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# SECOND ORDER CLOSURE OF HEAT AND MASS TRANSPORT EQUATIONS FOR SHEAR TURBULENCE

*SUMMARY*

*The paper contains a model of shear turbulent flow closure based on the second moment equations for velocity, temperature and concentration fields. The model is constructed on approximation of local – equilibrium turbulence. The solution of the obtained equation is represented in two factors, the first one describes homogeneous flow and the second takes into account the influence of Archimedes' forces, which depend on Richardson's numbers. The minimal number of empirical constants taken from theories of homogeneous turbulence is applied in the model.*

**Keywords**: *shear turbulence, heat and mass transport, turbulence flow equations*

*RÓWNANIE TRANSPORTU CIEPŁA I MASY Z APROKSYMACJĄ CZŁONÓW DRUGIEGO RZĘDU DLA PRZYPOWIERZCHNIOWYCH PRZEPŁYWÓW TURBULENTNYCH*

*Artykuł opisuje model przypowierzchniowego przepływu turbulentnego oparty na równaniu z członami drugiego rzędu dla prędkości, temperatury i pola koncentracji emisji. Model został skonstruowany z wykorzystaniem zależności przybliżających lokalną równowagę turbulencji. Rozwiązanie otrzymanych równań zawiera dwa rodzaje członów. Pierwsze z nich odnoszą się do przepływu w ośrodku jednorodnym, natomiast drugie opisują wpływ sił Archimedesa wywołanych przez pola temperatur i koncentracji i zależnych od liczb Richardsona. Model ma również tę zaletę, że korzysta z minimalnej ilości stałych empirycznych pojawiających się w teorii jednorodnych przepływów turbulentnych.*

**Słowa kluczowe**: *przypowierzchniowa turbulencja, transport ciepła i masy, równania przepływu turbulentnego*

## 1. INTRODUCTION

Emissions into the moving environment in three-dimensional area are one of actual problems. If it is given emissions into the moving environment, for example into atmosphere, then usually the environment itself has already had turbulence owing to turbulence generation in ground area of the basic flow. In the areas, which are in distance from a source of emission, this turbulence dominates over the turbulence generated by a source of emission and defines distribution of polluting substances. Problems in such positing usually are reduced to investigation of passive admixture transference, where the admixture does not influence the basic moving environment.

The problem positing in this paper, when an admixture moving together with the basic flow above a surface with non-uniform temperature entering into interaction with the basic movement and temperature, has great practical usage. In this case it is impossible to consider that an admixture is passive because of complex correlations of velocity of movement, temperature and admixture concentration.

Currently for the description of admixture dispersion processes a range of the mathematical models based on the equations of turbulent diffusion has been created. Generally these models are constructed for velocity and one of scalar value of temperature or concentration. In this work the mutual correlations of temperature, concentration and flow velocities are taken into account.

Volume forces have great influence on character of turbulence if they interconnect with pulsations of velocity. The simplest example is strong influence of gravity on flow with density pulsations. If density pulsations appear as the result of existence of the average density gradient in the same direction as the average velocity gradient or if flow actually arises because of a difference of average density, then the good correlation is appeared between pulsations of density and velocity and then influence of buoyancy forces may become very great. In case when the density increases in a vertical direction from below to upwards we will have the unstable flow. The interrelation of density and velocity may lead to transformation of potential energy into turbulent kinetic energy. On the contrary when the density decreases from below to upwards faster than it is necessary for preservation of hydrostatic balance of liquid then available turbulent energy can be transformed into potential energy. It means that turbulent mixing aspires to decrease a gradient of density and thus to raise the center of gravity of liquid volume.

## 2. EQUATIONS

In considering problem the main aim is to get expression for turbulent characteristics in the explicit form via average characteristics of turbulent flow. For the investigation we use the equations describing the changes of Reynolds's turbulent stresses [1, 2]

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$$\frac{\partial}{\partial \tau}\overline{u_i u_j} + U_k \frac{\partial}{\partial x_k}\overline{u_i u_j} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k}$$

$$-\overline{\frac{p}{\rho}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)} -$$

$$-\frac{\partial}{\partial x_k}\left[\nu \frac{\partial}{\partial x_i}\overline{u_i u_j} - \overline{u_i u_j u_k} - \overline{\left(\delta_{jk}u_i + \delta_{ik}u_j\right)\frac{p}{\rho}}\right] \quad (1)$$

$$+2\nu \overline{\frac{\partial u_j}{\partial x_k}\frac{\partial u_i}{\partial x_k}} + F_{ui} = 0$$

