MECHANICS
Vol. 25 No. 4 2006

Jerzy STOJEK[*]

ESTIMATION OF THE WEAR IN POSITIVE-DISPLACEMENT PUMPS BY THE TIME SERIES METHOD

SUMMARY

The article discusses the possibility to assess the wear characteristics in multi-piston positive-displacement pumps operating at constant delivery rate, basing on an analysis of the measured outlet pressure variations. Parameter estimates of the selected parametric models of pressure variations were determined. Comparing the values of parameters, the wear and tear characteristics were determined in the examined pumps.

Keywords*: positive-displacement pumps, diagnosis of machine operation, time series, parametric models*

OCENA STANU ZUŻYCIA WIELOTŁOCZKOWYCH POMP WYPOROWYCH METODĄ CIĄGÓW CZASOWYCH

W artykule opisano możliwości diagnozowania stanu zużycia wielotłoczkowych pomp wyporowych na podstawie zmierzonych przebiegów ciśnień w przewodzie tłocznym każdej z badanych pomp. Otrzymane ciągi czasowe opisane zostały modelem autoregresji z ruchomą średnią (ARMA). Wyznaczone w procesie identyfikacji estymaty parametrów modeli każdego z analizowanych ciągów zostały porównane z parametrami pompy idealnej (wzorcowej), po czym nastąpiła klasyfikacja stopnia zużycia, badanych pomp. W artykule opisano przyjęty przez autora model pompy idealnej, wyjaśniono pojęcie ciągu czasowego oraz modelu parametrycznego, opisano najczęstsze uszkodzenia występujące w pompach wielotłoczkowych. Przedstawiono najczęściej stosowane metody diagnozowania pomp.

Słowa kluczowe*: pompy wyporowe, diagnostyka maszyn, szeregi czasowe, modele parametryczne*

1. INTRODUCTION

Pressure is one of the most frequently measured signals in the systems of power hydraulics. The periodical measurements of pressure variations in individual system elements can be the source of valuable information on the wear and tear characteristics of operating parts. This signal is free from the drawbacks commonly encountered in, applied until now in diagnosis, vibration signals [1], whose value is greatly affected by the signals of disturbances, making the diagnostic process much more complicated. The problem of diagnosis of hydraulic systems usually consists in assessment of the wear and tear rate suffered by a most critical element running in this system. In common hydraulic drive and control systems, i.e. in the systems without any electro-hydraulic elements (servovalves), the most critical element is a positive-displacement pump. This fact has prompted the authors of this article to attempt at an estimation of the wear and tear rate in multi-piston positive-displacement pumps, basing on the analysis of time series of the measured pressure values.

2. DESCRIPTION OF THE EXAMINED OBJECT

The object of examinations was a set of six multi-piston axial-flow pumps designed for operation at a rated flow of Q_n = 360 dm^3/min and pressure p_n = 40 MPa. In practical application these pumps were running at a constant delivery rate in the system of hydraulic press power feed.

