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STRESS ANALYSIS IN SLOTTED SPRINGS

SUMMARY

In this paper the modern construction of slotted springs was presented. It was proven that maximal stresses in such springs under load have higher values than the stresses calculated with previous method. The new model of phenomena and calculating method based on it was proposed. The new method gives results closer to FEM analysis than the previous.

Keywords*: slotted springs, structural stresses*

ANALIZA NAPRĘŻEŃ W SPRĘŻYNACH SZCZELINOWYCH

W artykule przedstawiono nowoczesną konstrukcję sprężyny szczelinowej, wycinanej z tulei. Wykazano, że dotychczasowa metoda obliczania naprężeń maksymalnych w sprężynach tego typu obarczona jest dużym błędem. Zaproponowano nowy model obliczeniowy zjawiska i oparty na nim sposób liczenia maksymalnych naprężeń. Sposób ten daje wyniki bliższe wynikom analiz przy użyciu MES.

Słowa kluczowe*: sprężyny szczelinowe, naprężenia*

1. INTRODUCTION

Metal elastic elements amount relevant group of joints in machines construction. They can secure against propagation of vibrations between joined elements, are used as shock-absorbers, flexible shafts, pads against unscrewing in screw joints, safety-cocks in safety clutches, dissipaters and accumulators of energy, etc. Such a huge usefulness of metal elastic elements extorts using many different constructions. At the same time, technological advance enables executing such constructions that formerly were not executable or just economically not justified.

The metalworking that has become very popular in recent years is technology of laser cutting. Helical springs coiled from wire with round cross-section are possible to execute only when nominal diameter of spring is much bigger than wire diameter because minimal coiling diameter is determined by material properties. It signifies that there is non-crossing border of ratio between stiffness of coiled helical spring and its outer diameter. Thus it is an advantage of springs cut from bush that they can have higher stiffness on the same area. Another advantage of such constructions is simplicity of fastening because mounting of such spring can have various construction forms (i.e. thread, pin joint etc.), it forms monolith with spring and works both when spring is stretched and compressed.

There are descriptions of such type of springs in literature, but formulas describing stiffness and stresses in constructions of springs discussed there are charged with big mistake, reducing safety, because stresses calculated according to these formulas are significantly lower than the real ones.

In [1] author gives formula for stresses during work of spring shown on Figure 1 in following form

$$\sigma_{ef} = k_g \frac{P \cdot D_N}{ab^2} \beta \qquad (1)$$

were:

- k_g – stress concentration coefficient depending on ratio between radius of cutting and height of horizontal coil r/a,
- P – force loading the spring,
- D_N – nominal diameter of spring, $D_N = D - b$,
- a, b – successively height and thickness of horizontal coil,
- β – coefficient depending on ratio b/a.

In order to verify the formula given above series of FEM analysis were conducted.

