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VIBRATION ENERGY FLOW IN RIBBED PLATES**

SUMMARY

The paper presents obtained from literature review formulations on structural intensity calculations. The formulas involve the loads (forces and moments) and strains (linear and angular) which enabled the evaluation of structural surface intensity for beams, shells and plates considered here as simple constructional elements. The numerical method of intensity evaluation was based on complex modal analysis with use of finite element method. There are presented results of the calculations which lead to the assessment of distribution of structural intensity vectors on the surface of simply supported, ribbed rectangular steel plate. The model included the source of vibrations (force excitation) and sink of energy (damper) with known position of application. The changes of finite elements grid density enabled detailed analysis of total vibration energy flow in analysed plate. Such solved problem was intended to show the usability of structure surface intensity method in diagnostics of joints in mechanical constructions.

Keywords*: energy of vibrations, structural intensity*

PRZEPŁYW ENERGII DRGANIOWEJ W PŁYTACH UŻEBROWANYCH

W pracy przedstawiono zależności służące do obliczeń natężenia dźwięków strukturalnych, uzyskane na podstawie danych literaturowych. Podane zależności wiążące natężenie dźwięków strukturalnych z odkształceniami (liniowymi i kątowymi) oraz obciążeniami (siłami i momentami) belek i płyt pozwalają wyznaczyć wartości składowych wektora natężenia dla typowych jedno- i dwuwymiarowych elementów konstrukcyjnych. Omówiono metodę wyznaczania wartości natężenia na podstawie zespolonych parametrów modalnych uzyskiwanych za pomocą analizy modalnej z zastosowaniem metody elementów skończonych. Podano przykład obliczeniowy, którego celem było wyznaczenie rozkładu wartości wektora natężenia dla użebrowanej płyty prostokątnej swobodnie podpartej na obrzeżu. W modelu przyjęto siłę wymuszającą o charakterze sinusoidalnym oraz tłumik dynamiczny o zadanej charakterystyce tłumienia. Rozwiązanie problemu badawczego pomogło stwierdzić, czy metoda natężeniowa jest przydatna do diagnostyki miejsc połączeń elementów konstrukcji mechanicznych.

Słowa kluczowe*: energia drgań, strukturalne natężenie dźwięku*

1. INTRODUCTION

The quantity of structural intensity was introduced with the purpose of description of acoustic phenomena with the application of vector fields. It has found the exceptional application in investigations of vibration energy flow in deformable elastic bodies. Structural intensity represents the averaged in time net mechanical energy flow through the unit area perpendicular to the direction of flow [4]. Analysis is conducted for the thin-walled elements. Analysis in deeper parts of constructional element is out of practical meaning. Analysis of spatial distribution of structural intensity vector fields enables determination and location of paths, sources and sinks of energy of vibrations in mechanical systems. It gives the particular information on the streams of energy flow much advantageous than the other methods used earlier to such kind of analysis.

Development of the structural intensity applications was noted in the beginning of nineties when have appeared works showing the methods of numerical calculation [3, 6]. The fundamental dependencies were based on assumption of one or two dimensional structure as beam or plate and the modal model. The possibility of analysis of structural intensity is very promising due to the prospects of use of elaborated finite element models of mechanical structures. Practical numerical examination of vibration energy flow in complex structures haven't been solved satisfactory till now. The main problems are found in necessity of consideration of complex boundary conditions and complexity of the analyzed real structures.

2. STRUCTURAL INTENSITY IN THIN-WALLED ELEMENTS

For steady state of vibration the surface structural intensity can be evaluated as the complex quantity [2]:

$$\tilde{S}_{\sigma kl\,vl}(\omega) = I_k(\omega) + jJ_k(\omega) \tag{1}$$

where:

$\omega = 2\pi f$ – angular frequency,
f – frequency of vibrations,
$\tilde{S}_{\sigma kl\,vl}(\omega)$ – cross spectrum function of complex components of stress and particle velocity.