For the description of complex turbulent flows that have temperature and concentration we use the additional equations for the second order moments of temperature and concentration fields [3, 4]

$$\frac{\partial \overline{u_i t}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{u_i t}}{\partial x_k} + \overline{u_k t}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial T}{\partial x_k} +$$

$$+\frac{\partial}{\partial x_k}\left(-\nu \frac{\partial \overline{u_i t}}{\partial x_k} + \overline{u_k u_i t} + \overline{\frac{p}{\rho} t}\right) + 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial t}{\partial x_k}} + F_{ti} = 0 \quad (2)$$

$$\frac{\partial \overline{u_i q}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{u_i q}}{\partial x_k} + \overline{u_k q}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial Q}{\partial x_k} +$$

$$+\frac{\partial}{\partial x_k}\left(-\nu \frac{\partial \overline{u_i q}}{\partial x_k} + \overline{u_k u_i q} + \overline{\frac{p}{\rho} q}\right) + \quad (3)$$

$$+2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial q}{\partial x_k}} + F_{qi} = 0,$$

$$\frac{\partial \bar{t}^2}{\partial \tau} + U_k \frac{\partial \bar{t}^2}{\partial x_k} + 2\overline{u_k t}\frac{\partial T}{\partial x_k} - \frac{\partial}{\partial x_k}\left[-a\frac{\partial \bar{t}^2}{\partial x_k} + \overline{u_k t^2}\right] + \quad (4)$$

$$+2a\overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = 0$$

$$\frac{\partial \bar{q}^2}{\partial \tau} + U_k \frac{\partial \bar{q}^2}{\partial x_k} + 2\overline{u_k q}\frac{\partial \bar{Q}}{\partial x_k} + \quad (5)$$

$$+\frac{\partial}{\partial x_k}\left[-d\frac{\partial \bar{q}^2}{\partial x_k} + \overline{u_k q^2}\right] + 2d\overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = 0$$

$$\frac{\partial \overline{qt}}{\partial \tau} + \bar{U}_k \frac{\partial \overline{qt}}{\partial x_k} + \overline{u_k t}\frac{\partial \bar{Q}}{\partial x_k} + \overline{u_k q}\frac{\partial T}{\partial x_k} + \quad (6)$$

$$+\frac{\partial}{\partial x_k}\left(-\nu \frac{\partial \overline{qt}}{\partial x_k} + \overline{u_k qt}\right) + (a+d)\overline{\frac{\partial q}{\partial x_k}\frac{\partial t}{\partial x_k}} = 0$$

where:

- $\tau$ – time,
- $p$ – pressure,
- $U_i$, $u_i$ – components of average and pulsation velocities respectively to axes $x_i$,
- $T$, $t$ – average and pulsation temperatures,
- $Q$, $q$ – average and pulsation characteristics of concentration.

In the flow of general type there exist six components of tensor of Reynolds stresses $u_i u_j$ due to symmetry, three components of correlation $u_i t$ type and three components of correlation $u_i q$ type, two equations for $t^2$ and $q^2$ and equation for correlation of $tq$ type. Hence, it is necessary to solve fifteen partial differential equations. It is obvious that the equations contain some new unknown variables except average velocity and the second order moments. For defining some terms of equation system we use approximated semi empirical ratios. Expressions for change of energy of various components of pulsations are expressed in the form [3]:

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} = -k\frac{\sqrt{E}}{l}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}E\right)$$

$$\overline{\frac{P}{\rho}\frac{\partial t}{\partial x_k}} = -k_t \frac{\sqrt{E}}{l}\overline{u_i t} \quad (7)$$

$$\overline{\frac{P}{\rho}\frac{\partial q}{\partial x_k}} = -k_q \frac{\sqrt{E}}{l}\overline{u_i q}$$

For dissipation of pulsation energy and its analogues the following expressions are used:

$$2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \nu c_\nu \frac{\bar{u}_i^2}{l^2} + \frac{2}{3}c\,\delta_{ij}\frac{E^{3/2}}{l}$$

$$2\nu \overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = c_{\nu t} a\nu \frac{\overline{t^2}}{l^2} + c_t \frac{\sqrt{E}}{l}\bar{t}^2 \quad (8)$$

$$2\nu \overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = c_{\nu t}\,\nu \frac{\overline{q^2}}{l^2} + c_q \frac{\sqrt{E}}{l}\bar{q}^2$$

but for second order moments they have the following form

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \nu c_v \frac{\overline{u_i u_j}}{l^2},$$

$$2\nu\overline{\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}} = c_{vt} a\nu \frac{\overline{u_i t}}{l^2},$$

$$2\nu\overline{\frac{\partial q}{\partial x_k}\frac{\partial q}{\partial x_k}} = c_{vt}\, \nu \frac{\overline{u_i q}}{l^2}$$

It is assumed that shear turbulent flows are considered in Boussinesq approximation, i.e. changes of density are little and they are taken into account only in mass forces [4]:

$$\begin{aligned} F_{ui} &= -\beta g\left(\delta_{3i}\overline{tu_j} + \delta_{3j}\overline{tu_i}\right) + \alpha g\left(\delta_{3i}\overline{qu_j} + \delta_{3j}\overline{qu_i}\right) \\ F_{ti} &= g\delta_{3i}\left(-\beta\overline{t^2} + \alpha\overline{tq}\right) \\ F_{qi} &= g\delta_{3i}\left(-\beta\overline{tq} + \alpha\overline{q^2}\right) \end{aligned} \tag{9}$$