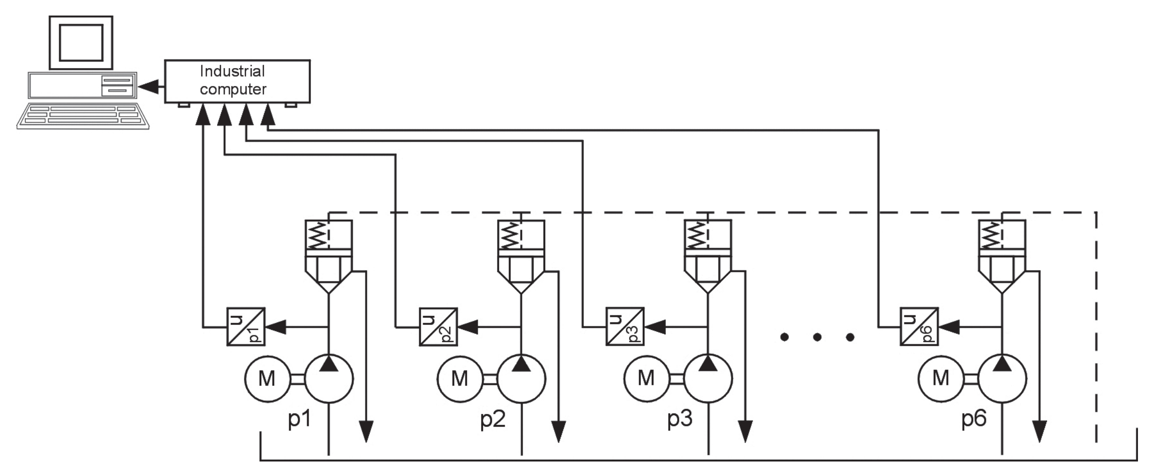


Fig. 1. Measurement system

---

[*] AGH University of Science and Technology in Cracow, Poland, Department of Process Control

Loading of each pump during its running was imposed by the operating cycle of a press. Switching of pump operation from the pressure run to a pressure-free run (and vice versa) was done by means of logical valves placed directly on the body of each pump.

To determine the wear and tear rate in the examined positive-displacement pumps, the pressure value was measured on an outlet of each pump (Fig. 1). During measurement, each pump was operating under light-duty conditions resulting from a drop in pressure due to the presence of oil flowing through a logical valve. The pressure was measured by means of piezoelectric pressure transducers which were co-working with an industrial computer, equipped with a 16-bit A/C transducer measurements were taken at a sampling frequency of f_p = 10 kHz; the time series of pressure values were stored in the memory of a PC computer, where they were subjected to further processing by means of a Matlab/Simulink programme.

3. MATHEMATICAL MODEL OF MULTI-PISTON, AXIAL-FLOW PUMP

To make assessment of the wear and tear rate in the examined pumps, a mathematical model of an ideal positive-displacement pump was designed. The model was assumed to be an object free from any faults, and it was used as a basis for further diagnostic analysis. As a parameter describing the pump operating characteristics was commonly adopted its delivery rate examined in function of the drive shaft angular displacement [3]. Remembering that each of the examined pumps had seven pistons (z = 7), spaced at an even distance in the scale of $\alpha = 51^{\circ}42'$, which at a rotational speed of the pump shaft n = 1500 rev/min gave the specific delivery q = 0.00025 m^3, the pump delivery for the actual angle of its shaft rotation φ was determined as:

$$\left.\begin{aligned} Q &= 0.00280499\sum_{k=0}^{3}\sin(\varphi + k\cdot 51^{\circ}42') \ \text{for} \quad 0^{\circ} \le \varphi \le 25^{\circ}71' \\ Q &= 0.00280499\sum_{k=0}^{2}\sin(\varphi + k\cdot 51^{\circ}42') \ \text{for} \ 25^{\circ}71' \le \varphi \le 51^{\circ}42' \end{aligned}\right\} \quad (1)$$

The system of equations (1) was processed in a Matlab/Simulink toolbox.

4. PARAMETRIC MODELS OF STATIONARY TIME SERIES

For the description of stationary time series, a model of autoregression (**AR**) is most frequently used. In this model, the current value of the examined process is a finite, linear combination of the values obtained previously for this process and of a residual which is white noise [2].

An **AR** model written in the form of difference equation is expressed by formula [2]

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \cdots + a_n y(n-na) = e(n) \quad (2)$$

or in shorter form by

$$y(t) = \frac{1}{A(z^{-1})} e(n) \quad (3)$$

where:

$y(n)$ – the value of time series at instant n,
$e(n)$ – unknown signal generating the time series- of a zero mean value and constant variation, i.e. the white noise,
$A(z^{-1})$ – characteristic polynomial of the delay operator z^{-1} (the, so called, polynomial of autoregression) of n_a order,
$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na},$$
$a_1, a_2, \ldots, a_{na}$ – the searched estimates of model parameters.

Owing to its simplicity, the **AR** model has been successfully applied in the description of simple physical phenomena. It has, however, an important drawback, viz. the obtained estimates of characteristic polynomial are burdened with a constant error. This means that there are differences (the, so called, residuals) between the real measuring signal and the signal obtained from a model describing this signal. The possibility of reducing significantly the error of description (the residual) in time series has been provided by a model of autoregression with moving average, i.e. the, so called, **ARMA** model (Fig. 2).

