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MODELLING AND CONTROL OF STRUCTURE-ACOUSTIC INTERACTION PROBLEMS VIA PIEZOCERAMIC ACTUATORS

SUMMARY

Active control of vibration suppression of a circular fluid-loaded plate is analytically studied. The purpose of this theoretical work is to present a general model of a planar vibrating structure located in a finite baffle and interacted with fluid as well as its time response on harmonic excitation when a control strategy is applied.

The structure under study is a vibrating circular plate of radius a, having a constant thickness h, to which centrally placed circular shaped piezoelectric ceramic patches of radius $a_1 < a$ are bonded. They are used to cancel the plate vibrations and related sound field when a controlling voltage is applied. It was assumed, that the plate clamped at the edge is excited on one side by a uniform periodic force with a constant amplitude F_0 and it radiates the acoustic waves into a surrounded fluid of density ρ_o. The control problem lies in using piezoceramic actuators working in a pair to reduce the plate vibrations. For the system under consideration the state-space model is constructed. The modern control theory is then applied to the system model using a linear quadratic regulator (LQR). The simulations of the active attenuation of the plate vibrations were made with a MATLAB/Simulink computer program. The results demonstrate that it is possible to achieve a significant reduction of the vibration amplitude with the use of a pair PZT actuators.

Keywords: *circular plate, vibration control, PZT actuators, fluid loading, feedback control*

MODELOWANIE I STEROWANIE STRUKTUR Z UWZGLĘDNIENIEM ODDZIAŁYWAŃ AKUSTYCZNYCH ZA POMOCĄ PIEZOCERAMICZNYCH UKŁADÓW WYKONAWCZYCH

Praca prezentuje analizę aktywnej metody redukcji drgań płyty kołowej oddziałującej z ośrodkiem. Celem pracy jest przedstawienie ogólnego modelu płaskiej struktury, umieszczonej w odgrodzie o skończonych wymiarach, z uwzględnieniem reakcji ośrodka oraz jej odpowiedzi czasowej na wymuszenie harmoniczne dla zastosowanej strategii sterowania.

Przedmiotem analizy jest cienka płyta kołowa o promieniu a, stałej grubości h, do której przyklejono centralnie dwa kołowe elementy PZT o promieniach $a_1 < a$. Przyłożenie do nich napięcia sterującego umożliwia redukcję drgań płyty i towarzyszącego im pola akustycznego. Założono, że płyta, utwierdzona na obwodzie, jest z jednej strony pobudzana do drgań za pomocą równomiernie rozłożonej siły harmonicznej o stałej amplitudzie i promieniuje fale akustyczne do otaczającego ośrodka o gęstości ρ_o. Problem dotyczący sterowania polega na zastosowaniu pary elementów PZT do redukcji drgań płyty. W tym celu rozważany model systemu został sprowadzony do przestrzeni stanów, a następnie, stosując liniową teorię sterowania, zaprojektowano regulator LQR. Symulacje aktywnej redukcji drgań płyty zostały przeprowadzone za pomocą programu MATLAB/Simulink, a otrzymane wyniki wskazują, że możliwa jest znacząca redukcja amplitudy drgań z zastosowaniem jednej pary elementów PZT.

1. INTRODUCTION

The application of classical control approaches to the problem of structural vibrations which lead to noise in the audible frequency range is promising alternative to conventional passive methods. Accurate modeling of the acoustic structural and coupling components is a necessary first step for predicting the dynamics of the structural acoustics systems and for the design of model-based controllers. The control of noise and vibration in such structural acoustic systems has been intensely investigated in recent years. A large number of studies on the *active vibration control* (AVC) and active noise and vibration control (ANVC) have been reported. In those studies the classical control, feedforward control, modern control and robust control have been used [3, 4, 15 and references cited inside]. Modern control theory has been also applied by the author [6–9] to reduce circular plate vibrations by using a linear – quadratic (LQR) and PI2D controllers. Those systems have been successfully implemented on an experimental plant, where assumed point control force has been put into practice by an electromechanical shaker attached to the plate [7, 9]

Point force input located at the middle of the plate seems to be quite an effective kind of control for suppressing the circular plate vibration and radiated acoustic pressure [8]. In this approach, the optimal control problem is solved by including in the performance index an additional term proportional to the squared far – field radiation pressure, besides the customary two terms which depend on the vector of state variables and on the control effort. The control input

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that minimises this performance index is derived by applying Hamilton's principle.

