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THE RESONANT VIBRATION OF THE ASYMMETRICAL ROTOR SUPPORTED IN THE SLIDE BEARINGS WITH THE FLOATING BEARING SHELL

SUMMARY

In the paper resonance vibrations of the asymmetric rotor supported on journal bearings are analysed. The profile of bushes is circular and additionally bushes are encased in floating ring with controlled elastic and dumping properties. It was proved that through modification of the elastic and dumping properties it is possible to limited the amplitude of resonance vibrations as well as it possible to control frequency at which frequency the resonance occurs. The diagrams with investigation results are given.

***Keywords**: asymmetric rotor, journal bearing, floating bearing shell, resonant vibration, kinematic excitation*

DRGANIA REZONANSOWE NIESYMETRYCZNEGO WIRNIKA Z PANEWKAMI PŁYWAJĄCYMI

W pracy zamieszczono rozważania na temat drgań rezonansowych asymetrycznego wirnika podpartego w łożyskach ślizgowych. Zarys panewek jest kołowy i dodatkowo osadzone są one w pływającym pierścieniu o regulowanych własnościach sprężysto-tłumiących. Wykazano, iż poprzez zmianę tych własności można ograniczać amplitudę drgań w rezonansie, a także sterować częstością, przy której występuje rezonans. W pracy zamieszczono odpowiednie wykresy.

***Słowa kluczowe**: wirnik asymetryczny, łożyskowanie czopu wału, pływające panwie łożyska, częstotliwość rezonansowa, wymuszenie kinematyczne*

1. INTRODUCTION

In connection with growing speed rotation of any machine as well as devices, designers try to eliminate unnecessary phenomenon which can disturb their desirable stable functioning. Rotating bearing shafts are essential structural elements and with their reliable function many constructions are dependent on them. The technology knows many methods for vibration damping called passive or active which could occur in the rotor rotating system. In this paper, the author would like to investigate the dynamic and resonant vibration asymmetric rotor system supported in the cylindrical slide bearings where kinematics function is applied. This kind of input can be frequently observed where rotation speeds of the working machines close to each other can be changed. Inspiration for this work, which were presented in 2004 at the Bearing Engineering Conference, gave very promising results in work [1] as well as in previous investigations where results are presented in work [2] and [3].

2. MODEL OF THE SYSTEM AND EQUATION OF MOTION

The model of the rigid rotor supported in the cylindrical slide bearings where the bearings shell are supported elastically (floating) is taken into account. The support can be any rubber material, composite material, magneto or electrorheology liquid where viscoelasticity quality can be changed. The whole system is controlled by kinematic input from the ground. The scope of this work was to limit the amplitude of resonant vibration and the possibility to move the resonant forward or back depending on the viscoelasticity quality of the elastic ring. In the system, the rotor asymmetry comes from the unsymmetrical position of the external load Q1. The model of the system, as well as supports, is presented in Figure 1.

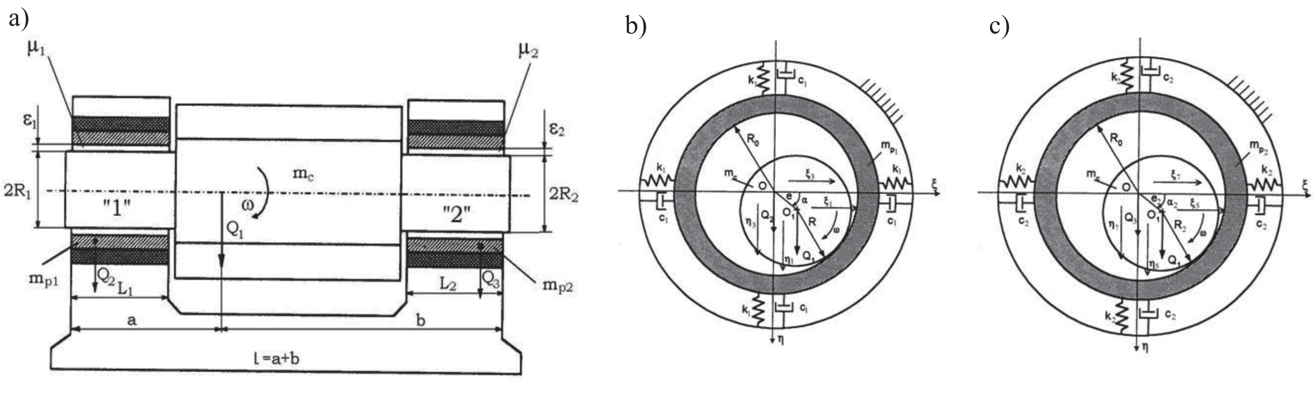

Fig. 1. Model of the system and bearing supports: a) rotor; b) support "1"; c) support "2"