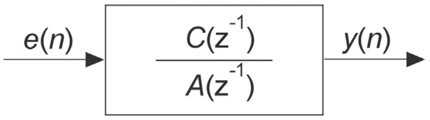


Fig. 2. An ARMA model of time series

A general form of **ARMA** model (4) and its developed form (5) are shown below [2]:

$$y(t) = \frac{C(z^{-1})}{A(z^{-1})} e(n) \quad (4)$$

$$\begin{aligned} &y(n) + a_1 y(n-1) + a_2 y(n-2) + \ldots + a_{na} y(n-na) = \\ &= e(n) + c_1 e(n-1) + \ldots + c_{nc} e(n-nc) \end{aligned} \quad (5)$$

where:

$C(z^{-1})$ – the polynomial of moving average of the delay operator z^{-1} of nc order; in the present studies it has been assumed that the moving average will be computed from the last four values of the white noise, which means nc = 4;
$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_{nc} z^{-nc};$$
$a_1, a_2, \ldots, a_{na}$, $c_1, c_2, \ldots, c_{nc}$ – the searched estimates of model parameters.

5. DETERMINATION OF ESTIMATES OF MODEL PARAMETERS

Basing on the measured pressure runs in the examined pumps and on the output signal from an ideal model pump, the next step consisted in describing the obtained time series with the adopted parametric models. At the beginning, the stationarity of each time series was checked, examining the runs of the respective covariance functions plotted for each of the time series (Fig. 3).

Before proceeding to identification of the parameters of characteristic polynomial and of the polynomial of moving average in **ARMA** model, an order of the characteristic polynomial (being also an order of the model), which would give the assumed matching error of time series described with the adopted model, was determined.

By increasing gradually the order of the characteristic polynomial, the loss function (matching error) was decreasing to a preset minimum value which, upon having been reached, did not result in any visible decrease of the value of the matching error with further increase of an order of equation.

Basing on the assumed value of the matching error δ ($\delta \leq 0.001$), the order of the polynomial $A(z^{-1})$ was determined for an ideal model pump. The assumed matching accuracy was achieved for an order of $na = 8$.

Next, parameters (a_1, ..., a_8) were estimated for characteristic polynomial and for a moving average (c_1, ..., c_4) using the method of minimum error mean-square.

Applying the test of forecast error whiteness (the test of residual errors), the consistency of the determined estimator of **ARMA** model parameters was checked. The run of autocorrelation function of the residuals of ARMA model confirms the consistency of the determined estimator (Fig. 4).

6. RESULTS

The determined estimates of the parameters of **ARMA** models of the individual pumps along with the date of their incorporation into a hydraulic system of the equipment and the value of final matching error δ are compiled in Table 1.