However, design of improver versions of electrodynamic actuators is still of continued interest [13], on the other hand, a significant drawback of the approach with the shaker lies in its relative inertia, gravity and bulky. Actuators are one of the most significant integral parts of controlled structures. Such features as light weight, high efficiency long live time and compact size are important issues. Piezoelectric actuators with proper design almost satisfy all of this requirements. In recent years, a great deal of work has been done on the development of control methodologies for structural and acoustic applications by using piezoelectric actuators.

The equations for relating strains to out of plane displacements was introduced by Lee and Moon [5]. During the past few years, a significant amount of research has been done in the field of control of flexible structures by the use of smart sensors and actuators. The effectiveness of using active control of a smart system has been demonstrated. However, most of the control designs have been applied to the beam-like or rectangular plate structures [1, 3, 15]. This paper focuses on modeling and control design issues concerning a clamped circular plate with a pair of thin piezoelectric ceramic actuators. Similar problems have been investigated by Van Niekerk *et al*. [19] and Tylikowski [16–18]. Van Niekerk, Tongue and Packard used a circular piezoelectric actuator to control the sound transmission through a concentric circular plate located in a duct. Since the actuator covered 60% of the plate surface area, it was very effective at controlling the first plate mode. More recently, Tylikowski obtained electromechanical models for circular [16, 18] and annular [17] plates, both excited by piezoelectric elements. By considering the structure to be composed of three regions, analytic displacement relations were obtained. Constants it the expressions were computed from the boundary and joint conditions. Those models were used to analise the performance and some features of such systems as: influence of external shunting capacity on plate response, influence of bonding layer, etc. The modeling of a circular plate with a surface-mounted circular piezoceramic elements in the center has been also considered in [10]. Instead of using analytical methods, a parametric system identification procedure has been employed to establish a mathematical model of the considered system. On the basis of this model, the control algorithm based on the pole-placement method has been developed and implemented on the real system.

The goal of this work is to develop a general model of a thin circular plate interacted with fluid, on which two circular piezoceramic patches are bonded (Fig. 1). It was assumed that the vibrating plate was clamped at the circumference in a planar finite baffle. The formal solution of the fluid-plate coupled equation is presented for the plate driven by a harmonic surface force with constant density. The control problem lies in using piezoceramic actuators working in pair to reduce the plate vibrations. For the system under consideration the state – space model is constructed. The modern control theory is then applied to the system model using a linear quadratic regulator (LQR).

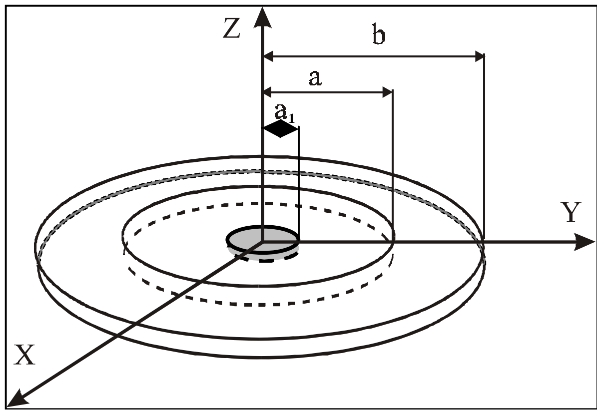

Fig. 1. Circular plate with circular shaped piezoelectric ceramic patches of radius a_1 located in finite baffle of radius b

2. FLUID-PLATE COUPLED EQUATION

The structure under study is a vibrating circular plate of radius a, having a constant thickness h, to which centrally placed circular shaped piezoelectric ceramic patches of radius a_1 are bonded. They are used to damp the plate vibrations when a controlling voltage is applied. The plate is clamped along its contour by a finite rigid co-planar baffle. Although structural damping caused by internal friction has usually little effect, the Kelvin–Voight term is used to model the natural damping of the plate motion. The plate is excited on one side by a uniform periodic force with a nearly constant amplitude F_0 generated by a loudspeaker and it radiates into free space filled with fluid of density ρ_o. The system model is formulated when taking into account the coupling effect between the structure and the acoustic medium.