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As a model of the system, the author has taken into consideration a flat model, which presents steady and constant pressure distribution in the oil film along the bearing. The dependence describes pressure distribution and hydrodynamic uplift forces taken on the base work [4]. Having specified the expression of hydrodynamic uplift forces on the base [5] the equation of motion of the system is described in form (1).

$$\ddot{\xi}_1 = \frac{a}{l}\frac{I_2}{I_1}\omega(\dot{\eta}_5 - \dot{\eta}_1) + \left(\frac{1}{m_c} + \frac{a^2}{I_1}\right)P_{1\xi} + \left(\frac{1}{m_c} - \frac{ab}{I_1}\right)P_{2\xi}$$

$$\ddot{\xi}_5 = \frac{b}{l}\frac{I_2}{I_1}\omega(\dot{\eta}_1 - \dot{\eta}_5) + \left(\frac{1}{m_c} - \frac{ab}{I_1}\right)P_{1\xi} + \left(\frac{1}{m_c} + \frac{b^2}{I_1}\right)P_{2\xi}$$

$$\ddot{\eta}_2 = \frac{b}{l}\frac{I_2}{I_1}\omega(\dot{\xi}_5 - \dot{\xi}_1) + \left(\frac{1}{m_c} - \frac{ab}{I_1}\right)P_{1\eta} + \left(\frac{1}{m_c} + \frac{b^2}{I_1}\right)P_{2\eta} + \frac{Q_1}{m}$$

$$\ddot{\eta}_1 = \frac{a}{l}\frac{I_2}{I_1}\omega(\dot{\xi}_1 - \dot{\xi}_5) + \left(\frac{1}{m_c} + \frac{a^2}{I_1}\right)P_{1\eta} + \left(\frac{1}{m_c} - \frac{ab}{I_1}\right)P_{2\eta} + \frac{Q_1}{m}$$

$$\ddot{\xi}_3 = \frac{1}{m_{p1}}\left(-k_1\xi_3 - c_1\dot{\xi}_3 - P_{1\xi}\right)$$

$$\ddot{\eta}_3 = \frac{1}{m_{p1}}\left[-k_1(\eta_3 - A\cos\nu t) - c_1(\dot{\eta}_3 + A\nu\sin\nu t) - P_{1\eta} + Q_2\right]$$

$$\ddot{\xi}_7 = \frac{1}{m_{p2}}\left(-k_2\xi_7 - c_2\dot{\xi}_7 - P_{2\xi}\right) \tag{1}$$

$$\ddot{\eta}_7 = \frac{1}{m_{p2}}\left[-k_2(\eta_7 - A\cos\nu t) - c_2(\dot{\eta}_7 + A\nu\sin\nu t) - P_{2\eta} + Q_3\right]$$

$$P_{1\xi} = P_{\beta 1}\cos\alpha_1 - P_{\tau 1}\sin\alpha_1$$

$$P_{1\eta} = P_{\beta 1}\sin\alpha_1 + P_{\tau 1}\cos\alpha_1$$

$$P_{2\xi} = P_{\beta 2}\cos\alpha_2 - P_{\tau 2}\sin\alpha_2$$

$$P_{2\eta} = P_{\beta 2}\sin\alpha_2 + P_{\tau 2}\cos\alpha_2$$

$$P_{\beta 1} = -\frac{12\mu_1 R_1 L_1}{\delta_1^2}\left[\frac{\beta_1^2(\omega - 2\dot{\alpha}_1)}{(1-\beta_1^2)(2+\beta_1^2)} + \frac{\beta_1\dot{\beta}_1}{(1-\beta_1^2)} + \frac{2\dot{\beta}_1}{(1-\beta_1^2)^{1.5}}\operatorname{arctg}\frac{\sqrt{1+\beta_1}}{\sqrt{1-\beta_1}}\right]$$