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In practical cases there is analyzed only the real part of structural intensity called active intensity. Only the real part is responsible for the energy transfer. The imaginary part is connected with the standing waves and represents the energy conservation in the system. Instantaneous value of real part of structural intensity $i_k(t)$ is time dependent vector quantity equal to the change of energy density in the infinitively small volume [6]. Its k-th component is given by the equation:

$$i_k(t) = \sigma_{kl}(t) v_l(t), \quad l = 1, 2, 3 \tag{2}$$

where:

$v_l(t)$ – l-th component of velocity vector,
$\sigma_{kl}(t)$ – kl-th component of stress tensor.

Averaged in time value of (2) represents the net energy flow in mechanical structure [6]

$$I_k = \langle i_k(t) \rangle \tag{3}$$

in the direction of k-th coordinate of rectangular frame of reference corresponding to analyzed constructional element.

Components of structural intensity vector are calculated for the beams, plates and shells as the functions of variables: bending and twisting moments, shear forces, linear and angular displacements.

2.1. Structural intensity in plates

Vibrations of thin plates are dominated by the bending and longitudinal waves. For longitudinal waves displacements do not vary with the depth of the plate. In case of bending vibrations there are added up displacements caused by the shear forces, bending and twisting moments. Structural intensity for a plate subjected to all kind of loads is the result of sum of three components. For the bending motion of the plate displacements are related to the in-plane displacements as follows:

$$u_x = \frac{h}{2}\frac{\partial u_z}{\partial x}, \quad u_y = -\frac{h}{2}\frac{\partial u_z}{\partial y} \tag{4}$$

In the numerical calculations by the means of finite element method the structural intensity is related to the neutral middle plane of plate. In general case the plate is subjected to the bending, tension and twisting motions.

$$\begin{aligned} I_x &= \frac{\omega}{2}\mathrm{Im}\left[\tilde{N}_x \tilde{u}_0^* + \tilde{N}_{xy} \tilde{v}_0^* + \tilde{Q}_x \tilde{w}_0^* + \tilde{M}_x \theta_y^* - \tilde{M}_{xy} \theta_x^* \right] \\ I_y &= \frac{\omega}{2}\mathrm{Im}\left[\tilde{N}_y \tilde{v}_0^* + \tilde{N}_{yx} \tilde{u}_0^* + \tilde{Q}_y \tilde{w}_0^* - \tilde{M}_y \theta_x^* + \tilde{M}_{yx} \theta_y^* \right] \end{aligned} \tag{5}$$

where:

$\tilde{N}_x, \tilde{N}_y$ – tension forces,
$\tilde{N}_{xy} = \tilde{N}_{yx}$ – internal forces,
$\tilde{Q}_x, \tilde{Q}_y$ – shear forces,
$\tilde{M}_{xy} = \tilde{M}_{yx}$ – torques,
$\tilde{M}_x, \tilde{M}_y$ – bending moments.

There are assumed small deformations which enable the superposition of independent displacements for flat finite element of plate or shell type.

3. MODAL APPROACH IN STRUCTURAL INTENSITY CALCULATION

Complex displacements and stresses needed for structural intensity evaluation can be obtained by the modal approach. Software used in computation with FE method uses the real stiffness and mass matrices. As a result are obtained the real displacements and stresses. Procedure of structural intensity calculation is based on complex response of the structure with the modal representation. Damping is considered in two forms. Structural internal damping is taken as modal damping. The additional damping, placed in known location is treated as external loading

$$\{\tilde{R}\} = -[\tilde{S}]\{\tilde{X}\} \tag{6}$$

where:

$[\tilde{S}]$ – additional stiffness matrix,
$\{\tilde{X}\}$ – displacement vector.

There was assumed excitation by the sinusoidal force in form $\{\tilde{F}\}e^{j\omega t}$. The equation of motion of structure including two models of damping is [3]

$$\omega^2 [M]\{\tilde{X}\} + [K]\{\tilde{X}\} = \{\tilde{F}\} + \{\tilde{R}\} \tag{7}$$

where $[M]$ and $[K]$ are the real mass and stiffness matrices.

