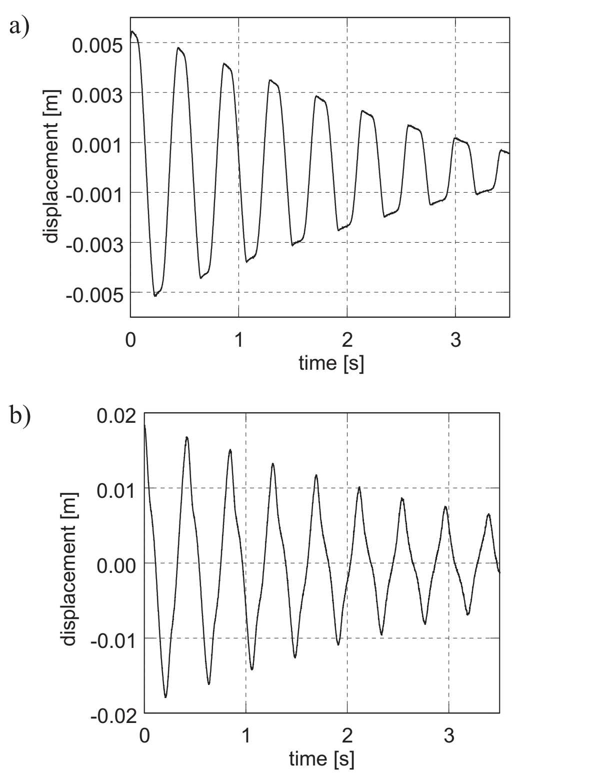

Fig. 5. Cable displacement observed at: a) the point where the damper is attached; b) a point remote from the damper

Assuming the Bingham model, we predict (section 2) that the point where the damper is attached is motionless for a short interval of time twice in one period of vibration, whereas the results of experiment show the motion with almost constant velocity at these intervals of time. We reach the conclusion that the Bingham model is not adequate to precisely explain the behaviour of the cable – MR damper system. In states of uniform motion the forces acting on the damper piston are balanced. Thus the damper force has the same value as the resultant cable force. On the graph showing the relationship between the force and velocity, the vertical segments appear for a characteristic value of velocity (Fig. 6). This value can be calculated from experimental data (Fig. 5a).

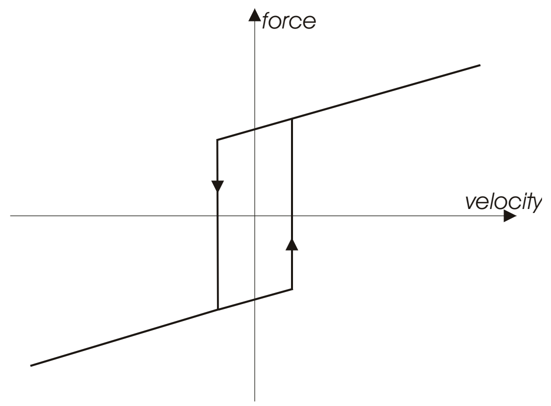

Fig. 6. Force *vs*. velocity for the MR damper with hysteresis

4. MODELLING OF THE CABLE – MR DAMPER SYSTEM

Because of inherent MR damper nonlinearities the exact description of the system motion is difficult. Generally, there is no simple way to solve the problem of motion of a continuous system with a nonlinear subsystem attached at any point. The cable – MR damper system is usually reduced to an appropriate discrete system by approximate methods. Two methods are often used to cable discretization. The first is the Ritz-Galerkin method and the other the finite element method (FEM).

In the Ritz-Galerkin method the displacement of the cable is approximated by a sum of series of independent functions satisfying the boundary conditions [5]. Finally, the expression for the displacement $w(x, t)$ assumes the form

$$w(x,t) = \sum_i \phi_i(x)\, q_i(t) \tag{6}$$

where:

$\phi_i(x)$ – known functions,
$q_i(t)$ – generalized coordinates.