$$P_{\tau 1} = \frac{6\pi\mu_1 R_1 L_1}{\delta_1^2}\frac{\beta_1(\omega - 2\dot{\alpha}_1)}{(1-\beta_1^2)^{0.5}(2+\beta_1^2)}$$

$$P_{\beta 2} = -\frac{12\mu_2 R_2 L_2}{\delta_2^2}\left[\frac{\beta_2^2(\omega - 2\dot{\alpha}_2)}{(1-\beta_2^2)(2+\beta_2^2)} + \frac{\beta_2\dot{\beta}_2}{(1-\beta_2^2)} + \frac{2\dot{\beta}_2}{(1-\beta_2^2)^{1.5}}\operatorname{arctg}\frac{\sqrt{1+\beta_2}}{\sqrt{1-\beta_2}}\right]$$

$$P_{\tau 2} = \frac{6\pi\mu_2 R_2 L_2}{\delta_2^2}\frac{\beta_2(\omega - 2\dot{\alpha}_2)}{(1-\beta_2^2)^{0.5}(2+\beta_2^2)}$$

where:

- μ_1, μ_2 – absolute viscosity of the lubricant factor for bearing “1” and “2”,
- R_1, R_2 – journal radius for bearing “1” and “2”,
- R_0 – bearing shell radius,
- L_1, L_2 – bearings length,
- $\delta_1 = \frac{\varepsilon_1}{R_1}$, $\delta_2 = \frac{\varepsilon_2}{R_2}$ – clearance for bearing “1” and “2”,
- ω – rotation speed,
- $\dot{\alpha}_1, \dot{\alpha}_2$ – tangential velocity,
- $\dot{\beta}_1, \dot{\beta}_2$ – radius velocity,
- β_1, β_2 – relative eccentricity,
- $P_{\beta 1}, P_{\beta 2}, P_{\tau 1}, P_{\tau 2}$ – the hydrodynamical uplift forces in the bearing “1” and “2”,
- α_1, α_2 – angle of the line between the journal centre and the centre of the bush bearing in the bearing “1”, “2”,
- m_{P1}, m_{P2}, m_c – the bearing shell weight and the rotor weight,
- I_1, I_2 – moment of interia,
- k_1, k_2 – stiffness of the elastic support,
- c_1, c_2 – dumping factors,
- a, b – coordinates of the external load Q1,
- A – amplitude of the ground movement,
- ν – frequency of the ground movement,
- A_i – maximum amplitude of the bearing journal centres “1” and “2” in direction ξ, η.

3. NUMERICAL RESULTS

The system of equations (1) are strongly out of line and also coupled to each other, therefore it is impossible to answer them in an analytical way. That is why for these analyses we are obliged to use digit simulation methods. Example results are presented in Figures 2 and 3. The simulation for different clearance, elastic rigid support and asymmetry are investigated.

