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IDENTIFICATION OF A BACKLASH USING THE MODELLING FUNCTION METHOD

SUMMARY

The physical value of a backlash in a kinematic pair of a mechanical system can be determined using a number of methods; still the problem is considered to be difficult. The identification may involve employing a modelling function, which allows also finding out about other properties of a given mechanical system. A modelling function constitutes a mathematical model for the relationship between the solids-joints interaction and the relative displacement and the relative velocity of the solids.

This study aims at identifying a backlash by means of one-dimensional modelling functions that do not take account of velocity. The functions depend on the parameters determined during the identification process. Their values result from the similarity of signals being a response of the analysed real system and its mathematical model (i.e. its modelling function). The method does not require selecting signals; any signal representing the actual operation of the system will be applicable. Signals are analysed over a time period. A special form of the modelling function should permit determining the values of the searched parameters, i.e. the backlash.

***Keywords**: backlash, identification, nonlinear elastic properties, computer simulations, mathematical model*

IDENTYFIKACJA LUZU KONSTRUKCYJNEGO Z WYKORZYSTANIEM METODY FUNKCJI MODELUJĄCEJ

Mimo istnienia wielu metod identyfikacji, wyznaczenie fizycznej wartości luzu w parach kinematycznych układów mechanicznych jest zadaniem trudnym. Metoda identyfikacji z wykorzystaniem funkcji modelującej pozwala na rozwiązanie tego zadania (umożliwia poznanie także innych właściwości układu). Funkcja modelująca stanowi model matematyczny zależności pomiędzy siłą oddziaływania brył na element łączący a przemieszczeniem względnym i prędkością względną tych brył. Funkcja modelująca jest zależna od parametrów, których wartości wyznaczane są w procesie identyfikacji. Wartości parametrów wyznaczane są z warunku podobieństwa sygnałów będących odpowiedzią badanego układu rzeczywistego i modelu matematycznego (funkcji modelującej). Metoda nie wymaga specjalnie dobieranych sygnałów, mogą to być sygnały wynikające z rzeczywistej pracy układu. Analiza sygnałów dokonywana jest w dziedzinie czasu. Specjalna postać funkcji modelującej powinna umożliwiać wyznaczenie wartości poszukiwanych parametrów – w rozważanym przypadku jest to luz konstrukcyjny.

W prezentowanych badaniach przeprowadzono identyfikację wartości luzu konstrukcyjnego w obrotowej parze kinematycznej. Wyznaczono wartość luzu sworznia łączącego łyżkę koparki jednonaczyniowej z ramieniem. Badania przeprowadzone były w laboratorium na oryginalnym osprzęcie koparki.

1. INTRODUCTION

The physical value of a backlash in a kinematic pair of a mechanical system can be determined using a number of methods; still the problem is considered to be difficult. The identification may involve employing a modelling function [1, 2, 3, 4], which allows also finding out about other properties of a given mechanical system. A modelling function constitutes a mathematical model for the relationship between the solids-joints interaction and the relative displacement and the relative velocity of the solids.

This study aims at identifying a backlash by means of one-dimensional modelling functions that do not take account of velocity. The functions depend on the parameters determined during the identification process. Their values result from the similarity of signals being a response of the analysed real system and its mathematical model (i.e. its modelling function). The method does not require selecting signals; any signal representing the actual operation of the system will be applicable. Signals are analysed over a time period. A special form of the modelling function should permit determining the values of the searched parameters, i.e. the backlash.

2. MODELLING FUNCTIONS

The modelling function $\tilde{S}$ constitutes a mathematical model of the relationship between the impact of a solid body on a joining element and the relative displacement and relative velocity of the solid. The modelling function is dependent also on the parameters λ_i, whose values are established in the process of identification. The identification enables one to determine selected values of the parameters λ_i of the modelling function so as to establish that the similarity of signals being a response of the real system and those of the mathematical model is sufficient (see Fig. 1).