Each coordinate $q_i(t)$ specifies the participation of the shape function $\phi_i(x)$ in resultant displacement but does not describe directly the displacement of any point of the continuous system. The efficiency of Ritz-Galerkin method depends on functions $\phi_i(x)$ which are selected. They are often assumed as modal functions determined for the cable with no damper. In practice, the finite number of modal functions is used in calculation. A drawback involved in approximation is that the truncated higher order modes contribute significantly to the motion of the point where the damper is attached. In some recent works [3, 4] the set of modal functions is supplemented with the function describing the static deflection of the string under concentrated force applied at the point where the damper is attached. The graph of this function looks like the triangle (Fig. 7). In this case we take into consideration the influence of higher order modes but in appropriate proportions between them. These proportions can be derived from the Fourier series expansion of triangle shape function.

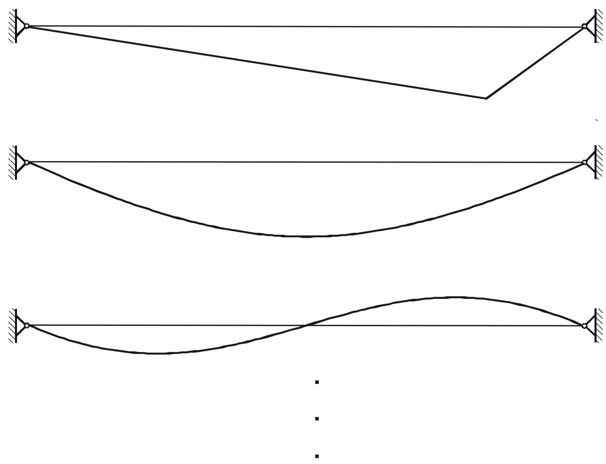

Fig. 7. Selected function used in Ritz-Galerkin method

The function describing the static deflection helps more precisely assess the displacement at the point where the damper is attached. Accordingly, the number of necessary shape function can be significantly less.

The damper is located near the end of the cable. In order to describe the displacement correctly, the flexural stiffness should be taken into account in this part of cable. It is difficult to do using Ritz-Galerkin method because the shape functions are assumed for the overall cable. This requirement can be easily fulfilled using FEM. In this method the displacement at any point of the cable is expressed in terms of finite number of displacements at nodal points multiplied by interpolation functions given for each element. The interpolation functions are generally low-degree polynomials. The advantage of FEM method is that the local properties of the system can be easier expressed than in the other methods.

5. FINITE ELEMENT MODEL OF THE CABLE – MR DAMPER SYSTEM

In order to create the FEM model of the system we apply 2-D line element with cubic interpolation functions. It has two nodes with translational (in transversal direction) and rotational degrees of freedom. Using this element we can respect the flexural stiffness in the neighbourhood of cable handles and damper handle.

The cable is divided into elements in such a way that the damper is attached in node. It is recommended that the sizes of adjoining elements in the neighbourhood of damper handle should be less than other elements.

Inside cable elements the displacement w_e is interpolated from the corresponding nodal values by the following relation

$$w_e = u_e^T \psi \tag{7}$$

where nodal displacements vector has the form

$$u_e = [w_I, \varphi_I, w_{II}, \varphi_{II}]^T \tag{8}$$

The following interpolation functions [5] are assumed:

$$\begin{aligned} \psi_1(x) &= 1 - 3\left(\frac{x}{h}\right)^2 + 2\left(\frac{x}{h}\right)^3 \\ \psi_2(x) &= h\left(\frac{x}{h} - 2\left(\frac{x}{h}\right)^2 + \left(\frac{x}{h}\right)^3\right) \\ \psi_3(x) &= 3\left(\frac{x}{h}\right)^2 - 2\left(\frac{x}{h}\right)^3 \qquad\qquad 0 \le x \le h \\ \psi_4(x) &= h\left(\left(\frac{x}{h}\right)^3 - \left(\frac{x}{h}\right)^2\right) \end{aligned} \tag{9}$$

They are the components of the following vector

$$\psi = [\psi_1, \psi_2, \psi_3, \psi_4]^T \tag{10}$$

The length of the element is equal to h. Nodal points are placed at ends of the element. Schematic diagram of element and graphs of interpolation functions are shown in Figure 8.