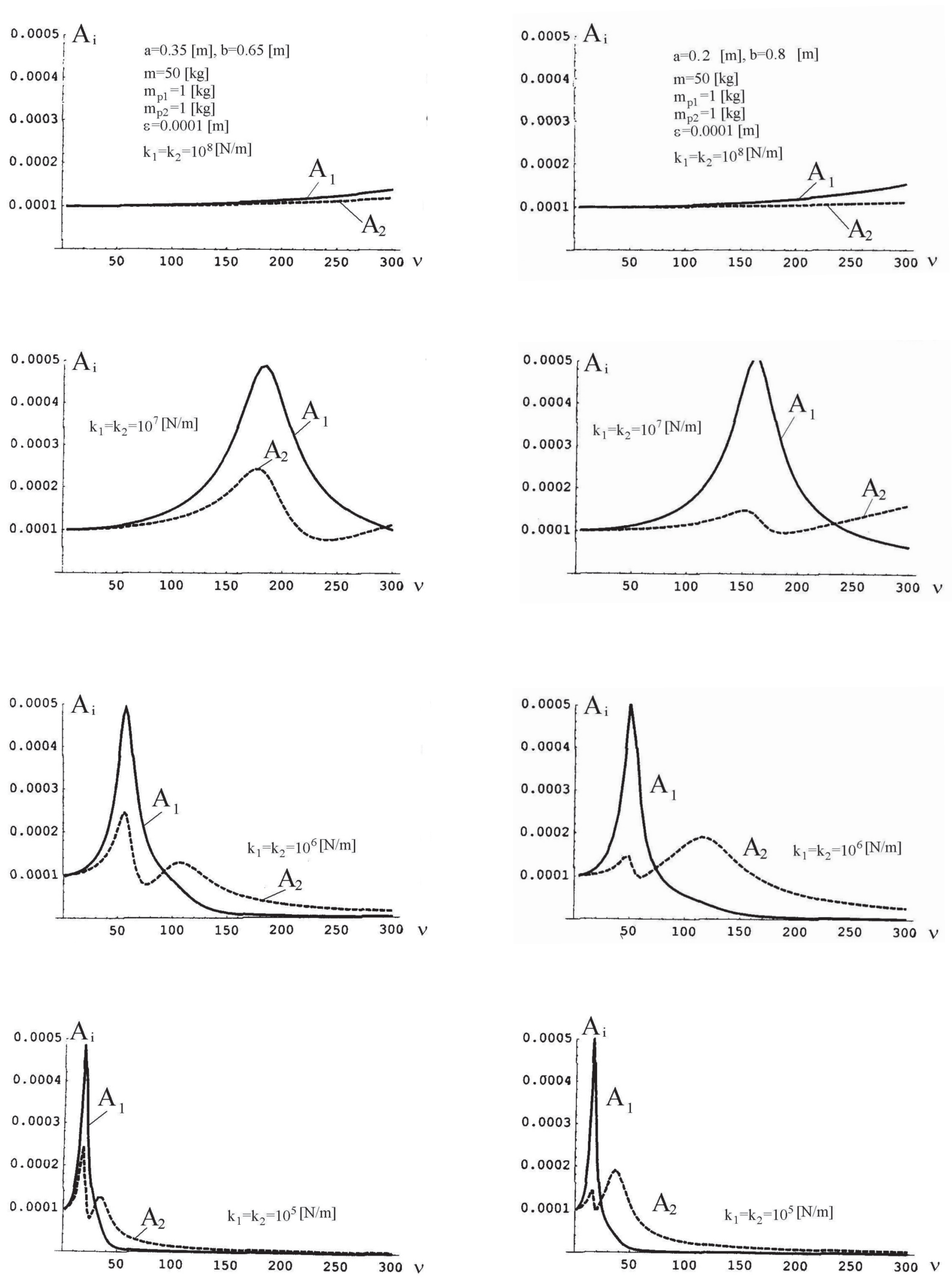

Fig. 2. Resonant curves for various stiffness of mobile support:
$\varepsilon = 0.0001$ m, A_1 – amplitude of journal vibration in bearing “1”, A_2 – amplitude of journal vibration in bearing “2”
R_1, $R_2 = 0.05$ m, L_1, $L_2 = 0.05$ m, $I_1 = 4.19$ kg·m^2, $I_2 = 0.0625$ kg·m^2, μ_1, $\mu_2 = 0.01$ Pa·s