The complex response of the structure is obtained from the calculations of eigenvalues. This is done with the post-processor program operating on values obtained from finite element software in purpose to compute the complex modal response in modal coordinates. The internal proportional damping is considered by use of complex eigenfrequencies. The real values of matrix $[\omega_0^2]$ are replaced with the values including modal loss factor η_i for the i-th mode and complex eigenfrequencies are equal to $\tilde{\omega}_{0i}^2 = (1 + j\eta_i)\,\omega_{0i}^2$.

Values of structural intensity by the definition (2) depend on particle velocity and stress. As the result of calculations one can get the displacements in nodes and stresses in the point inside the finite element. In the process of calculations they values are need to be found in the same points. Because the accuracy of stress calculations is less than for displacements, the values of structure response are taken in points of highest accuracy for stress. Displacements in theses points are counted with the help of shape function for finite element from related to them relative displacements of element. Shape function matrix is in general case related to the local system of coordinates for finite element. Vector of modal displacements have to be defined in the same coordinate system. Displacements inside finite element are also related to the local system of coordinates. Procedure application allows to find the vector containing the proper displacements of element centroides for given eigenvalue.

Complex displacements and stresses both related to the element centroide necessary for evaluation of structural intensity are calculated by modal superposition. Obtained as the calculation result the values of structural intensity are of complex nature and expressed in the local coordinate system connected with the finite element and need to be transformed into the general system of coordinates connected with the analyzed structure.

4. CASE OF RECTANGULAR RIBBED PLATE

For the purpose of structural intensity usability verification in application to the identification of the constructional element's joints the numerical experiment was performed. The model chosen for analysis was three dimensional structure of simply supported ribbed rectangular plate. The plate has mechanical properties as structural steel and dimensions of 1.5 m in width and 3 m in length with thickness of 10^{-2} m. The finite element method model was prepared using the NASTRAN software. Plate was divided into 4608 the same square shell elements of QUAD4 type. The excitation force and damping force were introduced to the model. The harmonic excitation force was attached to the plate in place indicated on figures by the star. The damping force proportional to the velocity of vibration was attached to the plate in place indicated on figures by the triangle. The magnitude of damping force was set to 10^3 N. The direction of its action was chosen perpendicularly to the plane of plate. There was chosen the model of simply supported plate. The only feasible motion was the rotation around the edges of the plate. There were eliminated the translation motions in any direction edges of plate. Such model was in accordance with the most technical cases of plate element mounting in practice.

4.1. Results of calculations

The calculation has been done for 100 first mode shapes. In cases of low number of mode shapes there were observed significant changes in distribution of vectors for the same density of net of elements. The positions of excitation and damping forces attachments were clearly shown by the structural intensity vectors distribution. It was assumed that for the number greater than 60 mode shapes the numerical model of plate is exact and in a proper way represents the vibration energy flow in the system. The magnitude of structural intensity vectors decease with increase of number of finite elements. This inclined to the conclusion that in energy flow analysis it is not sufficient to observe only the intensity vectors. The better measure of energy flow seamed to be the total energy flow through the closed area around the places of excitation and damping or through the whole width of the plate. For that reason the method of summation of structural intensity vector magnitude along the path surrounding the source or through the whole cross section of the element should be applied. Significant changes were observed for the area around the excitation and sink. This is due to the changes of stress values in the region of point force attachment. For the other cross sections there were observed only slight change in total energy values. The obtained results of calculations done for ribbed rectangular plate are shown on Figures 1–5.