In order to derive the element mass matrix, the kinetic energy of the element should be expressed as a quadratic form of the nodal velocities. This transformation can be expressed as

$$E_k = \frac{1}{2}\mu \int_0^h \dot{w}_e^2 \, dx = \frac{1}{2}\dot{u}_e^T M_e \dot{u}_e \tag{11}$$

The calculations yield the following mass matrix

$$M_e = \begin{bmatrix} \frac{156}{420}\mu h & \frac{22}{420}\mu h^2 & \frac{54}{420}\mu h & -\frac{13}{420}\mu h^2 \\ \frac{22}{420}\mu h^2 & \frac{4}{420}\mu h^3 & \frac{13}{420}\mu h^2 & -\frac{3}{420}\mu h^3 \\ \frac{54}{420}\mu h & \frac{13}{420}\mu h^2 & \frac{156}{420}\mu h & -\frac{22}{420}\mu h^2 \\ -\frac{13}{420}\mu h^2 & -\frac{3}{420}\mu h^3 & -\frac{22}{420}\mu h^2 & \frac{4}{420}\mu h^3 \end{bmatrix} \tag{12}$$

The element stiffness matrix can be determined in the similar way. The potential energy has two components. The first component is associated with tension and the second one with bending. Assuming small displacement, the cable tension is constant. We can obtain the quadratic form of the nodal displacements as the approximation of potential energy.

$$\begin{aligned} E_p &= \frac{1}{2}T\int_0^h \left(\frac{dw_e}{dx}\right)^2 dx + \frac{1}{2}EJ\int_0^h \left(\frac{d^2 w_e}{dx^2}\right)^2 dx = \\ &= \frac{1}{2}u_e^T K_e u_e \end{aligned} \tag{13}$$

where EJ is the flexural stiffness of the cable.

Performing the calculations discussed above we obtain the following element stiffness matrix

$$K_e = \begin{bmatrix} \frac{18}{15}\frac{T}{h} + 12\frac{EJ}{h^3} & \frac{1}{10}T + 6\frac{EJ}{h^2} & -\frac{18}{15}\frac{T}{h} - 12\frac{EJ}{h^3} & \frac{1}{10}T + 6\frac{EJ}{h^2} \\ \frac{1}{10}T + 6\frac{EJ}{h^2} & \frac{2}{15}Th + 4\frac{EJ}{h} & -\frac{1}{10}T - 6\frac{EJ}{h^2} & -\frac{1}{30}Th + 2\frac{EJ}{h} \\ -\frac{18}{15}\frac{T}{h} - 12\frac{EJ}{h^3} & -\frac{1}{10}T - 6\frac{EJ}{h^2} & \frac{18}{15}\frac{T}{h} + 12\frac{EJ}{h^3} & -\frac{1}{10}T - 6\frac{EJ}{h^2} \\ \frac{1}{10}T + 6\frac{EJ}{h^2} & -\frac{1}{30}Th + 2\frac{EJ}{h} & -\frac{1}{10}T - 6\frac{EJ}{h^2} & \frac{2}{15}Th + 4\frac{EJ}{h} \end{bmatrix} \tag{14}$$

Each term of the stiffness matrix is the sum of two components. The first component is associated with tension and the other with bending moment. Equating the components of the greatest term of stiffness matrix we calculate the critical length of the element.