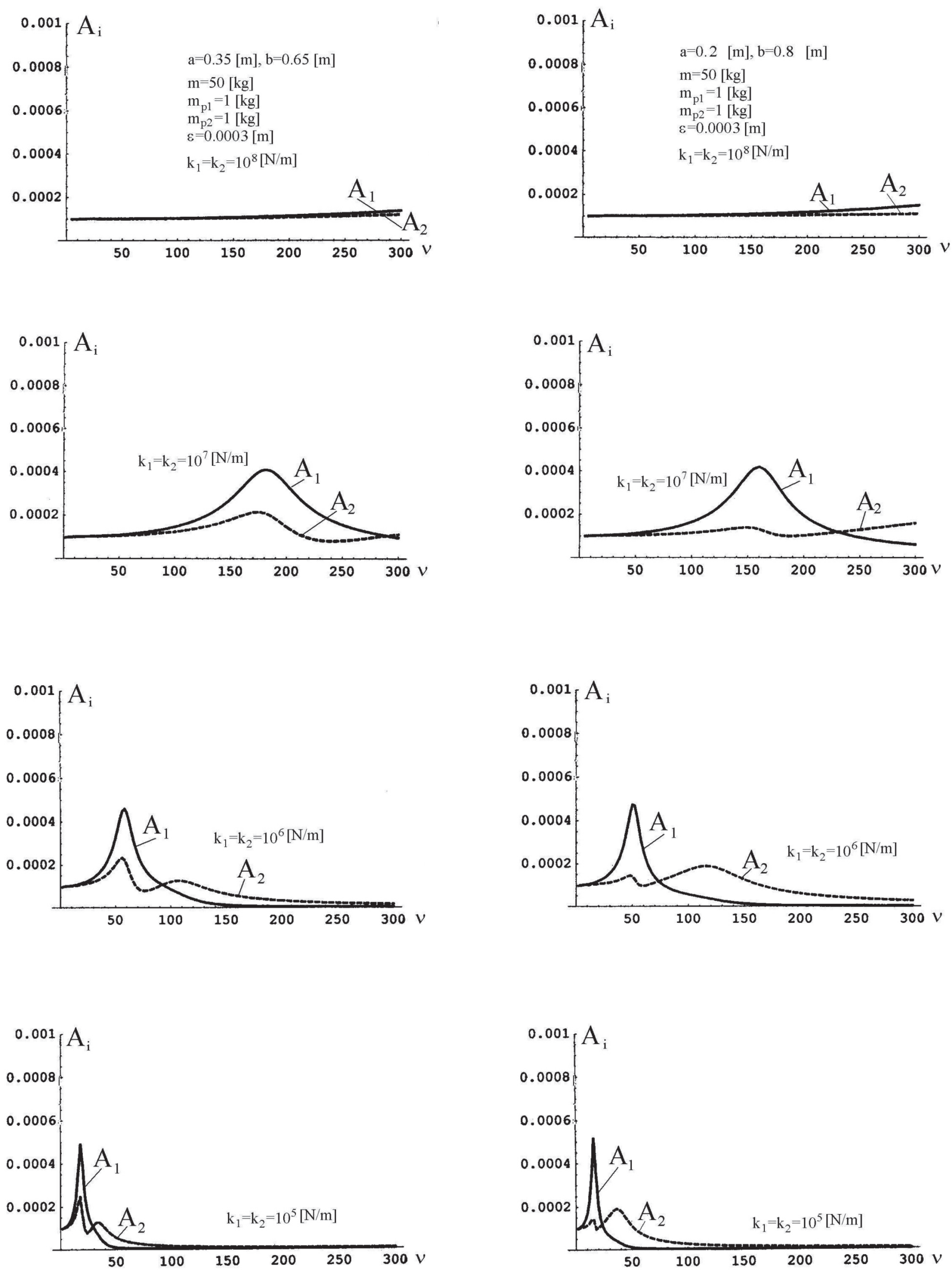

Fig. 3. Resonant curves for various stiffnesses of mobile support: ε = 0.0003 m, A_1 – amplitude of journal vibration in bearing "1", A_2 – amplitude of journal vibration in bearing "2"
R_1, R_2 = 0.05 m, L_1, L_2 = 0.05 m, I_1 = 4.19 kg·m^2, I_2 = 0.0625 kg·m^2, μ_1, μ_2 = 0.01 Pa·s

Because the motion of the journals centre are analysed in the Cartesian co-ordinate system ξ, η the expression of $\alpha_1, \alpha_2, \dot{\alpha}_1, \dot{\alpha}_2, \beta_1, \beta_2, \dot{\beta}_1, \dot{\beta}_2$ can be written in the form (2)

$$\beta_1 = \frac{\sqrt{(\xi_1 - \xi_3)^2 + (\eta_1 - \eta_3)^2}}{\varepsilon_1}$$

$$\dot{\beta}_1 = \frac{\left((\xi_1 - \xi_3)(\dot{\xi}_1 - \dot{\xi}_3) + (\eta_1 - \eta_3)(\dot{\eta}_1 - \dot{\eta}_3)\right)}{\beta_1 \varepsilon_1^2}$$