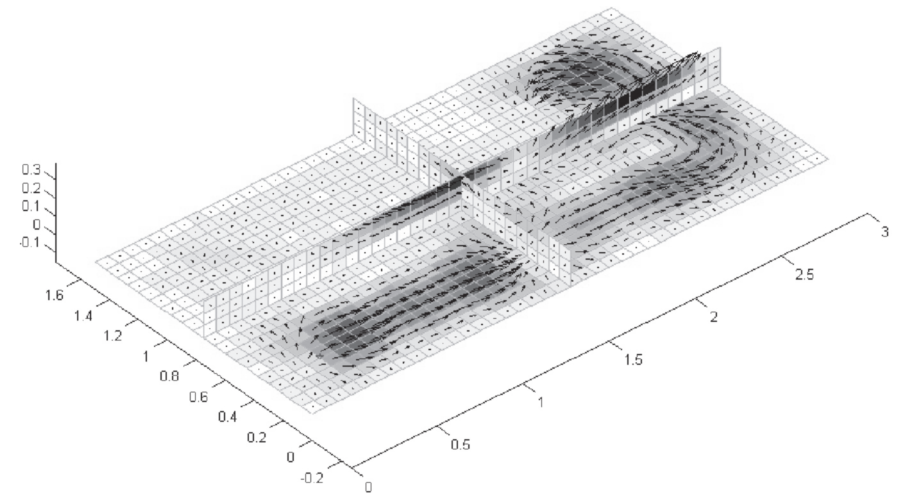

Fig. 1. Distribution of structural intensity vectors for simply supported ribbed rectangular plate. Gray scale is used to vector value representation

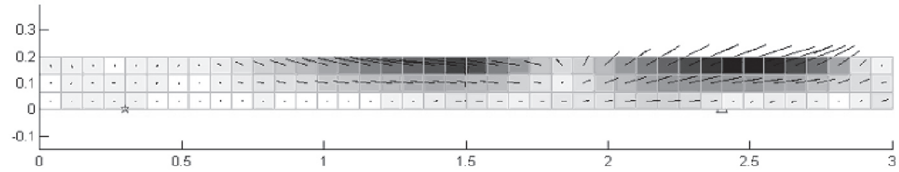

Fig. 2. Distribution of structural intensity vectors for longer rib of simply supported ribbed rectangular plate

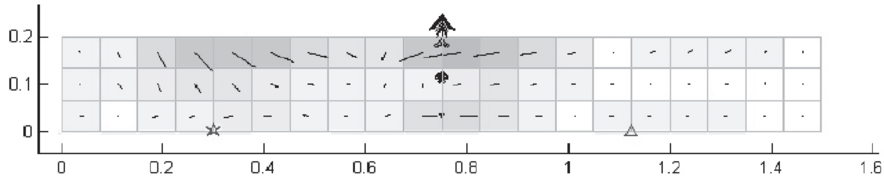

Fig. 3. Distribution of structural intensity vectors for short rib of simply supported ribbed rectangular plate

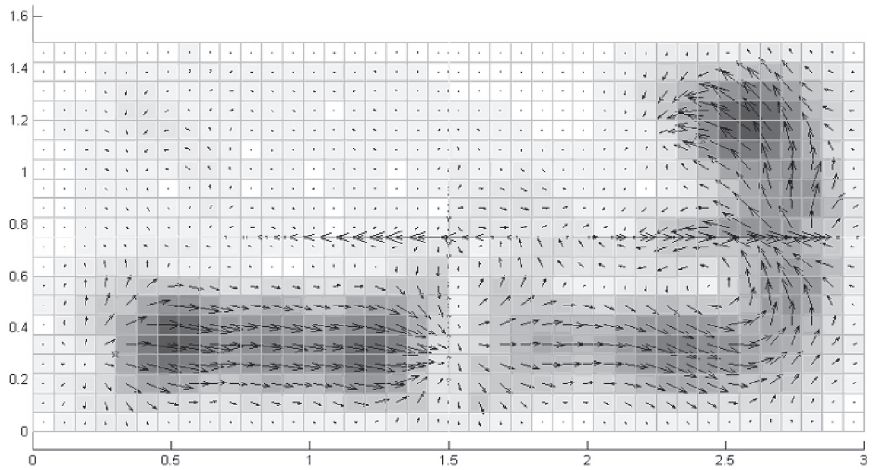

Fig. 4. Distribution of structural intensity vectors for simply supported ribbed rectangular plate. Gray scale is used to vector value representation