The models for similar systems reported in literature differ in the manner through the strains produced by the bonding piezoactuators are coupled with the dynamics of the underlying structures. To simplify the discussion, the initial assumptions are similar to that presented by Van Niekerk *et al.* [19]: the piezoceramic material is perfectly bonded to the plate and the bonding layer is neglible; the piezoceramic does not add any mass to the structure; the pezoceramic is effective in the radial direction; the circular plate deforms symmetrically in pure bending. In the case being considered, the applied loading of the circular plate are independent of the angle φ, (axially symmetrical vibrations), thus we can write the governing differential equation of the forced motion of the plate as follows [11, 14]:

$$B\nabla^4 w(r,t) + \rho h \frac{\partial^2}{\partial t^2} w(r,t) = \\ = f(r,t) - R\frac{\partial}{\partial t}[\nabla^4 w(r,t)] - p(r,t,0) \tag{2.1}$$

where:

$$\nabla^4 = \nabla^2\nabla^2,$$

$\nabla^2 = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]$ – the Laplace operator,

$B = Eh^3/12(1-\nu^2)$ – the bending stiffness of the plate,

E, ρ, ν, R – the Young's modulus, density, Poisson's ratio and Kelvin–Voigt damping coefficient for the plate.

It should be pointed out that the material parameters listed above might have changed a little in the region occupied by the piezoelectric elements and they should be identified experimentally. The displacement $w(r, t)$ and its derivative $\partial w(r, t)/\partial r$ satisfy the boundary condition for a clamped plate: they both equal zero at the edge of the plate. The coupling between the plate dynamics and external acoustic field is incorporated by including the backpressure p as a surface force acting on the plate.

The pair of circular PZT actuators bonded to the surface of the plate generate external bending moments M_p. For the analytical development being undertaken in this paper, it is assumed that an external surface force $f(r, t)$ in Eq. (2.1) can be expressed as follows

$$f(r,t) = f_w(r,t) + f_s(r,t) \tag{2.2}$$

where $f_w(r, t)$ is a surface force, modelling the external excitation, generated by a loudspeaker:

$$f_w(r,t) = F_0 e^{-i\omega t} \quad \text{for} \quad 0 \le r \le a \tag{2.3}$$

and the second term of (2.2) represents the control force due to application of voltage to the ceramic patches, which can be expressed as:

$$f_s(r,t) = \nabla^2 M_p, \quad 0 \le r \le a_1 \tag{2.4}$$

The plate equation (2.1) can be re-expressed now as

$$B\nabla^4 w(r,t) + R\frac{\partial}{\partial t}[\nabla^4 w(r,t)] + \rho h \frac{\partial^2}{\partial t^2} w(r,t) = \\ = f_w(r,t) + f_s(r,t) - p(r,t,0) \tag{2.5}$$

For the considered circular plate with the axially-symmetrically located circular piezoactuators of radius a_1, the external moment generated by the piezodiscs in response to an applied voltage (out of-phase) is given by [see 16, 19]

$$M_p = \frac{\kappa}{h_p} d_{31} u(t) H(a_1 - r) \tag{2.6}$$

In the above expression $H(a_1–r)$ is the Heaviside'a function, which has a value of 1 in the region covered by the piezoactuator and zero elsewhere; the constant κ depends on geometric and material properties of plate and piezoactuator, $u(t)$ is the applied voltage, d_{31} is the piezoelectric strain constant, and h_p is the piezoactuator thickness. The goal in the control problem is to determine a control force, which, when applied to the plate (realized via a voltage $u(t)$ for piezoactuators), leads to a reduced level of vibrations.

In considered case, the equivalent external load can be calculated as follows [16, 19]:

$$f_s(r,t) = \frac{\partial^2 M_p}{\partial r^2} + \frac{2}{r}\frac{\partial M_p}{\partial r} \quad 0 \le r \le a_1 \tag{2.7}$$

By denoting $\delta(.)$ and $\delta'(.)$ the Dirac delta distribution and the derivative of Dirac distribution one can obtain

$$f_s(r,t) = \kappa' u(t)\left[\delta'(a_1 - r) + \frac{2}{r}\delta(a_1 - r)\right] \tag{2.8}$$