$$\dot{\alpha}_1 = \frac{\left((\dot{\eta}_1 - \dot{\eta}_3)(\xi_1 - \xi_3) - (\dot{\xi}_1 - \dot{\xi}_3)(\eta_1 - \eta_3)\right)}{\beta_1^2 \varepsilon_1^2}$$

$$\cos\alpha_1 = \frac{(\xi_1 - \xi_3)}{\beta_1 \varepsilon_1}; \quad \sin\alpha_1 = \frac{(\eta_1 - \eta_3)}{\beta_1 \varepsilon_1}$$

$$\beta_2 = \frac{\sqrt{(\xi_5 - \xi_7)^2 + (\eta_5 - \eta_7)^2}}{\varepsilon_2} \tag{2}$$

$$\dot{\beta}_2 = \frac{\left((\xi_5 - \xi_7)(\dot{\xi}_5 - \dot{\xi}_7) + (\eta_5 - \eta_7)(\dot{\eta}_5 - \dot{\eta}_7)\right)}{\beta_2 \varepsilon_2^2}$$

$$\dot{\alpha}_2 = \frac{\left((\dot{\eta}_5 - \dot{\eta}_7)(\xi_5 - \xi_7) - (\dot{\xi}_5 - \dot{\xi}_7)(\eta_5 - \eta_7)\right)}{\beta_2^2 \varepsilon_2^2}$$

$$\cos\alpha_2 = \frac{(\xi_5 - \xi_7)}{\beta_2 \varepsilon_2}; \quad \sin\alpha_2 = \frac{(\eta_5 - \eta_7)}{\beta_2 \varepsilon_2}$$

where:

$\varepsilon_1 = R_0 - R_1$ – absolute clearance (bearing “1”),
$\varepsilon_2 = R_0 - R_2$ – absolute clearance (bearing “2”),
ξ_3, η_3 – displacements of the sleeve centre of mass m_{p1} in ξ, η directions (bearing “1”),
ξ_1, η_1 – displacements of the journal centre in ξ, η directions (bearing “1”),
ξ_7, η_7 – displacements of the sleeve centre of mass m_{p2} in ξ, η directions, (bearing “2”),
ξ_5, η_5 – displacements of the journal centre in ξ, η directions (bearing “2”),
$\dot{\xi}_1, \dot{\eta}_1$ – journal centre velocities in ξ, η directions (bearing “1”),
$\dot{\xi}_3, \dot{\eta}_3$ – sleeve centre of mass m_{p1} velocities in ξ, η directions,
$\dot{\xi}_5, \dot{\eta}_5$ – journal centre velocities in ξ, η directions (bearing “2”),
$\dot{\xi}_7, \dot{\eta}_7$ – sleeve centre of mass m_{p2} velocities in ξ, η directions.

4. CONCLUSIONS

– Changes of visco-elastic properties in the mobile journal bearing can be controlled “forwards” and “backwards” by the apperance of resonances.
– The above mentioned effect can be achieved for weaker and stronger asymmetry of the applied load.
– Decreasing stiffness of the support leads to considerable chop of vibration amplitudes (outsaide the resonance). Practically, the amplitudes of journal tend to zero.

References

[1] Czołczyński K.: *High Stifness Gas Journal Bearings In Grinding Machines.* Machine Dynamics Problems, vol. 5, 1993, 65–87
[2] Starczewski Z.: *Self-Excited Vibration of a Rotor Supported in Journal Bearings with Flexibly Mounted Bushes.* Zeitschrift für Angewandte Mathematik und Mechanik, vol. 80, 2000, 301–302
[3] Przybyłowicz P.M., Starczewski Z.: *Simulation of Resonant Characteristics of a Rotor System Supported on Journal Bearings with Flexibly Mounted Bushing.* V Warsztaty Naukowe Polskiego Towarzystwa Symulacji Komputerowej, Zbiór referatów, 2000, 94–100
[4] Kurnik W., Starczewski Z.: *Hydrodynamical Forces in a Journal Bearing Corresponding to Combined Plane Journal Motion.* Machine Dynamics Problems, vol. 4, 1984, 89–102
[5] Kurnik W., Starczewski Z.: *Stability and Vibration of an Unsymmetric Rotor/Bearing Sysytem.* Zeitschrift für Angewandte Mathematik und Mechanik, vol. 74, 1994, 4, T46–T48