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OPTIMAL REDUCTION OF VIBRATIONS OF A ONE- OR TWO-AXIS GYROSCOPIC PLATFORM ON BOARD OF A FLYING OBJECT

SUMMARY

The paper presents a method of optimal reduction of vibrations of a one-or two-axis gyroscopic platform mounted on board of a flying object. The included algorithm allows one to select optimal parameters of the platform as well as optimal coefficients of the controller, and, in consequence, minimise the deviations from the preset motion. The investigations focused on the platform operation affected by short-duration external disturbances, manoeuvres of the aerial vehicle, and dry or viscous friction in the bearings.

Keywords: *optimal reduction of vibrations, gyroscopic platform, pre-programmed motion*

OPTYMALNA REDUKCJA DRGAŃ PLATFORMY JEDNO- I DWUOSIOWEJ
NA POKŁADZIE OBIEKTU LATAJĄCEGO

W pracy przedstawiona jest metoda optymalnej redukcji drgań jedno- i dwuosiowej platformy żyroskopowej umieszczonej na pokładzie obiektu latającego. Przytoczony jest algorytm doboru optymalnych parametrów zarówno samej platformy, jak i współczynników regulatora, dzięki którym minimalizowane są błędy między ruchem zadanym a rzeczywistym. Badania przeprowadzone zostały w warunkach zakłóceń działających na wspomniane platformy, tj. przy krótkotrwałych zakłóceniach zewnętrznych, manewrach obiektu latającego oraz uwzględnieniu tarcia suchego i wiskotycznego w łożyskach.

1. INTRODUCTION

Today's unmanned aerial vehicles have numerous applications. They can be used in military missions for missile homing, reconnaissance of the enemy's territory or detecting changes in the deployment of the enemy's land and marine forces, etc. The aim of civil missions, on the other hand, is, for instance, to observe power lines, control traffic or report changes in the water level during violent storms and under other unfavourable weather conditions. Such applications require employing special-purpose heads with units responsible for homing, tracing or detecting (see Fig. 1). Their operation needs to be stable unaffected by the board vibrations. Thus, a reference system common to all these units is necessary. Stability is maintained with the aid of stabilisation systems, for instance, gyroscopic platforms.

Depending on the design and functions, gyroscopic platforms are mounted in one, two, three and even four frames with two, three, four and five axes to assure complete freedom of motion. This paper will discuss only one- and two-axis structures.

The design of a one-axis gyroscopic platform is analogous to that of a free gyroscope. Both possess the same number of degrees of freedom, the only difference being that the outer frame is constituted by the platform. As can be seen from Figure 2, the structure and properties of a gyroscopic platform are generally the same as those of a gyroscope on a Cardan universal joint with a system of interframe misalignment correction. On top of the platform one can mount various devices such as sensors, a television camera or a target coordinator. It is required that their motion be independent of any angular displacements of the base (i.e. the vehicle board) in relation to the Oz_p axis.

One-axis gyroscopic platforms and two-axis gyroscopic platforms are used for similar purposes, yet, in the latter case, angular motion around two axes is possible. A simplified diagram of a power two-axis gyroscopic platform is presented in Figure 3.

Similarly, it is possible to mount on top of a two-axis platform various navigational devices or sensors of other subsystems responsible for control that are unaffected by the angular motions of the base. The principle of operation of each stabilisation channel in a two-axis gyroscopic platform, whether at the initial setting stage or during actual operation, is analogous to that of a one-axis platform. The main difference results from the fact that in a two-axis platform there exist strong interdependence between the channels, which causes errors, and, consequently, a decrease in precision and accuracy. Thus, it is crucial to impose additional requirements concerning the selection of parameters for the particular elements of the stabilisation channels.

The work presents equations of motion for controlled one- and two-axis gyroscopic platforms and an algorithm for the selection of optimal control parameters. Moreover, the authors discuss the results of simulations of a gyroscopic platform operating under the conditions of external disturbances, i.e. board vibrations, and vehicle manoeuvres.