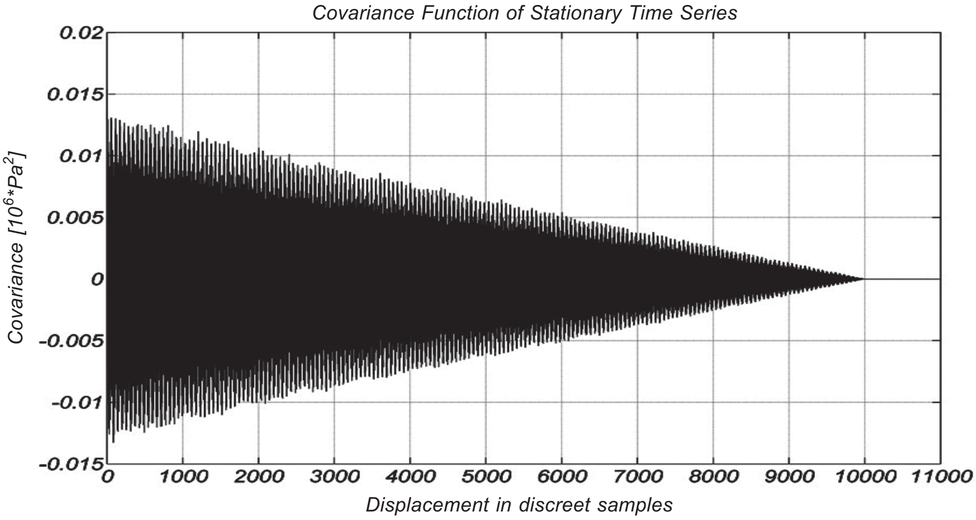


Fig. 3. Stationary check of time series for pump no. 6

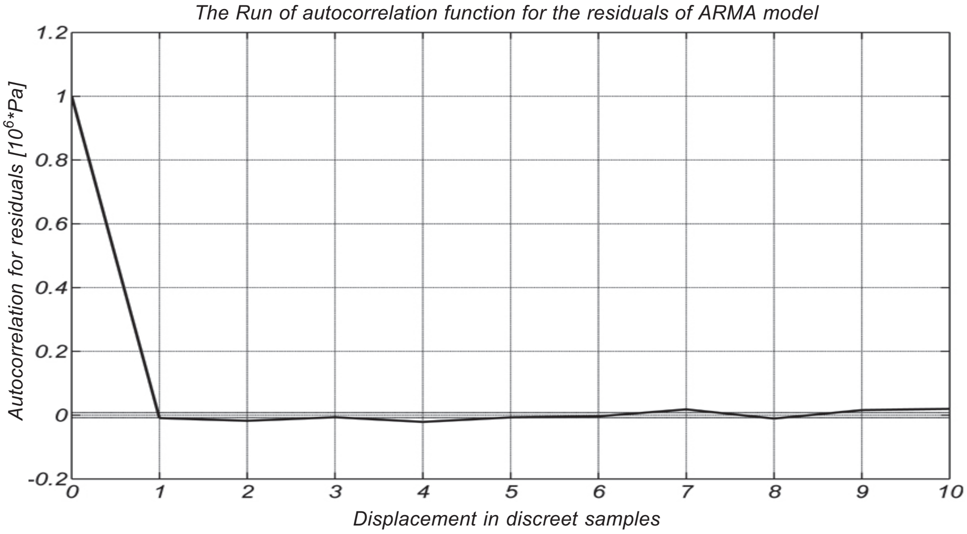


Fig. 4. The run of autocorrelation function for the residuals of ARMA model

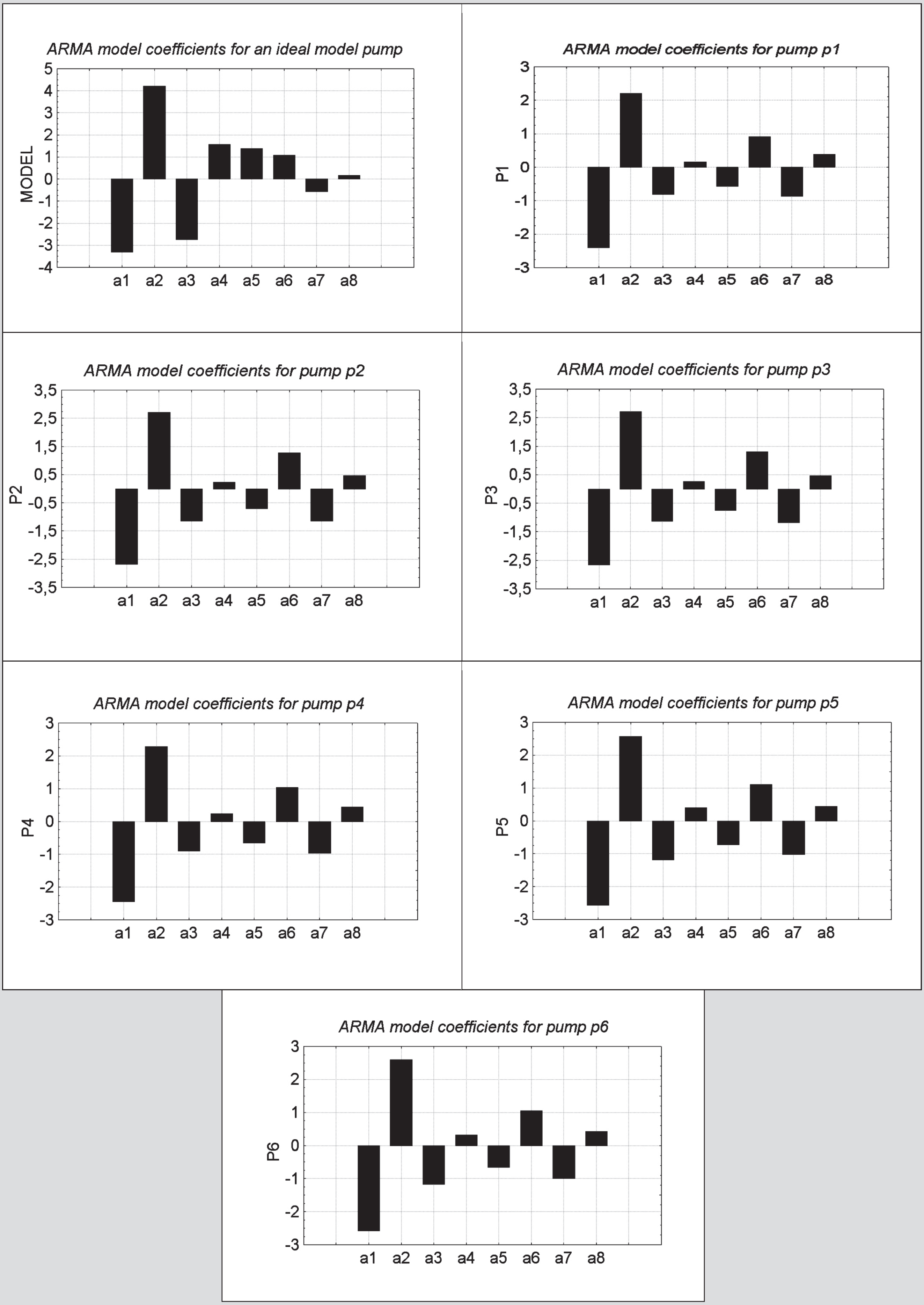
Table 1

Pump model	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–3.2989	4.2068	–2.7498	1.5664	–1.3855	1.07013	–0.5601	0.1681
	c_0	c_1	c_2	c_3	c_4	$\delta = 0.000011$			
	1	–1.6624	0.5902	–0.094	0.1679				
Pump no. 1	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.402	2.2046	–0.8135	0.1536	–0.5709	0.9148	–0.8642	0.3916
	c_0	c_1	c_2	c_3	a_4	Engaged in: 1998 (master pump) $\delta = 0.0011$			
	1	–2.268	2.1179	–0.7972	0.0275				
Pump no. 2	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.6731	2.7186	–1.1402	0.2374	–0.7072	1.2675	–1.1506	0.4639
	c_0	c_1	c_2	c_3	a_4	Eengaged in: 1995 $\delta = 0.00118$			
	1	–2.4959	2.5346	–1.122	0.1461				
Pump no. 3	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.6692	2.7047	–1.1317	0.2564	–0.7478	1.3085	–1.1769	0.4727
	c_0	c_1	c_2	c_3	a_4	Engaged in: 1999 $\delta = 0.0009$			
	1	–2.4865	2.5051	–1.0864	0.126				
Pump no. 4	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.449	2.2899	–0.8991	0.2393	–0.6632	1.0372	–0.9701	0.4364
	c_0	c_1	c_2	c_3	a_4	Engaged in: 1995 $\delta = 0.00107$			
	1	–2.2679	2.1673	–0.8822	0.0876				
Pump no. 5	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.5658	2.5737	–1.19	0.4005	–0.7239	1.0998	–1.0168	0.4361
	c_0	c_1	c_2	c_3	a_4	Engaged in: 1999 (after regeneration) $\delta = 0.0012$			
	1	–2.2954	2.2286	–0.942	0.1245				
Pump no. 6	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	1	–2.5847	2.5983	–1.1636	0.3189	–0.6507	1.0559	–0.9883	0.4323
	c_0	c_1	c_2	c_3	a_4	Engaged in: 1995 $\delta = 0.0011$			
	1	–2.3672	2.3157	–0.9778	0.1026				

The graphical representations of the determined coefficients of characteristic polynomials of **ARMA** models (plotted separately for each pump) are shown in Table 2.