As the arguments of the modelling function are dependent on time ($x(t)$, $x(t)$), the function can be treated as a complex function of time (1).

$$\begin{matrix} x = x(t) \\ \dot{x} = \dot{x}(t) \end{matrix} \quad \tilde{S} = \tilde{S}(t, \lambda_i) \tag{1}$$

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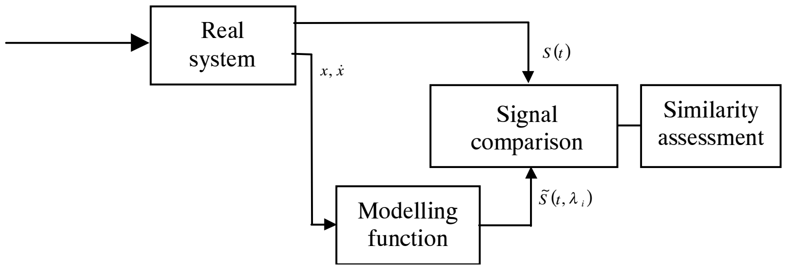

Fig. 1. Schematic diagram of the identification process

The form of the modelling function makes it possible to specify which of the parameters are relevant to a system under consideration. The parameters should be included in the structure of the modelling function. When identifying a backlash, one can apply a modelling function in the following form:

$$\begin{aligned}\tilde{S}(t,\lambda_i) &= \lambda_1\left[x(t)+\operatorname{sign}(\dot{x}(t))\cdot\lambda_4\right]+\\ &+\lambda_3\left[x(t)+\operatorname{sign}(\dot{x}(t))\cdot\lambda_4\right]^3+\\ &+\lambda_2\left[h+\operatorname{sign}(\dot{x}(t))\cdot\lambda_4\right]\cdot\left[1-\frac{(x(t)-x_0)^2}{h^2}\right]\end{aligned} \tag{2}$$

where:

$$x_0=\frac{1}{2}\left(x_A(t)+x_b(t)\right),$$

$$h=\frac{1}{2}\left(x_B(t)-x_A(t)\right),$$

$x_A(t)$, $x_B(t)$ – consecutive extreme values of a displacement signal (Fig. 2),
λ_1 – denotes linear elastic properties,
λ_2 – indirectly denotes attenuation,
λ_3 – denotes nonlinear elastic properties,
λ_4 – denotes the backlash in a system,
$x(t)$ – displacement signal,
$\dot{x}(t)$ – velocity signal.

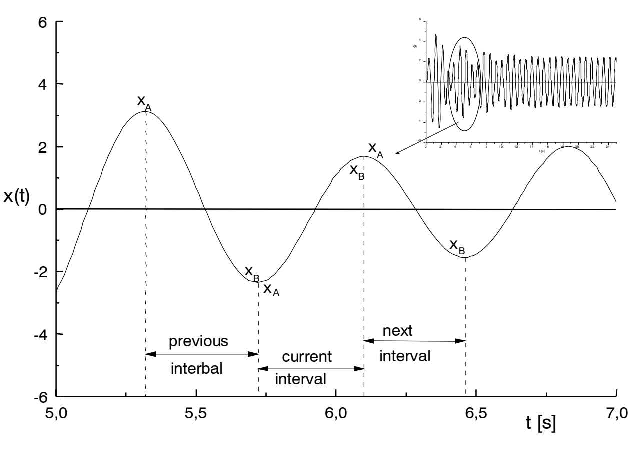

Fig. 2. Extreme values of a displacement signal

3. COMPUTER SIMULATIONS

To check the method suitability, computer simulations were conducted on a mathematical model representing a real system. The input signals were generated in a computer using some predetermined procedures. The experiment was carried out for the system presented in Figure 3.

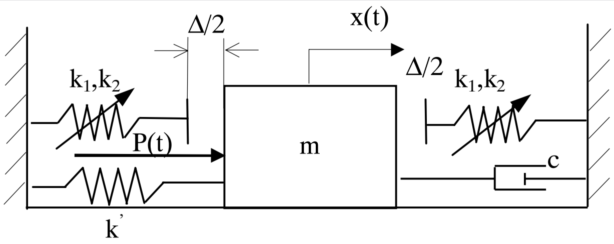

Fig. 3. A schematic diagram of a mechanical system

The mathematical model is constituted by the following equations:

$$\begin{array}{lll} 1^o & m\ddot{x}+c\dot{x}+k'x=P(t) & \text{when } |x|\le\Delta \\ 2^o & m\ddot{x}+c\dot{x}+k'x+k_1(x-\operatorname{sign}(x)\Delta)+ & \\ & +k_2(x-\operatorname{sign}(x)\Delta)^3=P(t) & \text{when } |x|>\Delta \end{array} \tag{3}$$

The system was excited by applying the force $P(t)$ in form (4) shown in Figure 4.

$$P(t)=2500\sin(7t)+800\sin(10t)\quad[\mathrm{N}] \tag{4}$$

The response of the system is displacement, velocity, and force $S(t)$ acting on a body of mass m (Fig. 5).