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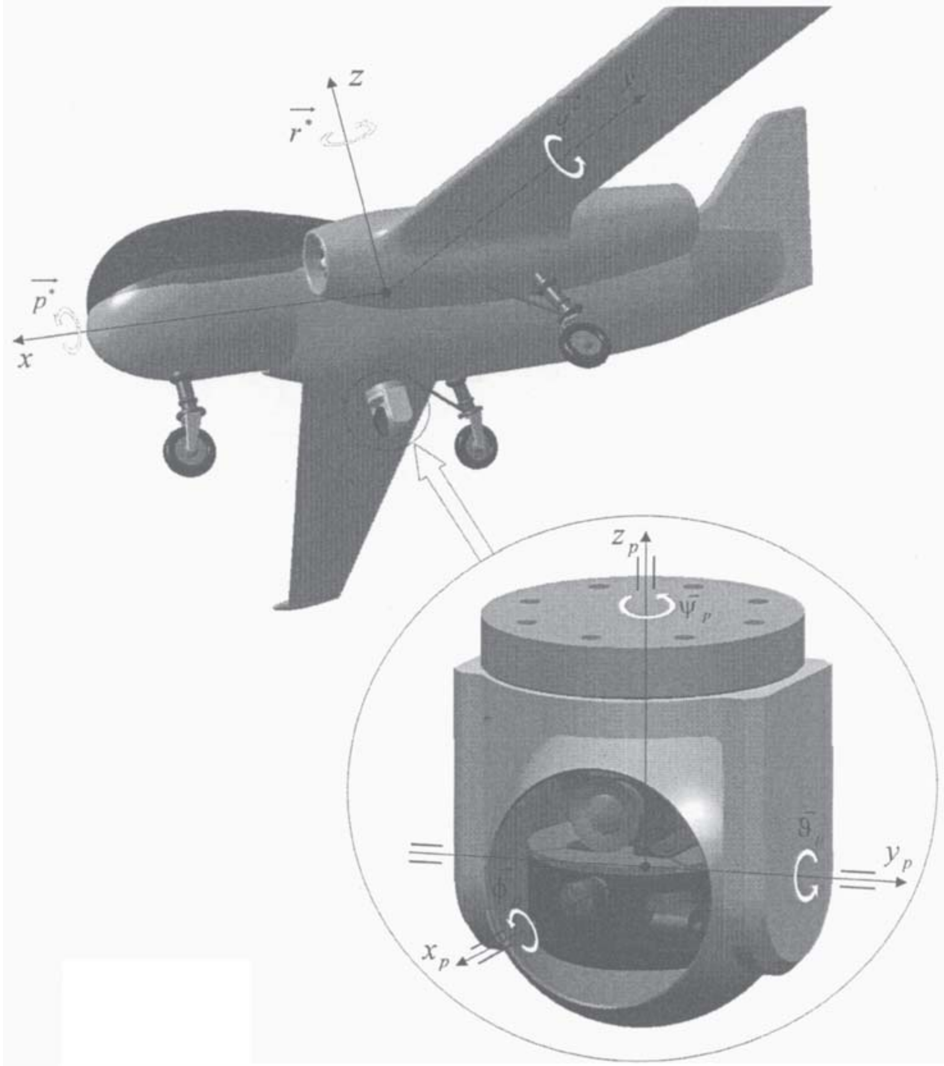