The third component of the right side of equation (2.4) represents the acoustic fluid-loading acting on the plate as additional force. The acoustic waves propagating through the fluid must satisfy the wave equation

$$\nabla^2 p(r,z,t) = \frac{1}{c^2}\frac{\partial^2 p(r,z,t)}{\partial t^2} \tag{2.9}$$

where ∇^2 is the two-dimensional Laplacian in cylindrical coordinates

$$\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} \tag{2.10}$$

and c is sound velocity in the fluid. At the fluid-structure interface, the pressure must satisfy the boundary condition [12]

$$\left.\frac{\partial p(r,z,t)}{\partial n}\right|_{z=0} = -\rho_o \frac{\partial^2}{\partial t^2} w(r,t) = -\rho_o \ddot{w}(r,t) \tag{2.11}$$

with n denoting the normal to the structure.

3. SYSTEM DISCRETIZATION

To approximate the plate dynamics, a Fourier–Bessel expansion of the plate displacement is used to discretize the infinite dimensional system (2.5). As showed in [4, 14], the plate displacement can be approximated by

$$w^N(r,t) = \sum_m^N s_m(t) w_m(r) \tag{3.1}$$

where N is considered to be a finite number suitably large for the accurately modelling the system dynamics and $w_m(r)$ is the (0, m) plate mode described as follows [11, 14]

$$w_m(r) = u_{0m}\left[J_0\left(\gamma_m \frac{r}{a}\right) - \frac{J_0(\gamma_m)}{I_0(\gamma_m)} I_0\left(\gamma_m \frac{r}{a}\right)\right] \tag{3.2}$$

$J_0(x), I_0(x)$ designate the cylinder functions, $\gamma_m = k_m a$ is the m-th root of the frequency equation and $s_m(t)$ is the corresponding modal amplitude in time t. In a similar way let us expand the right side of the plate equation of motion (2.4) into series:

$$f_w^N(r,t) = \sum_m^N r_m(t) w_m(r) \tag{3.3}$$

$$f_s^N(r,t) = \sum_m^N u_m(t) w_m(r) \tag{3.4}$$

$$p^N(r, z=0, t) = \sum_m^N z_m(t) w_m(r) \tag{3.5}$$

Inserting above expansions into the equation (2.5), multiplying both sides by the orthogonal eigenfunction $w_n(r)$, and integrating over the surface of the structure S, the governing equation of motion can be re-expressed as

$$\sum_{m=1}^N \begin{bmatrix} \ddot{s}_m(t) + 2\mu\omega_m^2 \dot{s}_m(t) + \omega_m^2 s_m(t) = \\ = r_m(t) + u_m(t) + z_m(t) \end{bmatrix} \tag{3.6}$$

where:

$$\left.\begin{matrix} r_m(t) \\ u_m(t) \\ z_m(t) \end{matrix}\right\} = \iint_S f_j(r,t) w_m(r) dS, \; j = w, s, p; \; m = 1, 2, ..., N \tag{3.7}$$

mean the modal generalised forces.

4. DERIVATION OF MODAL GENERALISED FORCES

To derive the modal generalised forces $r_m(t)$, $u_m(t)$, $z_m(t)$ it is necessary to integrate the analytical expressions according to the formula (3.7). The excitation force (2.3) we can express as $f_w(r, t) = f_w(r)r(t)$. Regarding Eq. (3.7) and using the orthonormality property $\int_0^a w_m(r) w_n(r) r dr = \delta_{mn}$, as a result of integration we get

$$r_m(t) = k_{wm} r(t) \tag{4.1}$$

where

$$k_{wm} = \frac{a F_0 J_1(\gamma_m)}{\gamma_m J_0(\gamma_m) \rho H} \tag{4.2}$$

For the control force $f_s(r, t) = f_s(r)u(t)$ described by (2.4), the result is:

$$u_m(t) = k_{sm} u(t) \tag{4.3}$$

$$k_{sm} = \frac{a[J_0(\gamma_m) I_0\left(\gamma_m \frac{a_1}{a}\right) - J_0\left(\gamma_m \frac{a_1}{a}\right) I_0(\gamma_m)]}{a I_0(\gamma_m)} - \\ - \frac{a_1 \gamma_m [J_0(\gamma_m) I_1\left(\gamma_m \frac{a_1}{a}\right) + J_1\left(\gamma_m \frac{a_1}{a}\right) I_0(\gamma_m)]}{a I_0(\gamma_m)} \tag{4.4}$$