By writing equations (1)–(6) for pure shear developed turbulent flow, neglecting turbulent diffusion and closing these equations by semi empirical hypotheses (7)–(9) we get the following system of equations:

$$\begin{aligned} &\overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \\ &+ k\frac{\sqrt{E}}{l}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}E\right) + \frac{2}{3}c\,\delta_{ij}\frac{E^{3/2}}{l} - \\ &-\beta g\left(\delta_{3i}\overline{tu_j} + \delta_{3j}\overline{tu_i}\right) + \alpha g\left(\delta_{3i}\overline{qu_j} + \delta_{3j}\overline{qu_i}\right) = 0 \\ &\overline{u_k t}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial T}{\partial x_k} + \\ &+ k_t\frac{\sqrt{E}}{l}\overline{u_i t} + g\delta_{3i}\left(-\beta\overline{t^2} + \alpha\overline{tq}\right) = 0 \\ &\overline{u_k t}\frac{\partial T}{\partial x_k} + c_t\frac{t^2\sqrt{E}}{l} = 0 \\ &\overline{u_k q}\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial Q}{\partial x_k} + \\ &+ k_q\frac{\sqrt{E}}{l}\overline{u_i q} + g\delta_{3i}\left(-\beta\overline{tq} + \alpha\overline{q^2}\right) = 0 \end{aligned} \tag{10}$$

where:

$E$ – kinetic turbulent energy;

$l$ – scale of turbulence.

The solution of equation system (10) regarding to pulsation characteristics consists of two factors. The first of them corresponds to flow in homogenous environment but the second one takes into account Archimedes' forces caused by temperature and concentration fields:

$$\begin{aligned} &E = E_0\varphi, \quad \overline{u_1^2} = \left(\overline{u_1^2}\right)_0\Omega_1, \quad \overline{u_2^2} = \left(\overline{u_2^2}\right)_0\Omega_2 \\ &\overline{u_3^2} = \left(\overline{u_3^2}\right)_0\Omega_3 \\ &\overline{u_1u_3} = \left(\overline{u_1u_3}\right)_0\Omega_4, \quad \overline{u_2u_3} = \left(\overline{u_2u_3}\right)_0\Omega_5 \\ &\overline{u_1u_2} = \left(\overline{u_1u_2}\right)_0\Omega_6 \\ &\overline{tu_1} = \left(\overline{tu_1}\right)_0\Omega_7, \quad \overline{tu_2} = \left(\overline{tu_2}\right)_0\Omega_8 \\ &\overline{tu_3} = \left(\overline{tu_3}\right)_0\Omega_9, \quad \overline{t^{-2}} = \left(\overline{t^{-2}}\right)_0\Omega_{10} \\ &\overline{qu_1} = \left(\overline{qu_1}\right)_0\Omega_{11}, \quad \overline{qu_2} = \left(\overline{qu_2}\right)_0\Omega_{12} \\ &\overline{qu_3} = \left(\overline{qu_3}\right)_0\Omega_{13}, \quad \overline{q^{-2}} = \left(\overline{q^{-2}}\right)_0\Omega_{14} \\ &\overline{qt} = \left(\overline{qt}\right)_0\Omega_{15} \end{aligned} \tag{11}$$

Note that parameters of averaged flows are known. The expressions for homogenous environment have the following basic form:

$$\left(\overline{u_3^2}\right)_0 = \frac{2}{3}\left(1 - \frac{c}{k}\right)\frac{1}{c^{2/3}}l^2\left[\left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left(\frac{\partial U_2}{\partial x_3}\right)^2\right],$$

$$\left(\overline{u_1^2}\right)_0 = \frac{2}{3}\frac{c}{k}\frac{1}{c^{2/3}}l^2\left[\left(\frac{k}{c} - 1\right)\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{k}{c} + 2\right)\left(\frac{\partial U_1}{\partial x_3}\right)^2\right],$$

$$\left(-\overline{u_1u_3}\right)_0 = l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial U_1}{\partial x_3}\right),$$

$$\left(-\overline{u_1t}\right)_0 = 2\frac{c^{1/3}}{k_t}\left(1 + \frac{k}{k_t}\right)l^2\left(\frac{\partial U_1}{\partial x_3}\right)\left(\frac{\partial T}{\partial x_3}\right),$$

$$\left(-\overline{u_3t}\right)_0 = \frac{k}{k_t}l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial T}{\partial x_3}\right),$$

$$\left(-\overline{u_1q}\right)_0 = 2\frac{c^{1/3}}{k_q}\left(1 + \frac{k}{k_q}\right)l^2\left(\frac{\partial U_1}{\partial x_3}\right)\left(\frac{\partial Q}{\partial x_3}\right),$$

$$\left(-\overline{u_3q}\right)_0 = \frac{k}{k_q}l^2\sqrt{\left(\frac{\partial U_2}{\partial x_3}\right)^2 + \left(\frac{\partial U_1}{\partial x_3}\right)^2}\left(\frac{\partial Q}{\partial x_3}\right),$$

$$E_0 = \frac{1}{c^{2/3}}l^2\left[\left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left(\frac{\partial U_2}{\partial x_3}\right)^2\right].$$