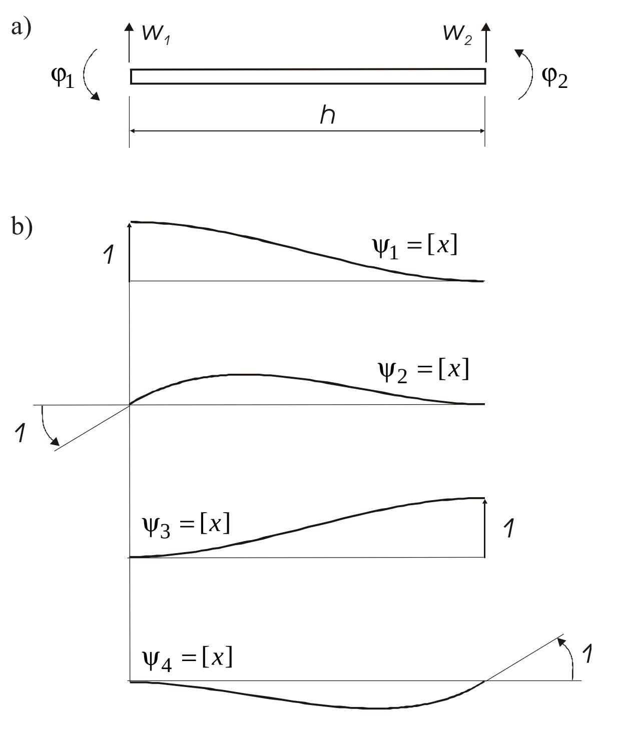

Fig. 8. Schematic diagram of an element (a) and cubic interpolation functions (b)

When the element has the critical length, the axial tension and the bending contribute almost equally to the potential energy of the element. If the element is significantly longer, the tension plays the major role and *vice versa*.

The motion of the overall system is defined in terms of nodal displacements and rotations. They were arranged in ascending order and were called $u_1, u_2, ..., u_N$. Equations of motion can be written in the compact form as a matrix equation

$$M\ddot{u} + Ku = QF(u) \tag{15}$$

where:

$u = [u_1, u_2, ..., u_N]^T$ – nodal displacement vector,
M and K – mass and stiffness matrixes,
Q – force coefficients vector,
$F(u)$ – the damper force.

The mass matrix M and stiffness matrix K for the complete system can be obtained from element mass matrices and element stiffness matrices by the assembling process. The vector of nodal forces for the complete system was obtained by calculating the virtual work of the damper force applied to the cable.

For control applications, the equations of motion may be equivalently written in the state-space form

$$\dot{y} = Ay + BF(y) \tag{16}$$

where state matrix A and vector B have the form:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}Q \end{bmatrix} \tag{17}$$

State vector y is defined as $y = [u_1, u_2, ..., u_N, \dot{u}_1, \dot{u}_2, ..., \dot{u}_N]^T$, and I is the unit matrix.

In the case when the damper model has the internal state coordinates (for example in the Bouc-Wen model), the state vector y should be extend and the equation (15) should be supplemented with damper state equations.

Since the damper is attached at the node, the force depends only on displacement and velocity of this node. Using more complex model of damper, the force depends also on damper state coordinates.

6. NUMERICAL CALCULATIONS AND DISCUSSION

In order to verify the results obtained from experiments, the calculations based on different damper models have been performed. In calculations the state equations (16) were supplemented with the relationship between state parameters and damping force. Since damper has nonlinear properties, the equations of motion became nonlinear. The equations were solved numerically using MATLAB. According to the general considerations discussed in sections 2 and 3, two models of damper force have been taken into account.

The first was the Bingham model. It is frequently applied in calculation concerning the MR-dampers. According to this model, force is derived from the formula

$$F = f_c sign(\dot{u}_d) + c_0 \dot{u}_d \tag{18}$$

where:

f_c – maximal value of friction component,
c_0 – viscous damping coefficient,
$\dot{u}_d$ – the velocity of the point where the damper is attached.

Schematic graph of the force is shown in Figure 3. The function *sign* in formula (18) is often approximated by *arctg* function. This exchange is advantageous taking into account both experiment results and numerical calculations. After substituting values of parameter (for RD-1005-3 damper of Lord Corporation) the formula was used in calculation. The damper force as a function of velocity is presented in Figure 9.

The results of calculation are shown in Figure 10. The subplot (a) in Figure 10 shows the displacement time history at the point where the damper is attached (at the distance of 1.5 m from the handle 1). The subplot (b) in Figure 10 shows the displacement time history at the point placed at a distance of 3 m from the handle 2. In the light of section 2, there is essential difference between these two displacements. Twice per each period of vibration, the point where the damper is attached comes to a stop for a short time whereas the second point of the cable continues the motion in this time. The displacements obtained from numerical simulation are consistent with the results of experiment with the exception of the short periods of time while the velocity is equal to zero (the damper piston remains immobile).