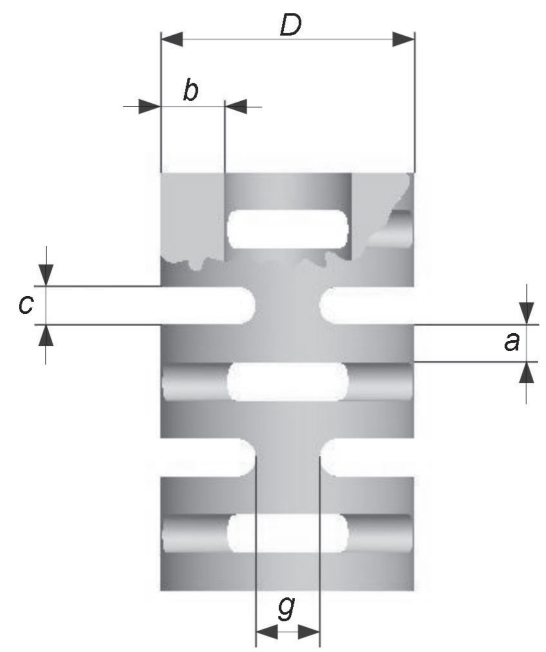

Fig. 1. Geometrical parameters of slotted spring

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Geometrical parameters in first analysis (Fig. 1) were as follows: a = 3 mm, b = 5 mm, D_N = 15 mm, g = 3 mm. Material properties are typical for steel: E = 206 000 MPa, ν = 0.3. The value of force loading the spring: P = 2000 N. Static analysis was conducted on model build from structural elements SOLID92 in environment of ANSYS software [4, 5]. One front of spring was fixed, the other one was loaded by force P. The results of analysis in form of reduced stresses according to Huber–Mises–Hencky (HMH) hypothesis are shown on contour plot Figure 2.

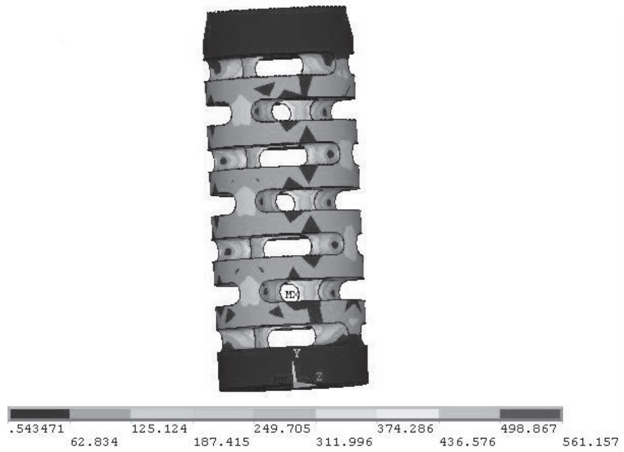

Fig. 2. Distribution of reduced stresses in spring (g = 3 mm), MPa

As it is shown on Figure 2 the highest substitutional stresses in spring have values exceeding 550 MPa. Stresses calculated according to the formula (1) for data given above amounts: σ_{ef} = 346 MPa. Therefor these stresses differ from the real ones about 60%.

During analysis of the geometry of spring it reveals that in rule (1) there is no coefficient taking into consideration the influence of thickness of supports g (Fig. 1) on stress state in spring material. This parameter has big influence both on load capacity of spring and on stresses and stiffness.

Distribution of substitutional stresses in spring different from the one described above with parameter g was shown on Figure 3.

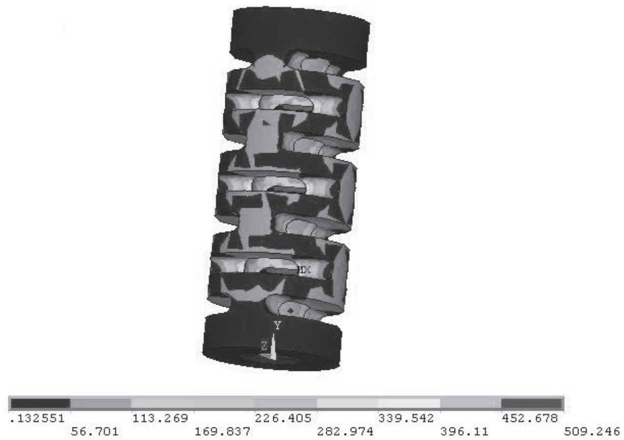

Fig. 3. Distribution of reduced stresses in spring (g = 4 mm), MPa

As we can see from contour plot on Figure 3 the maximum reduced stresses in spring with g = 4 mm achieve values σ_{HMH} = 509 MPa. According to formula (1) maximum stresses in both springs are equal and amount σ_{ef} = 345.8 MPa.

The example described above shows that present method of calculating stresses in slotted springs is not sufficiently accurate for technical applications and there is need to find different, more accurate calculation model.