The third component of the right side of Eq. (2.5) can be calculated as follows [6, 8]

$$z_m(t) = \frac{b}{a} \varepsilon_1 \frac{1}{J_0(\gamma_m)} \sum_{n=1}^N \ddot{s}_n(t) c_{mn} \tag{4.5}$$

where $\varepsilon_1 = \dfrac{\rho_0}{\rho H k_0}$ represents the fluid-loading parameter and $c_{mn} = \sum_{l=0}^{\infty} W_{ml} \chi_{0l}(-ih) W_{nl}^T$, with the characteristic function of a circular plate $W_{ml} = \int_{-1}^{1} w_m(\eta) S_{0l}(-ih, \eta) \eta d\eta$ is expressed in spheroidal coordinate system. The coefficients c_{mn}, can be transformed into cylindrical coordination system by assuming $\xi_0 = 0$ with the use of the following expression $r = b[(1-\eta^2)]$[6].

5. TRANSFORMATION INTO STATE-SPACE FORM

As a result of previous calculations the following equation describing the behaviour of m-ith mode of considering system is obtain

$$\ddot{s}_m(t) + 2\mu\omega_m^2 \dot{s}_m(t) + \omega_m^2 s_m(t) = \\ = k_{wm} r(t) + k_{sm} u(t) + \sum_n^N \frac{d^2}{dt^2} s_n(t) c_{mn} \tag{5.1}$$

The matrix notation greatly simplifies the mathematical representation of the system, and provides a form of problem expression which is readily amenable to computer solution. So, we write Eq. (5.1) in the matrix form

$$(\mathbf{I}+\mathbf{C})\ddot{\mathbf{s}}(t)+2\mu\mathbf{\Omega}^2\dot{\mathbf{s}}(t)+\Omega^2\mathbf{s}(t)=\mathbf{K_s}\mathbf{u}(t)+\mathbf{K_w}\mathbf{r}(t) \quad (5.2)$$

In above expression **I** denotes identity matrix, $\mathbf{K_S}$ i $\mathbf{K_W}$ are the coefficient vectors, calculated for each mode with expression (4.2) and (4.4) respectively, **C** represents fluid-plate interaction matrix , $\mathbf{\Omega} = \mathbf{diag}[\omega_1, \omega_2, ..., \omega_N]$.
The modal model presented above can now be expressed in the state space format. To do so, let us define the state vector

$$\mathbf{x}(t)=\begin{bmatrix}\mathbf{s}(t)\\ \dot{\mathbf{s}}(t)\end{bmatrix} \quad (5.3)$$

Equation (5.2) can be expressed as

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{V}\mathbf{r}(\mathbf{t}) \quad (5.4)$$

where the dot denotes differentiation with respect to time, **x** is the (nx1) state vector, **u** is (mx1) control vector, and **A** is (nxn) state matrix, **B** is the (nxm) control input matrix, **V** is (1xn) disturbance matrix:

$$\mathbf{A}=\begin{bmatrix}0 & 1\\ -(\mathbf{I}+\mathbf{C})^{-1}\Omega^2 & -2\mu(\mathbf{I}+\mathbf{C})^{-1}\Omega^2\end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}0\\ (\mathbf{I}+\mathbf{C})^{-1}\mathbf{K}_s\end{bmatrix} \quad (5.5)$$

$$\mathbf{V}=\begin{bmatrix}0\\ (\mathbf{I}+\mathbf{C})^{-1}\mathbf{K_w}\end{bmatrix}$$

The above state-space model of the considered system will be used in the process of designing optimal feedback control so as to suppress the plate vibrations.

6. COMPUTER SIMULATION OF THE FEEDBACK CONTROL

The goal of control problem is to determine a voltage $u(t)$ which, when applied to the piezoactuators, leads to a significantly reduced level of vibration. For the system (5.4) one possible approach is to obtain a solution by applying the well known linear-quadratic regulator (LQR). LQR method consists on using a control law [2]

$$\mathbf{u}(t)=-\mathbf{K}\mathbf{x}(t) \quad (6.1)$$

which minimise the cost function given by

$$J=\frac{1}{2}\int_0^\infty\left(\mathbf{x}^\mathbf{T}\mathbf{Q}\mathbf{x}+\mathbf{u}^\mathbf{T}\mathbf{R}\mathbf{u}\right)dt \quad (6.2)$$