Functions taking into account influence of stratification on a turbulent flow have the following form:

$$\Psi=\varphi^2+\varphi\left[Rt\left(\frac{k}{c_t}+\frac{k}{c_s}\frac{\Pr}{Sc}+2\right)-Rq\left(\frac{k}{c_q}+\frac{k}{c_s}\frac{Sc}{\Pr}+2\right)\right]+$$

$$+Rt^2\frac{k}{c_s}\frac{\Pr}{Sc}\left(\frac{k}{c_t}+2\right)+Rq^2\frac{Sc}{\Pr}\frac{k}{c_s}\left(2+\frac{k}{c_q}\right)-$$

$$-Rt\cdot Rq\left[\frac{k}{c_t}\frac{k}{c_q}+2\cdot\frac{k}{c_s}\left(\frac{c_t}{c_q}+\frac{c_q}{c_t}\right)\right],$$

$$\Omega_3=\frac{\varphi}{\Psi}\Bigg\{\varphi^2+\varphi\left[Rt\left(\frac{k}{c_t}+\frac{k}{c_s}\frac{\Pr}{Sc}\right)-Rq\left(\frac{k}{c_q}+\frac{k}{c_s}\frac{Sc}{\Pr}\right)\right]+$$

$$+\frac{k}{c_s}\left(Rt^2\frac{k}{c_t}\frac{\Pr}{Sc}+Rq^2\frac{k}{c_q}\frac{Sc}{\Pr}\right)-Rt\cdot Rq\frac{k}{c_t}\frac{k}{c_q}\Bigg\},$$

$$\Omega_1=\frac{\varphi}{\frac{2}{3}\left(2+\frac{c}{k}\right)\Psi}\Bigg\{\frac{2}{3}\left(2+\frac{c}{k}\right)\varphi^2+$$

$$+\varphi\cdot Rt\left[\frac{2}{3}\left(2+\frac{c}{k}\right)\cdot\left(\frac{k}{c_t}+\frac{k}{c_s}\frac{\Pr}{Sc}\right)+4\right]-$$

$$-\varphi\cdot Rq\left[\frac{2}{3}\left(2+\frac{c}{k}\right)\cdot\left(\frac{k}{c_q}+\frac{k}{c_s}\frac{Sc}{\Pr}\right)+4\right]+$$

$$+Rt^2\frac{k}{c_s}\frac{\Pr}{Sc}\left[\frac{2}{3}\left(2+\frac{c}{k}\right)\frac{k}{c_t}+4\right]+$$

$$+Rq^2\frac{k}{c_s}\frac{Sc}{\Pr}\left[\frac{2}{3}\left(2+\frac{c}{k}\right)\frac{k}{c_q}+4\right]-$$

$$-Rt\cdot Rq\left[4\frac{k}{c_s}\left(\frac{c_t}{c_q}+\frac{c_q}{c_t}\right)+\frac{2}{3}\left(2+\frac{c}{k}\right)\frac{k}{c_t}\frac{k}{c_q}\right]\Bigg\},$$

$$\Omega_2=\Omega_1,$$

$$\Omega_4=\frac{\varphi^{3/2}}{(\varphi+Rt-Rq)\Psi}\times$$

$$\times\Bigg\{\varphi^2+\varphi\left[Rt\left(\frac{k}{c_t}+\frac{k}{c_s}\frac{\Pr}{Sc}-\frac{1}{\Pr}\right)-Rq\left(\frac{k}{c_q}+\frac{k}{c_s}\frac{Sc}{\Pr}-\frac{1}{Sc}\right)\right]+$$

$$+Rt^2\frac{k}{c_s}\frac{1}{Sc}\left(\frac{k}{c_t}-1\right)+Rq^2\frac{k}{c_s}\frac{1}{\Pr}\left(\frac{k}{c_q}-1\right)-$$

$$-Rt\cdot Rq\left[\frac{k}{c_s}\left(\frac{c_t}{c_q}\frac{1}{\Pr}+\frac{c_q}{c_t}\frac{1}{Sc}\right)-\frac{k}{c_t}\frac{k}{c_q}\right]\Bigg\},$$