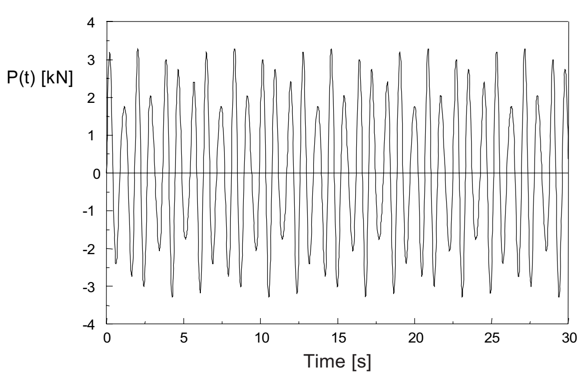

Fig. 4. An external force exciting a motion

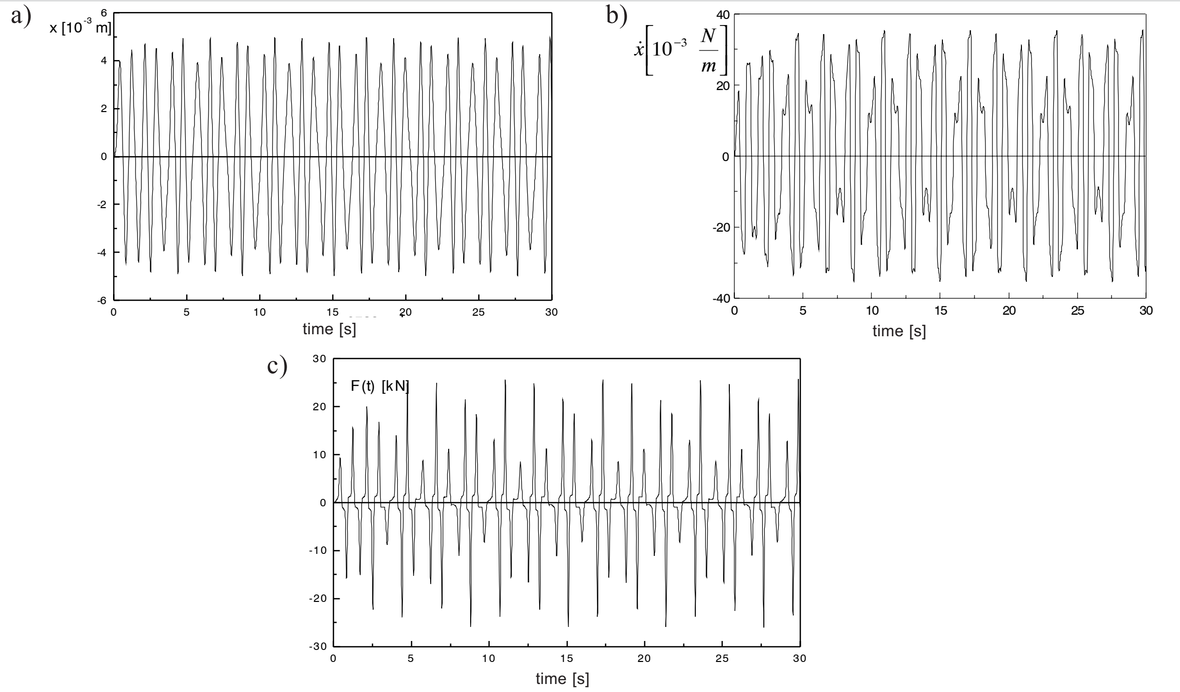

Fig. 5. The system response: a) displacement; b) velocity; c) force

The force acting on the mass *m* is a resultant of the external force exciting a motion and the forces of interacting elastic and damping elements, and it is calculated from the relationship

$$S(t) = P(t) - m\ddot{x}(t) \tag{5}$$

Figure 6 is a graphic representation of the force $S(t)$ dependent on the displacement and the modelling function for optimal values of the parameters λ_i.

Since it was a computer-assisted experiment, only the real values were known. It was essential to establish the identification error. All the values are presented in Table 1.