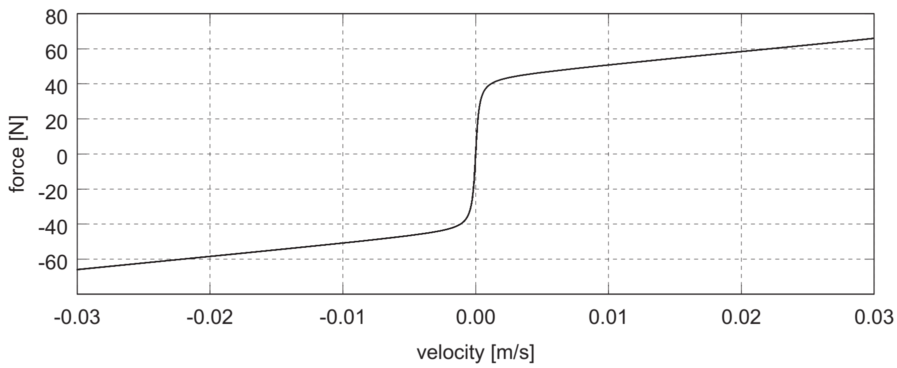

Fig. 9. Force *vs.* velocity

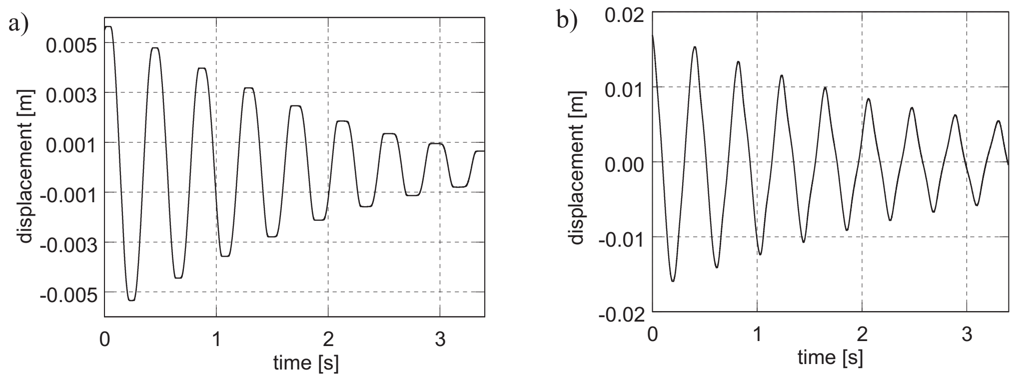

Fig. 10. Time histories of displacement: a) the point where the damper is attached; b) a point remote from the damper

In order to adjust the simulations to experimental data, the damper model with hysteresis was used to determine the damper force in calculations. The characteristic value of velocity defining the width of hysteresis was calculated on the basis of experimental results. The remaining parameters of the model were the same as in the previous calculations. The damper force versus velocity is depicted in Figure 11.

The results of calculation are shown in Figure 13. The subplot (a) in Figure 12 shows the displacement time history at the point where the damper is attached (at the distance of 1.5 m from the handle 1). The subplot (b) in Figure 12 shows the displacement time history at the point placed at the distance of 3 m from the handle 2. Both displacements in Figure 12 are almost the same as those obtained from experimental data (Fig. 5). Analysing the displacement of the point where the damper is attached one can observe short intervals while the point moves with almost constant velocity. This phenomenon was discussed in section 3.

In order to emphasize the differences between the damper piston movement in two cases when the Bingham model was applied and when the model with hysteresis was used, the appropriate displacements are more precisely shown in Figure 13.