$$\Omega_5=\Omega_4,\quad \Omega_6=\frac{\Omega_4}{\sqrt{\varphi}},$$

$$\Omega_9=\frac{1}{\Psi}\left\{\varphi^{3/2}\left[\varphi+\frac{k}{c_s}\left(Rt\frac{\Pr}{Sc}-Rq\frac{c_t}{c_q}\right)\right]\right\},$$

$$\Omega_7=\frac{\varphi}{(\varphi+Rt-Rq)(1+\Pr)\Psi}\times$$

$$\times\Bigg\{\varphi^2(1+\Pr)+\varphi\cdot Rt\left[\frac{k_t}{c_t}+\frac{k}{c_s}\frac{\Pr}{Sc}(1+\Pr)\right]-$$

$$-\varphi\cdot Rq\left(\frac{k_t}{c_q}+\frac{k_q}{c_s}+1-\frac{\Pr}{Sc}+\frac{k}{c_s}\frac{c_t}{c_q}\right)+$$

$$+Rt^2\frac{k}{c_s}\frac{k_t}{c_t}\frac{\Pr}{Sc}+Rq^2\frac{k}{c_s}\left(\frac{k_q}{c_q}+\frac{c_t}{c_q}-1\right)-$$

$$-Rt\cdot Rq\left[\frac{k}{c_s}\left(\frac{\Pr}{Sc}-\frac{c_q}{c_t}\frac{1}{Sc}\right)+\frac{k_t}{c_t}\frac{k}{c_q}\right]\Bigg\},$$

$$\Omega_8=\Omega_7,\quad \Omega_{10}=\frac{\varphi}{\Psi}\left[\varphi+\frac{k}{c_s}\left(Rt\frac{\Pr}{Sc}-Rq\frac{c_t}{c_q}\right)\right],$$

$$\Omega_{13}=\frac{1}{\Psi}\left\{\varphi^{3/2}\left[\varphi+\frac{k}{c_s}\left(Rt\frac{c_q}{c_t}-Rq\frac{Sc}{\Pr}\right)\right]\right\},$$

$$\Omega_{11}=\frac{\varphi}{(\varphi+Rt-Rq)(1+Sc)\Psi}\times$$

$$\times\Bigg\{\varphi^2(1+Sc)-\varphi\cdot Rq\left[\frac{k_q}{c_q}+\frac{k}{c_s}\frac{Sc}{\Pr}(1+Sc)\right]+$$

$$+\varphi\cdot Rt\left(\frac{k_t}{c_s}+\frac{k_q}{c_t}+1-\frac{Sc}{\Pr}+\frac{k}{c_s}\frac{c_q}{c_t}\right)+$$

$$+Rq^2\frac{k}{c_s}\frac{k_q}{c_q}\frac{Sc}{\Pr}+Rt^2\frac{k}{c_s}\left(\frac{k_t}{c_t}+\frac{c_q}{c_t}-1\right)-$$

$$-Rt\cdot Rq\left[\frac{k}{c_s}\left(\frac{Sc}{\Pr}-\frac{c_t}{c_q}\frac{1}{\Pr}\right)+\frac{k_q}{c_q}\frac{k}{c_t}\right]\Bigg\},$$

$$\Omega_{12}=\Omega_{11},\quad \Omega_{14}=\frac{\varphi}{\Psi}\left[\varphi+\frac{k}{c_s}\left(Rt\frac{c_q}{c_t}-Rq\frac{Sc}{\Pr}\right)\right],$$

$$\Omega_{15}=\frac{\varphi}{(Sc+\Pr)\Psi}\times$$

$$\times\left[\varphi\cdot(Sc+\Pr)+Rt\frac{k}{c_s}\Pr\left(\frac{c_q}{c_t}+1\right)-Rq\frac{k}{c_s}Sc\left(\frac{c_t}{c_q}+1\right)\right],$$

$$\varphi=\frac{1}{3}\left[1-Rt\left(\lambda_1+\frac{k}{c_s}\frac{\Pr}{Sc}\right)+Rq\left(\lambda_2+\frac{k}{c_s}\frac{Sc}{\Pr}\right)\right]+$$
$$+\left(\sqrt{\Phi}-\Theta\right)^{1/3}-\left(\sqrt{\Phi}+\Theta\right)^{1/3},$$

$$\Theta=\left(\frac{\omega}{3}\right)^3-\frac{\omega\zeta}{6}+\frac{\phi}{2},\quad \Phi=\Theta^2+\left(\frac{\zeta}{3}-\frac{\omega^2}{9}\right)^3,$$

$$\lambda_1=\frac{2}{3}\left(\frac{k}{c}-1\right)+\frac{k}{c_t}+3,\quad \lambda_2=\frac{2}{3}\left(\frac{k}{c}-1\right)+\frac{k}{c_q}+3,$$

$$\lambda_3=\frac{k}{c_s}\frac{2}{3}\left(2+\frac{k}{c}\right)\left(\frac{c_t}{c_q}+\frac{c_q}{c_t}\right)+\frac{k}{c_t}\frac{k}{c_q},$$