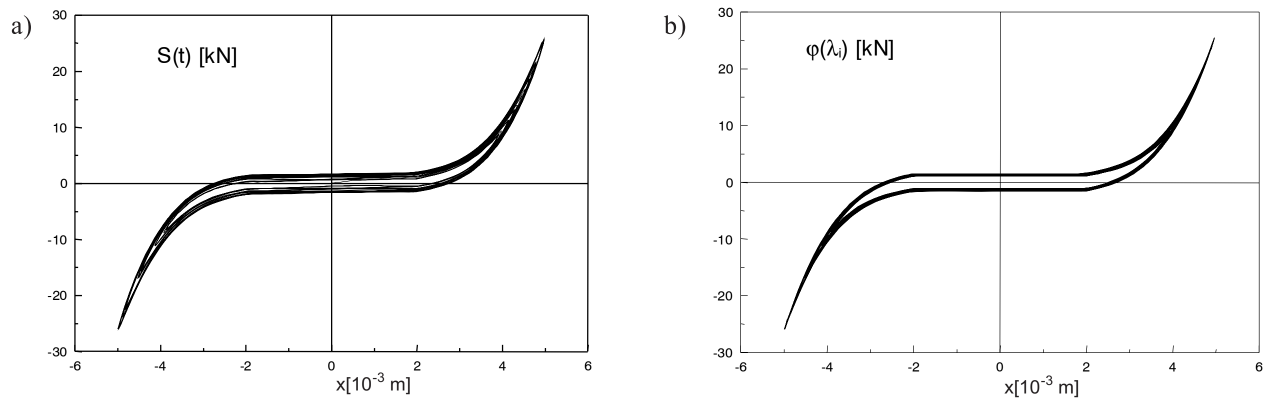

Fig. 6. Interaction force acting between: a) the system and the mass; b) the modelling function

Table 1. Values used in experiment

Units	kg	$\frac{N}{m}$	$\frac{N}{m}$	$\frac{N}{m}$	$\frac{Ns}{m}$	$\frac{N}{m}$	$\frac{N}{m^3}$	m
Real values	m	k_1	k'	k_1+k'	c		k_2	Δ
	30	1410	90	1500	50		800	$2 \cdot 10^{-3}$
Experimental data				λ_1	$\tilde{c}$	λ_2	λ_3	λ_4
				1674.4	47.69	526.12	784.23	$2.013 \cdot 10^{-3}$
Identyfication error				11.63%	4.62%		1.97%	0.65%

4. IDENTIFYING A BACKLASH IN A ROTARY KINEMATIC PAIR

The paper is concerned with the identification process of a design backlash in a rotary kinematic pair. The aim was to determine the value of a backlash for a pin joining the digging bucket and the arm of a backhoe excavator. The tests were conducted under laboratory conditions on real machine elements, i.e. backhoe excavator K-111 (Fig. 7).

Fig. 7. The test facility

The backlash measured directly with a slide calliper was reported to be considerable, i.e. 2.1 mm. Signals detected by sensors were converted into digital form using A/C transducers and then registered. The data were further processed according to the identification procedures.

The testing involved measuring and registering three types of signals: servo-motor displacement, angular rotation of the digging bucket, and force acting on the servo-motor piston rod. The other quantities required for the analysis were calculated taking account of the kinematic model of the excavator elements (see Ref. [4] for details).

To identify the parameters of the excavator members, the modelling function in form (2) was applied. In the process of identification, the registered signals (Fig. 8) were used to determine the values of the parameters λ_i being sought. Table 2 shows the identification results.

The next figure, i.e. Figure 9 presents the force acting on the piston rod and the modelling function for the optimal values of the parameters λ_i.

The aim of the identification process was to establish a backlash in a kinematic pair. The other values of the parameters obtained in the process of identification (Tab. 2) are of no practical importance. The value of the backlash determined in the analysis was 2.2 mm.