2. SPRING GEOMETRY ANALYSIS

The following definitions were established for simplicity: the horizontal coils will be called rings and the elements supporting them will be called supports. Let us assume that considered spring is compression spring. The force applied to upper front of spring has to be carried to lower front. Thereby stress distributes in material of upper front and through the first two supports it flows to the first ring. From the first ring the load is carried through the next two supports to the next ring and so on until it reaches lower front. From equilibrium conditions it results that vertical component of stresses in supports multiplicated by their cross-section and quantity of supports joining the same two rings has the value of entire force acting on spring. Because there are two supports on the same level, each of them carries the half of entire load. Considering only three neighbouring rings joined with two pairs of supports twisted against each other by 90^o it can be confirmed that it is configuration with two perpendicular planes of symmetry, crossing each other at spring axis. It means that supports are not bent to sides as a result of pressing the spring. Therefore only quarter of ring with supports propping up it can be modelled as a fixed on its edges beam with supports shifted parallely to each other (Fig. 4). It was assumed that angular deflections of rings are small, what is in compliance to reality.

Fig. 4. Simplified model of quarter of the unfolded ring

As it is known in the middle of such beam (point k) the value of bending moment is equal to zero. If we cut this beam in the middle we achieve two beams with one end fixed and the second one free. In order to keep the same values of moments in these cut beams as in the one not cut the vertical forces in value of quarter of entire load have to be applied to free ends of beams (Fig. 5).

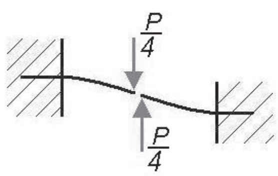

Fig. 5. Further analysis of simplified model

Because in real conditions configuration is not flat but spatial therefore stresses appearing in considered element will have complex nature. The highest stress concentration will take place in support neighbourhood. These stresses are results of bending, twisting and shearing of the ring as well as bending and compression of support.

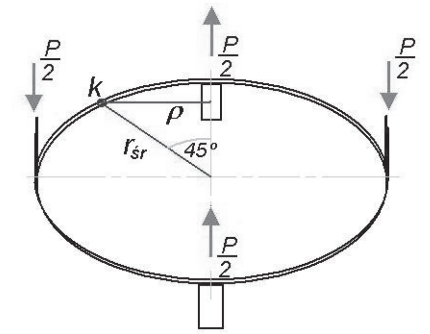

Fig. 6. Spatial model of the ring with supports

Figure 6 presents considered configuration in space. It illustrates formulas (2), (3), (6)–(8) given below. These formulas let calculate values of stresses mentioned above.

Bending stresses in ring:

$$\sigma_{gp} = \frac{M_g}{W_x}; \; M_g = \frac{P}{4}\left(\rho - \frac{g}{2}\right); \; W_x = \frac{ba^2}{6};$$

$$\rho = r_{śr}\cos 45^\circ; \; r_{śr} = \frac{D-b}{2};$$

$$\sigma_{gp} = \frac{3P\left(\rho - \frac{g}{2}\right)}{2ba^2} \tag{2}$$

Shear stresses in ring

$$\tau_t = \frac{P}{4ab} \tag{3}$$

Torsional stresses in bar with rectangular cross-section are defined by formula given by de Saint-Venant [3]

$$\tau_{Ms} = \frac{M_s}{\mu ab^2} \tag{4}$$

where:

- a – longer side of bar section,
- b – shorter side,
- μ – coefficient is given as

$$\mu = \frac{a}{3a + 1{,}8b} \tag{5}$$

$$M_s = \frac{P}{4} r_{śr}(1 - \cos 45^\circ) \tag{6}$$

Compressive stresses in support

$$\sigma_N = \frac{P}{2gb} \tag{7}$$

Bending stresses in support

$$\sigma_{gs} = \frac{3P \cdot r_{śr}(1 - \cos 45^\circ)}{gb^2} \cong \frac{0.879\,\mathrm{P}r_{śr}}{gb^2} \tag{8}$$

Only (2)–(4) formulas refer to stresses in ring, in the place where by round it turns into a support. Formulas (7) and (8) refer to stresses in the middle part of support.