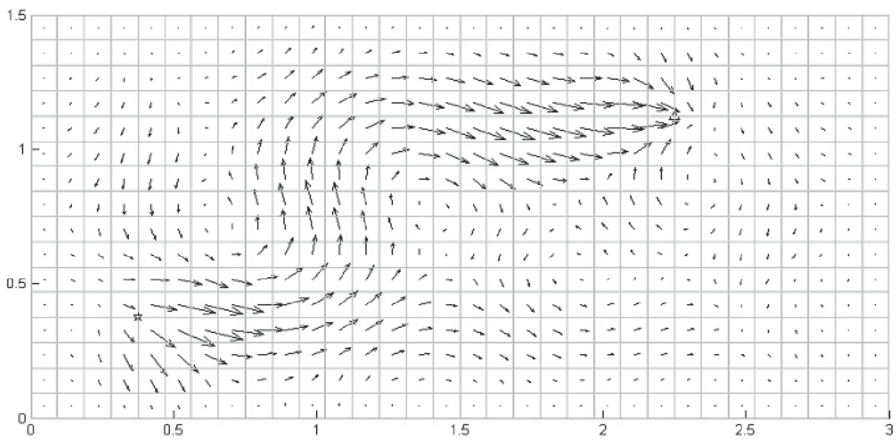

Fig. 5. Distribution of structural intensity vectors for rectangular plate without ribs

The main target of the analysis of the rectangular plate with ribs was the testing the distribution of structural intensity vectors especially in the regions of ribs attachment.

For the comparison of results there is presented distribution of intensity vectors for plane rectangular plate of the same dimensions. Obtained distribution of intensity vectors for ribbed plate is significantly different form result got for plate without ribs. Distributions of intensity vectors in places near by the places of excitation and damping are similar for both cases of plates. The positions of excitation and damping forces attachments were clearly shown. However in all cases the distribution of intensity vectors in the middle part has similar shape. The distribution of vectors has shown distinctly the direction of energy flow from the excitation to the damper. For the plates there were observed the vortices of vectors field in the region far from excitation and damping.

Comparing obtained for the rectangular plates results of calculations presented here as the distribution of structural intensity vectors over the area of plate one can notice that there is no similarity with the distribution of displacements for each mode shape. This conclusion results from the fact that mode shapes are connected to the standing waves formed in the plate. In the system with small internal damping there is not observed the energy flow or at least it is very low. The effect of energy flow between the structure elements is observed only for systems with high internal damping caused e.g. by the local abrupt changes of properties or localized damping force.

5. CONCLUSIONS

Presented method of structural intensity vector calculation enables its evaluation for chosen frequency range and mode shapes [6]. Derived dependencies connect the structural intensity with linear and angular displacements, forces and moments exerted to the structures like beams, plates and shells. The calculations are done with the application of complex modal parameters counted by the use of the numerical modal analysis based on finite element method.

Distribution of intensity vectors for rectangular plate with two perpendicular plates allows for conclusions:

– In region far from connections, external damping or excitation the directions of intensity vectors are ordered, parallel and steady regarding the vector direction and magnitude. It means steady regular flow of vibration energy in one direction defined by the sense of vectors.
– Ribs disturb the direction of energy flow forcing it to the directions parallel to the ribs. This is shown by the comparison with the.
– In regions separated by the ribs without excitation or damping the vorticity of vector field can be observed. This effect of vortices represented by the rotational distribution of intensity vectors means the circulation and conservation of vibration energy in the mechanical system.
– The energy flow can be observed also in the ribs. The presented case of analysis gives too less information for the assessment of energy flow in ribs.

The distribution of structural intensity vectors gives the qualitative characteristic of vibration energy transportation in mechanical systems. Introduction of an additional measure in form of integral of magnitude structural intensity vector component perpendicular to the certain closed surface e.g. element cross section enables the quantitative assessment of energy transfer paths and its balance in the structure.

The main disadvantage of the method is its poor convergence. The number of modes which are used in calculation process should be properly chosen and is much bigger than for standard modal analysis. In case of analysed plates the number of mode shapes should be grater than 60. The abrupt changes of stress which occur in points close to the excitation or discontinuities of structure can not be well described by the limited number of lower modes. This makes the main problem of calculation method exactitude.

The method of analysis of structural intensity distribution enables the investigation in the regions of high concentration of vibration energy flow which consequently is exposed to the risk of damage or is propagating the sound waves to the environment. It can be also considered as the identification of the regions for application of additional damping in purpose of lowering of vibration level and resulting noise radiation.

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