A comparison of the coefficients of **ARMA** model of the examined pumps and of an ideal pump (model pomp) are shown on Figure 5.

Table 2



ARMA model coefficients for an ideal model pump

ARMA model coefficients for pump p1

ARMA model coefficients for pump p2

ARMA model coefficients for pump p3

ARMA model coefficients for pump p4

ARMA model coefficients for pump p5

ARMA model coefficients for pump p6

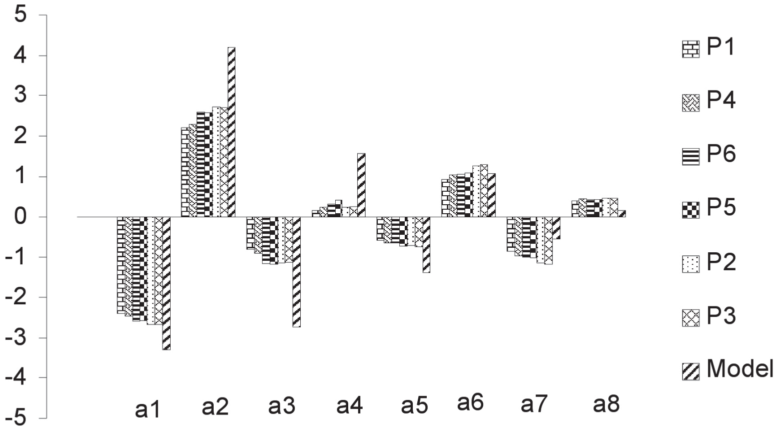


Fig. 5. A comparison of the coefficients of characteristic polynomial $A(z^{-1})$ of ARMA model of the examined pumps and of an ideal pump. p1 – pump no. 1 (1998), p2 – pump no. 2 (1995), p3 – pump no. 3 (1999), p4 – pump no. 4 (1995), p5 – pump no. 5 (1999 Reg.), p6 – pump no. 6 (1995), mp – model pump

7. SUMMARY

Comparing the plotted coefficients of characteristic polynomials $A(z^{-1})$ of **ARMA** models of the examined pumps with the respective diagrams plotted for an ideal (model) pump, it has been stated that the most severe wear and tear was suffered by pump no. 1. This high wear and tear rate was caused by the fact that operating as a master pump in the press feeding system, it was working under alternate loading throughout the whole working cycle. The lowest wear and tear rate was recorded in pump no. 3 which had the shortest time of operation in the system and medium loading throughout the working cycle. Compared with pumps which were engaged in 1995, i.e. were operating from the very beginning of the press start up, very good performance characteristics revealed pump no. 2, the least loaded throughout the whole working cycle.

An analysis of the wear and tear rate in positive-displacement pumps carried out by the method of time series confirms an applicability of this method in diagnosis of hydraulic pumps using pressure as a measuring signal. Analysing the values of the coefficients of characteristic polynomial, it was concluded that with short delay times the values of these coefficients ($a_1 - a_5$ in the **ARMA** model) computed theoretically were higher than the values obtained for the examined pumps. On the other hand, with long delay times, the values of the coefficients ($a_6 - a_8$ for **ARMA**) computed theoretically, because of signal processing disturbances and errors entering the system, assumed lower values than the values obtained for the examined pumps.

References

[1] Gao Y., Kong X., Zhang Q.: *Wavelet Analysis for Piston Pump Fault Diagnosis.* National Conference on Fluid Power, Las Vegas 2002

[2] Mańczak K., Nahorski Z.: *Komputerowa identyfikacja obiektów dynamicznych.* (Computer identification of dynamic objects). Warszawa, PWN 1983

[3] Stryczek S.: *Napęd i sterowanie hydrauliczne.* Volume I. (Hydraulic drive and control), WNT 1990