$$\omega=Rt\left(\lambda_1+\frac{k}{c_s}\frac{\Pr}{Sc}\right)-Rq\left(\lambda_2+\frac{k}{c_s}\frac{Sc}{\Pr}\right)-1,$$

$$\zeta=Rt^2\left[\lambda_1\left(\frac{k}{c_s}\frac{\Pr}{Sc}+1\right)-1\right]+Rq^2\left[\lambda_2\left(\frac{k}{c_s}\frac{Sc}{\Pr}+1\right)-1\right]-$$
$$-Rt\cdot Rq\cdot\left[\frac{k}{c_s}\left(\frac{Sc}{\Pr}+\frac{\Pr}{Sc}\right)-2+\lambda_1+\lambda_2+\lambda_3\right]+$$
$$+Rt\left(\frac{1}{\Pr}-\frac{k}{c_s}\frac{\Pr}{Sc}-\frac{k}{c_t}\right)-Rq\left(\frac{1}{Sc}-\frac{k}{c_s}\frac{Sc}{\Pr}-\frac{k}{c_q}\right),$$

$$\phi=(Rt-Rq)\times$$
$$\times\left\{\left[Rt^2\frac{k}{c_s}\frac{\Pr}{Sc}(\lambda_1-1)+Rq^2\frac{k}{c_s}\frac{Sc}{\Pr}(\lambda_2-1)\right]-Rt\cdot Rq\cdot\lambda_3\right\}+$$
$$+Rt^2\frac{k}{c_s}\frac{1}{Sc}\left(1-\frac{k_t}{c_t}\right)+Rq^2\frac{k}{c_s}\frac{1}{\Pr}\left(1-\frac{k_q}{c_q}\right)+$$
$$+Rt\cdot Rq\cdot\frac{k}{c_s}\left(\frac{k}{c_s}-\frac{c_t}{c_q}\frac{1}{\Pr}-\frac{c_q}{c_t}\frac{1}{Sc}\right).$$

Note that some functions coincide due to symmetry of the initial equations.

$$Rt=\frac{2}{3}\frac{\beta g\,\dfrac{\partial T}{\partial x_3}}{\Pr\left(\dfrac{k}{c}-1\right)\left[\left(\dfrac{\partial U_1}{\partial x_3}\right)^2+\left(\dfrac{\partial U_2}{\partial x_3}\right)^2\right]},$$

$$Rq=\frac{2}{3}\frac{\alpha g\,\dfrac{\partial Q}{\partial x_3}}{Sc\left(\dfrac{k}{c}-1\right)\left[\left(\dfrac{\partial U_1}{\partial x_3}\right)^2+\left(\dfrac{\partial U_2}{\partial x_3}\right)^2\right]},$$

where:

$Rt, Rq$ – Richardson's numbers depending on temperature and concentration respectively,

$Pr, Sc$ – Turbulent Prandtl's and Schmidt's numbers respectively depending on physical properties of a liquid.

All constants $c_q$, $c_t$, $c_s$, $k_t$, $k_q$ are determined via $k$ and $c$ found as

$$k=\sqrt{\frac{c}{k}}\left[\frac{2}{3}\left(\frac{k}{c}-1\right)\right]^{3/4},\quad c=\left(\frac{c}{k}\right)^{3/2}\left[\frac{2}{3}\left(\frac{k}{c}-1\right)\right]^{3/4},$$

where the constant $\frac{k}{c}=7$ is determined from the theory of isotropic turbulence as coefficient of anisotropy not depending on types of flow. Thus the obtained expressions allow to close Reynolds's equations for complex flows and to calculate turbulent pulsation characteristics of flow.

## 3. BASIC EQUATION

To study the interaction of fields of velocity, temperature and admixture concentration, the advanced turbulent flow will be considered in the three-dimensional aerodynamic channel. To simulate the problem it is used Reynolds' three-dimensional non-stationary equations, turbulent heat and concentration transference:

$$\frac{\partial U_i}{\partial\tau}+U_j\frac{\partial U_i}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left\langle-u_ju_i\right\rangle-\delta_{i3}g\rho'$$

$$\frac{\partial T}{\partial\tau}+U_j\frac{\partial T}{\partial x_j}=\frac{\partial}{\partial x_j}\left(-\overline{u_jt}\right)$$

$$\frac{\partial Q}{\partial\tau}+U_j\frac{\partial Q}{\partial x_j}=\frac{\partial}{\partial x_j}\left(-\overline{u_jq}\right)\qquad(12)$$

$$\frac{\partial U_j}{\partial x_j}=0$$

$$\rho'=-\beta\cdot T+\alpha\cdot Q$$

For calculation it is taken that the resulting velocity $U_g$, which is parallel to a wall, locates on harsh borders at a point $x^c$ just behind a viscous sublayer. This resulting velocity is expressed via dynamic velocity by the logarithmic law of a wall

$$\frac{U_g}{U_*} = \frac{1}{\kappa} \ln\left(y^c G\right),$$

where:

$y^c = x^c U_*/\nu$,

$\kappa = 0{,}4$ – Karman's constant,
$G = 9$ – a coefficient of wall roughness,
$U_*$ – a dynamic velocity on a wall,
$x^c$ – such that the condition $30 \leq y^c \leq 100$ is fulfilled.