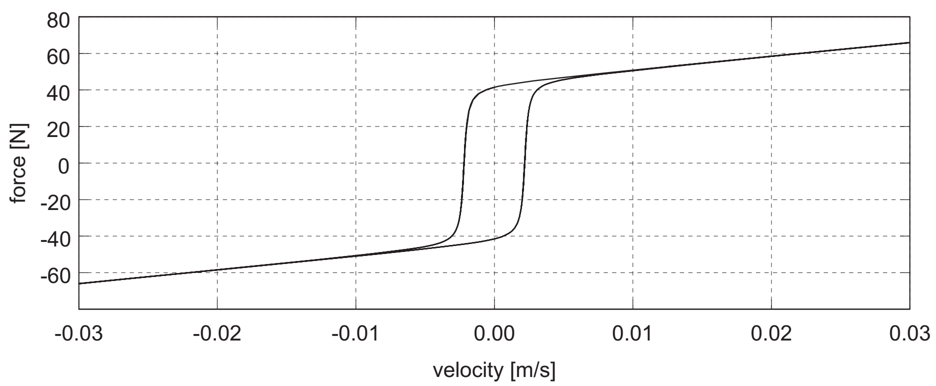

Fig. 11. Damper force as a function of velocity

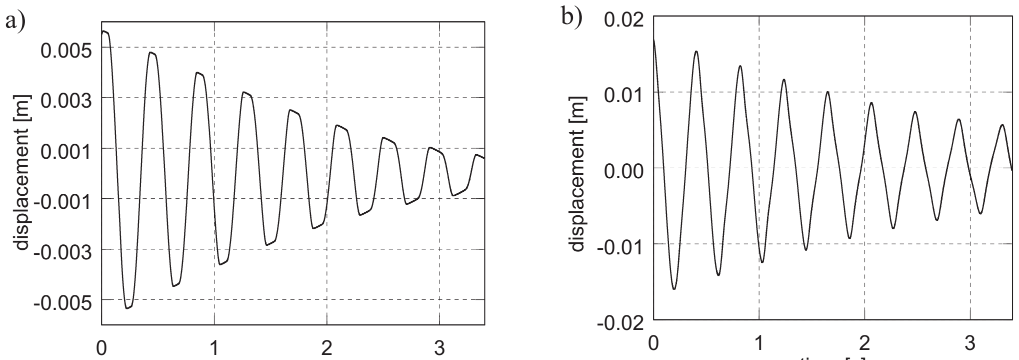

Fig. 12. Time histories of displacement: a) the point where the damper is attached; b) a point remote from the damper

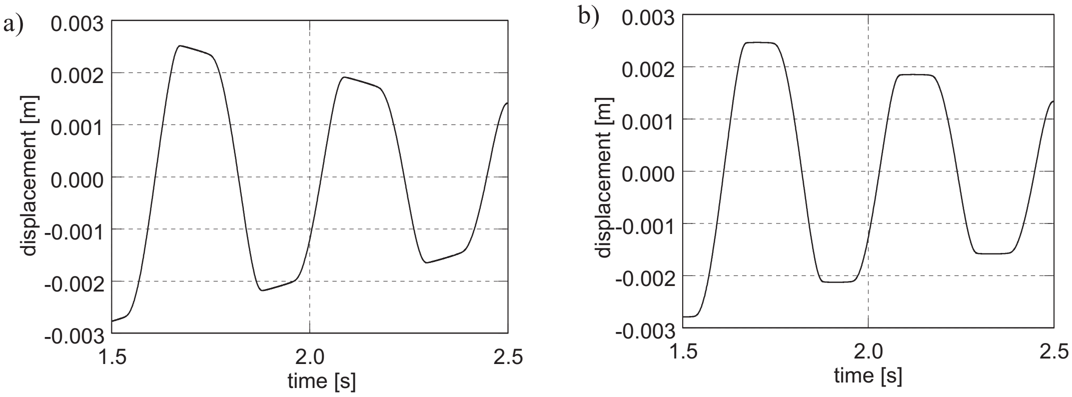

Fig. 13. Displacements of the damper piston: a) hysteresis model; b) Bingham model

7. CONCLUDING REMARKS

MR-dampers are often used as actuators in control systems specially designed to eliminate cable vibrations [6].

This paper addresses the very important phenomenon appearing in cable MR-damper system that can disturb the performance of simple control algorithms. Measurements and calculations prove that for small velocity of damper piston, less than critical value constraining the hysteresis area, the force switching occurs in a different manner than for other values of velocity. This phenomenon appears first of all for small vibrations. In this case the simple control algorithm that does not take into account the phenomenon described in this paper can be useless. At larger velocity, control algorithms can work with less efficiency.

In order to solve the problem of damping control, the exact description of the damper motion is very important. The mathematical model presented in our study describes the motion of the system with high precision. Thanks to finite element approach used in calculations, the influence of cable flexural stiffness in the vicinity of end clamps and damper handle were taken into account.

Acknowledgement

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