It is sensible to make steel spring of any shape but under one circumstance. If the spring reaches yield stress in any point and its strain is smaller than strain of simple bar with the same height – it does not make sense, because it is easier to produce simple bar than complex shaped spring. After performing many analysis it was confirmed that executing slotted springs of geometrical proportions different than $1 \le g/a \le 1.5$ and $1 \le a/c \le 3$ makes no sense for reasons mentioned above. Also it was assumed that rounds between ring and support are full rounds as it is the best solution regarding fatigue strength.

The state of stress in the place where the ring joins the support was taken into account in mathematical model. There is complex state of stress in this section.

In order to calculate the value of substitutional stresses the Huber–Mises–Hencky hypothesis was used according to which substitutional stresses are given by formula [2]

$$\sigma_{HMH} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2)} \tag{9}$$

In considered case formula (9) equals

$$\sigma_{HMH} = \sqrt{\sigma_{gp}{}^2 + 3(\tau_T + \tau_{Ms})^2} \tag{10}$$

The formula (10) does not take into account stress concentration depending on geometry. In order to take it into consideration two empirical coefficients δ and α were introduced. Their values are based on many numerical analysis. First one from them takes into account inequality of stresses distribution in section and depends on ratio b/D. Second one takes into account stress concentration and depends on ratio a/c.

Finally formula to calculate maximum substitutional stresses in slotted spring gets shape

$$\sigma_{E\max} = \alpha\delta\sigma_{HMH} \tag{11}$$

where:

- α – stress concentration coefficient receives values:
  $\alpha = 1$ for $a/c = 1$,
  $\alpha = 1.3$ for $a/c = 1.5$,
  for indirect values of ratio a/c interpolation has to be performed;
- δ – inequality of stress distribution coefficient given by formula

$$\delta = 1 + \sqrt{\frac{b}{D}}.$$

Comparison of maximum substitutional stresses from FEM analysis with values of stresses calculated using formula (1) and values of stresses calculated according to formulas given by author are shown in Table 1.

Table 1

No.	P [N]	D [mm]	b [mm]	a [mm]	g [mm]	c [mm]	σ_{HMH} [MPa]	σ_{ef} [MPa]	$\sigma_{E\,\max}$ [MPa]	$\frac{\sigma_{HMH}}{\sigma_{ef}}$	$\frac{\sigma_{HMH}}{\sigma_{E\,\max}}$
1	2000	20	5	3	3	3	560	348	513	1.6	1.09
2	2000	20	5	3	4	3	510	348	477	1.46	1.07
3	2000	20	5	3	3	2	700	388	667	1.8	1.05
4	500	20	1	3	3	3	1390	1852	1105	0.75	1.26
5	500	20	2	3	3	3	464	517	410	0.9	1.13
6	2000	20	3	3	3	3	1105	999	963	1.1	1.15
7	2000	20	4	3	3	3	785	565	680	1.4	1.15
8	500	15	1	3	3	2	1255	1522	1097	0.82	1.14
9	500	15	1.5	3	3	2	500	700	595	0.7	0.84
10	1000	25	2.5	3	3	2	1080	957	1008	1.12	1.07
11	2000	25	5.5	3	3	2	820	403	787	2.04	1.04

3. CONCLUSIONS

Results placed in Table 1 show that calculations according to formula (11) are considerably more accurate than calculations according to formula (1). It also indicates reasonableness of proposed mathematical model.

The calculating method proposed by author is more time-consuming than method given by E.I. Rivin, but it is worth using for its high accuracy. The proposed method takes into account some geometrical features that have high influence on state of stresses and they are not taken into account in formula [1].

High precision of dimensions and lack of self stresses in slotted spring material are followed by a very high accuracy of characteristic, not possible to achieve in helical springs coiled from wire. Thanks to these features, slotted springs can be applied in systems, where high accuracy of positioning is necessary. But because of very unequal stress distribution as it was indicated during FEM analysis and explained above only slotted springs with the following geometrical proportions are useful in machines construction:

– ratio between thickness of support and height of ring $a \le g \le 1.5a$,
– ratio between height of ring and height of slot $c \le a \le 3c$.

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