For temperature and concentration we take the conditions

$$\frac{\partial T}{\partial x_2} = 0, \quad \frac{\partial Q}{\partial x_2} = 0$$

on the lateral walls:

$$T = T_s, \quad \frac{\partial Q}{\partial x_3} = 0 \tag{13}$$

on the lower wall

$$T = T_b$$

on the heated up surface

$$\frac{\partial Q}{\partial x_3} = Q_s$$

on the source of concentration.

On the input we put the conditions:

$$U_1 = U_0, \quad U_2 = 0,$$
$$U_3 = 0, \quad T = T_v, \quad \frac{\partial Q}{\partial x_1} = 0 \tag{14}$$

On the output we take:

$$\frac{\partial U_1}{\partial x_1} = 0, \quad U_2 = 0, \quad U_3 = 0$$
$$\frac{\partial T}{\partial x_1} = 0, \quad \frac{\partial S}{\partial x_1} = 0 \tag{15}$$

The system of the equations (12) is closed by the turbulence model (11) and solved by the numerical method in view of the boundary conditions (13)–(15).

## 4. NUMERICAL RESULTS

The stated model is applied to solve the problem described in paper [5]. The movement of stratified air in rectangular three-dimensional area is considered in experiment. The height is $H_3$ = 60 cm, the width is $H_2$ = 182 cm and the length is $H_1$ = 360 cm. There is the linear source with length $H_2$ = 152 cm in front of the heated up square plate. This source is across of flow. From the source the admixture is given to the basic movement of flow. The parameters of basic flow: the average velocity of flow is $U_0$ = 1,25 m/s, the temperature of flow is $T_v$ = 43°C, the temperature of the heated up plate is $T_b$ = 121°C, the temperature outside the heated up plate is $T_s$ = 4°C.

The characteristics of averaged fields of velocity, the spatial distribution of temperature and the transfer of concentration are obtained. On Figure 1 the profiles of longitudinal velocity $U_1/U_m$ on different distances from the beginning of the heated up surface are given, where 1 is the profile of flow on the beginning of the heated up plate, 2 is the profile on the middle of the heated up plate, 3 is the profile on the end of the heated up plate. Above the heated up surface there is the deformation of flow caused by thermal convection. Longitudinal velocity on the top of the channel is accelerated, and on directly above the heated up surface the velocity is decreased.

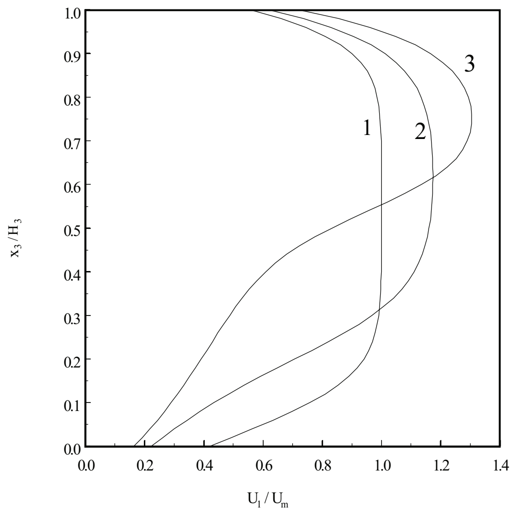


**Fig. 1.** Profiles of longitudinal velocity on different distances from the beginning of the heated up surface: 1 – flow on the beginning of the heated up plate; 2 – flow on the middle of the heated up plate, 3 – flow at the end of the heated up plate

On Figure 2 the field of vectors of velocities in cross section of flow is given at the end of the heated up plat, where secondary flows is formed. Figure 3 presents the spatial distributions $U_1/U_m$ at the same section. On Figure 3 we can see that the longitudinal component of velocity above the heated up surface is decreased but on lateral edges of the channel velocity is increased. Figure 4 shows contour distributions of concentration of the admixture in longitudinal section that is in the middle of the channel along the basic flow. On Figure 5 we can see contour distributions of concentration of the admixture in represented cross section. By comparing of Figure 5 and Figure 2 we can see that field of velocities and field of concentration are correlated. The distribution of temperature in cross section in end of heated up plate is shown on Figure 6. The results of modeling coincide with experimental data. They describe the basic laws of transfer of an admixture above the heated up surface.