where **Q**, **R** denote weighting matrices chosen as follows:

$$\mathbf{R}=\left[\frac{\beta}{u_{\max}}\right],\ \mathbf{Q}=\begin{bmatrix}\Omega^2 & \mathbf{0}\\ 0 & \alpha\Omega^2\end{bmatrix} \quad (6.3)$$

and α and β are the weight coefficients. The problem is to determine the gain matrix **K** which facilitates our requirements. The optimal solution is [2]

$$\mathbf{K}=\mathbf{R}^{-1}\mathbf{B}^\mathbf{T}\mathbf{P} \quad (6.4)$$

where the matrix **P** is the unique, positive definite solution to the algebraic Riccati equation

$$\mathbf{A}^\mathbf{T}\mathbf{P}+\mathbf{P}\mathbf{A}-\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^\mathbf{T}\mathbf{P}+\mathbf{Q}=0 \quad (6.5)$$

In simulations the model including the four first modes of the aluminium plate of a 0.46 m diameter and 1 mm thickness was applied. Since the system (5.4) has all its poles in the left half plane and is both stabilisable and detectable, the feedback gains are obtained by solving the Riccati equation and by applying the Schur tuning technique.

For nominal parameters presented in Table 1, the state-space model of the system is discretized by using a sampling time period of 0.0001 s. Figures show the time response of the system with and without the control feedback obtained with the MATLAB/Simulink computer program.

Figure 2 presents the results obtained at 70 Hz, which corresponds to the resonance frequency of the first mode. It can be seen that the controller that is started after 0.6 second, provides substantial damping within very short time (attenuation of 80%) in case when the piezo-actuator covers 1% of controlled plate surface (a_1/a = 0.1). As can be expected, this result is better for piezodiscs of greater radius: if piezo-actuator covers 9% of plate surface (a_1/a = 0.3), the vibration cancellation exceeds 95% (Fig. 3). Similar simulation results can be observed for sinusoidal disturbance of 200 Hz, closed to second plate mode, i.e. (0.2), but obtained attenuation is worsen (Figs. 4 and 5).

Table 1. The physical material properties used in simulations

Para-meter	Density [Kg/m^3]	Young's modulus [N/m^2]	Poisson's ratio	Thickness [m]	Radius [m]	Strain Constant [N/V]
Plate	$\rho = 2700$	$E = 7.1*10^{10}$	$\nu = 0.33$	h = 0.001	a = 0.23	–
PZT	$\rho = 7600$	$E = 6*10^{10}$	$\nu = 0.3$	h = 0.0001	a_1 = 0.02	$d_{31} = 190*10^{-12}$

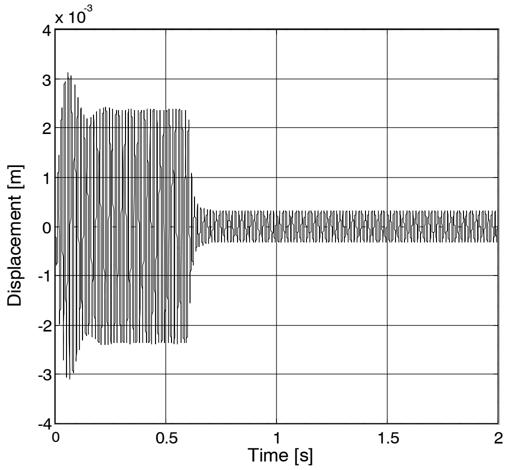

Fig. 2. Simulation results for sampling time 0.0001 s obtained for $a_1/a = 0.1$ The time response of the open-loop system (0–0.6 s) and the closed-loop system (0.6–2 s) for sinusoidal disturbance of 70 Hz