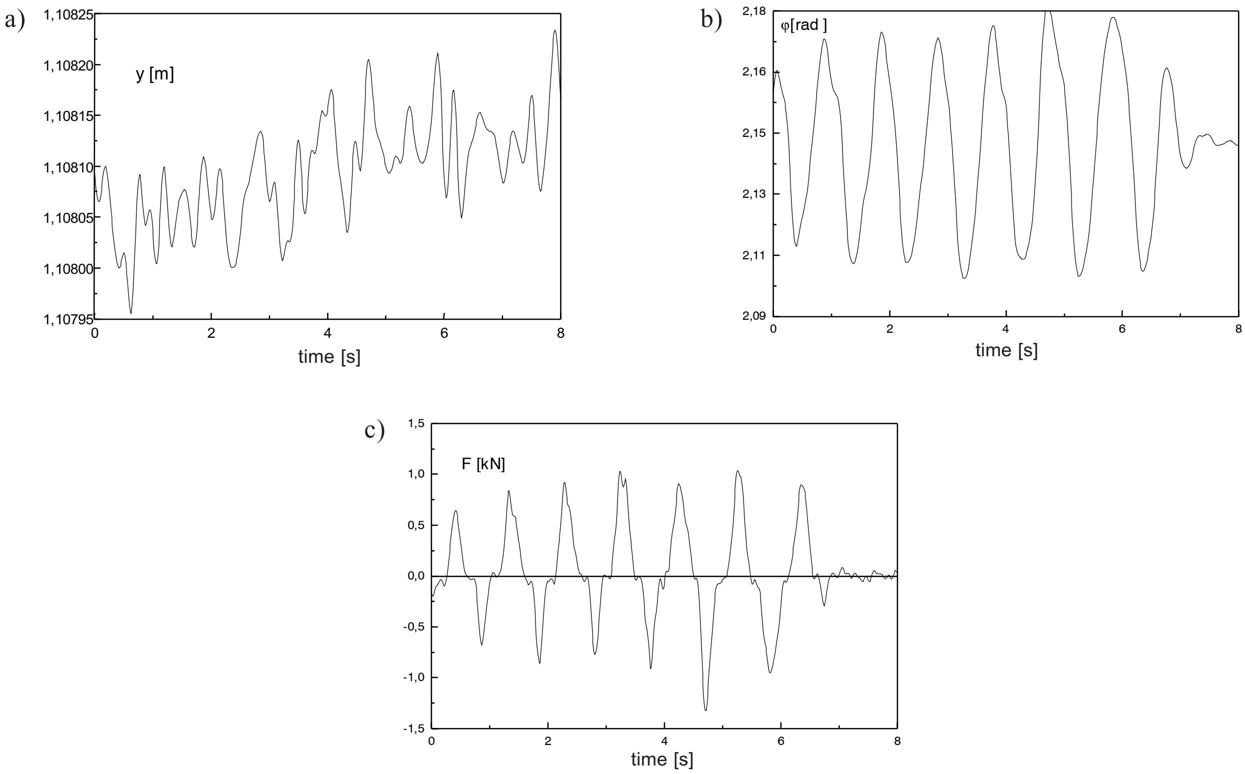

Fig. 8. Signals registered in the excavator elements: a) displacement of the servo-motor; b) angular rotation of the bucket; c) force acting on the servo-motor piston rod

Table 2. Parameters obtained in the process of identification

$\lambda_1 \left[\frac{kN}{m}\right]$	$\lambda_2 \left[\frac{kN}{m}\right]$	$\lambda_3 \left[\frac{kN}{m}\right]$	λ_4 [m]
135.421	18.729	0.461	0.0022

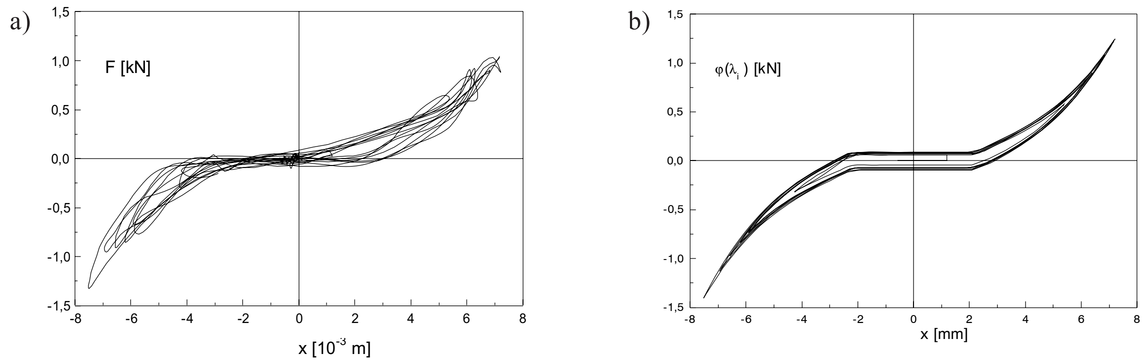

Fig. 9. Representation of: a) the real function; b) the modelling function

Comparing the identification result with the value obtained by measurement, we reported a 4.5% relative error.

5. CONCLUSION

The work deals with a method for identification of a backlash in a mechanical system. The method enables one to establish properties and parameters that are difficult to identify. To illustrate the problem we investigated a design backlash. The experimental data confirm that the method is suitable for various practical applications.

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