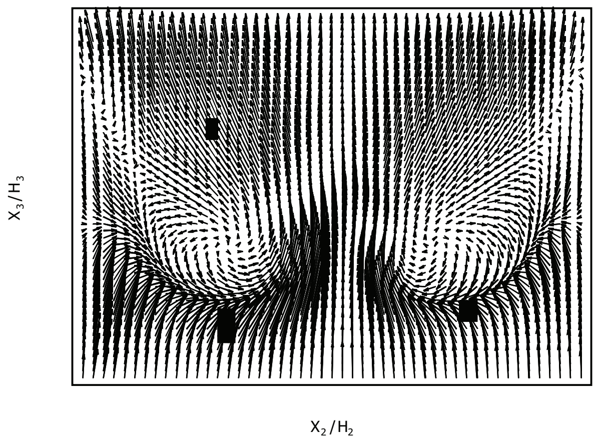


**Fig. 2.** Velocities vectors field in cross section of flow at the end of the heated up plat

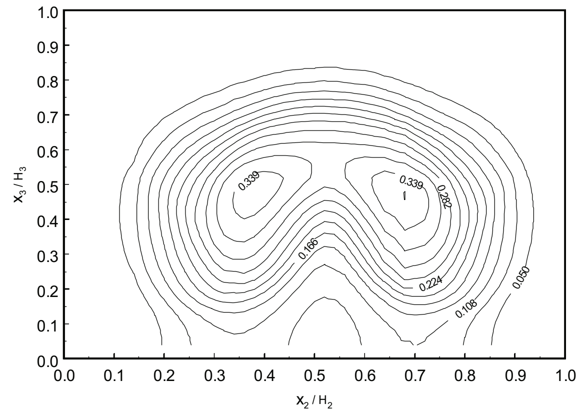


**Fig. 5.** Contour distributions of concentration of the admixture in represented cross section

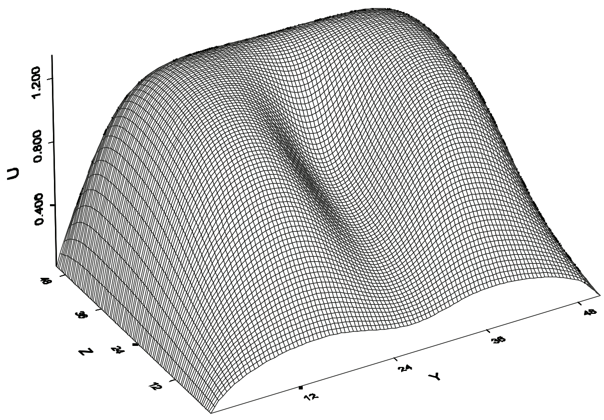


**Fig. 3.** Longitudinal component of velocity above the heated up surface

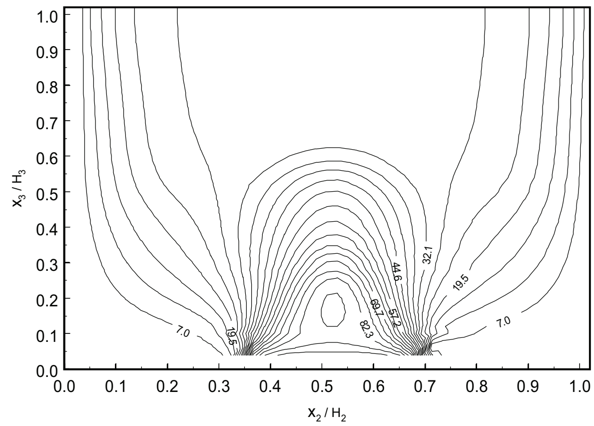


**Fig. 6.** The distribution of temperature in cross section in end of heated up plate

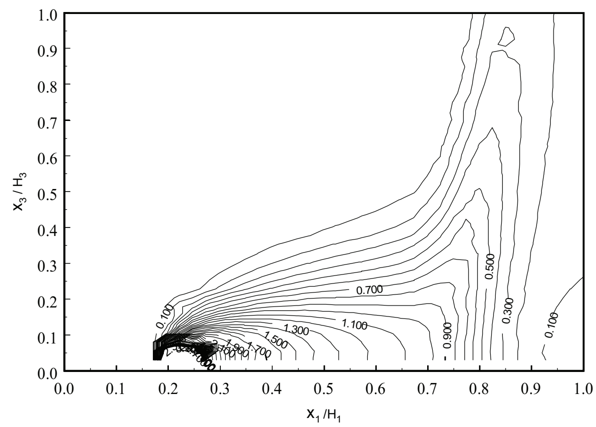


**Fig. 4.** Contour distributions of concentration of the admixture in longitudinal section in the middle of the channel along the basic flow

Thus, the model of turbulence allows to solve the problems of the transfer of admixture above a temperature-nonuniform spreading surface and to calculate fields of average velocity, temperatures and concentration. Additionally the mutual correlations of these fields are taken into account. This model of turbulence can be applied for modeling turbulent transfer of admixture in the stratified medium.

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