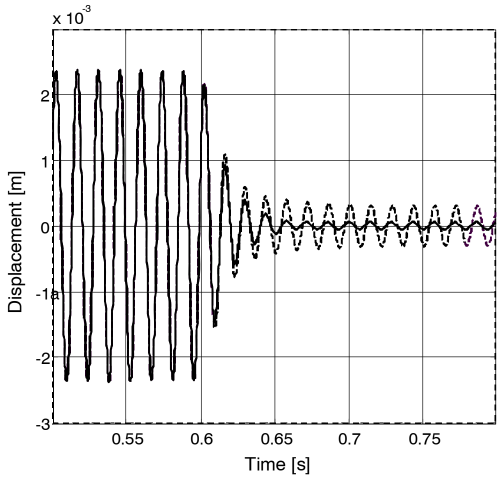

Fig. 3. Simulation results for sampling time 0.0001 s obtained for two ratio of a_1/a.
$a_1/a = 0.3$ (solid line); $a_1/a = 0.1$ (dashed line). The part of the time response of the open-loop system (0–0.6 s) and the closed-loop system (0.6–2 s) for sinusoidal disturbance of 70 Hz

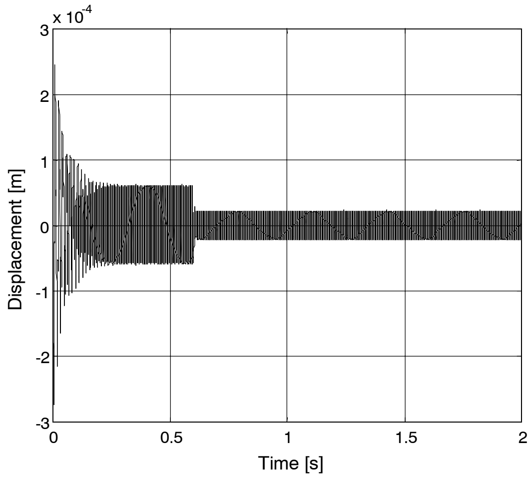

Fig. 4. Simulation results for sampling time 0.0001 s and $a_1/a = 0.3$. The time response of the open-loop system (0–0.6 s) and the closed-loop system (0.6–2 s) for sinusoidal disturbance of 200 Hz

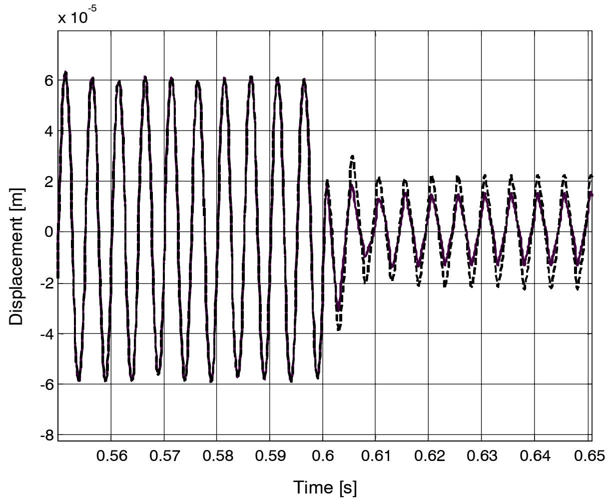

Fig. 5. Simulation results for sampling time 0.0001 s obtained for two ratio of a_1/a.
$a_1/a = 0.3$ (solid line); $a_1/a = 0.1$ (dashed line). The part of the time response of the open-loop system (0–0.6 s) and the closed-loop system (0.6–2 s) for sinusoidal disturbance of 200 Hz

7. CONCLUSION

The paper is concerned with the problem of active attenuation of plate vibrations in contact with fluid. The aim of this work was to investigate the feasibility of using a pair of PZT actuators, positioned symmetrically on each side of the circular plate to suppress its vibrations. It was assumed that the plate was clamped at the circumference in the planar finite baffle and radiated into a lossless homogeneous liquid medium. The mathematical model of the plate included the influence of the acoustic wave radiated by the plate on its vibrations. The acoustic pressure was calculated on the basis of the known distribution of vibration velocity in a series of eigenfunctions, using properties of the oblate spheroidal coordinates. Primary excitation is provided by a low frequency loudspeaker installed centrally at the bottom of the cylinder and 2-layer piezo disk elements (PZT) are used as the actuators. For the designing process this continuous model was reduced and approximated by the linear state-space model while the orthogonal series method was applied. An optimal reduction of the plate vibrations was obtained using a linear quadratic regulator (LQR). The simulations of the active attenuation of the plate vibrations were made with a MATLAB/Simulink computer program. The results demonstrate that it is possible to achieve a significant reduction of the vibration amplitude with the use of a pair PZT actuators, for the low frequency bandwith.

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