Fig. 1. View of a detecting head stabilised by a gyroscopic platform

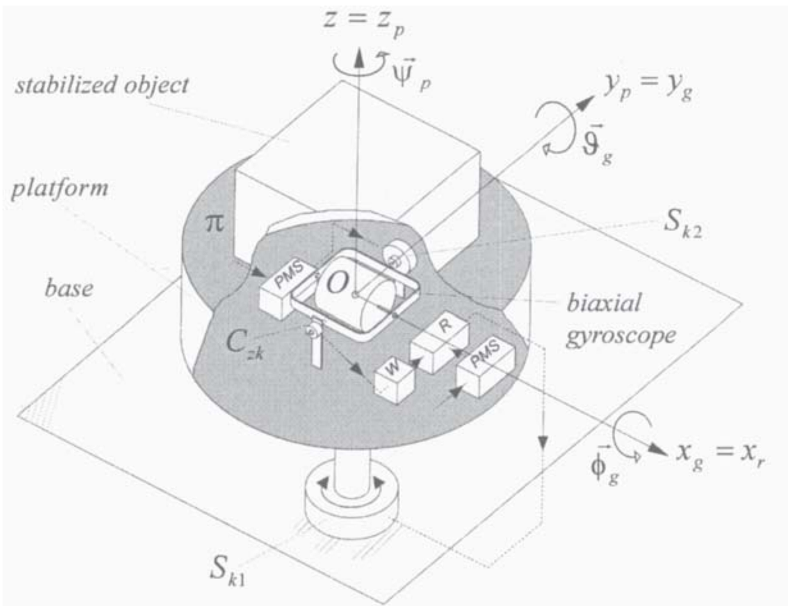

Fig. 2. Schematic diagram of a power one-axis gyroscopic platform

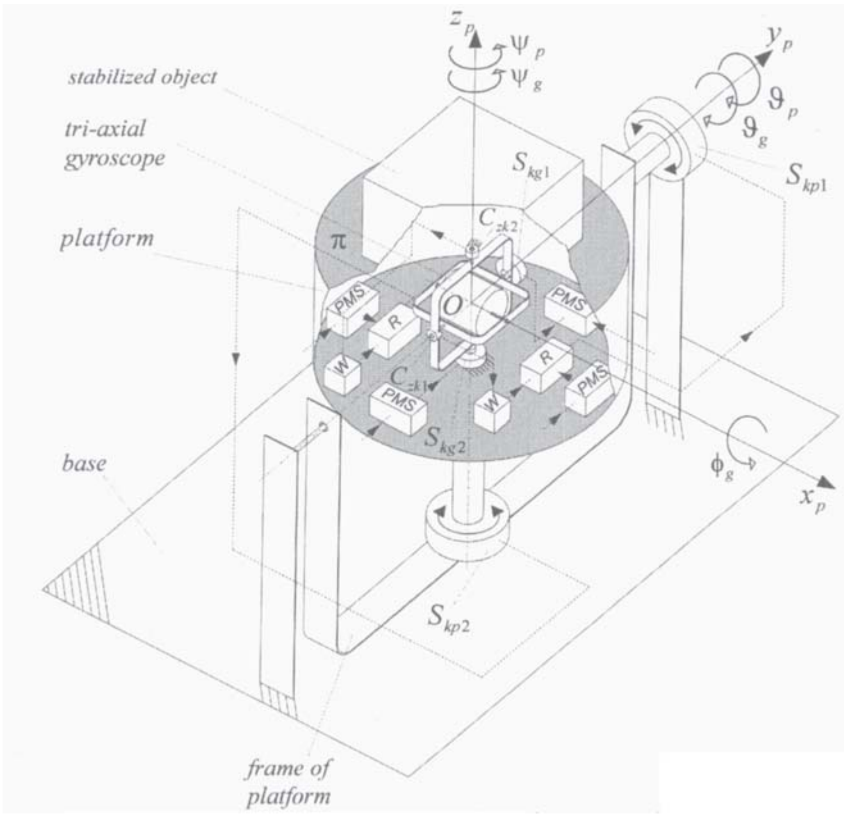

Fig. 3. Schematic diagram of a two-axis gyroscopic platform

2. EQUATIONS OF MOTION FOR A ONE-AXIS GYROSCOPIC PLATFORM ON BOARD OF A FLYING OBJECT

Linearised equations of motion for a power one-axis gyroscopic platform (on the basis of which the control law will be formulated) are given by Eqs. (1a) and (1b). Thus, if we consider a case of negligible angular deflections of the axes of the gyroscopes and the platform elements from their initial positions, neglecting the velocity products as quantities of a lower order and assuming that the gyroscopes are astatic and the inertia of their frames is negligible, we have:

$$J_{gk}\left(\ddot{\vartheta}_g+\dot{q}^*\right)+J_{go}n_g\left(\dot{\psi}_p+r^*\right)=M_{k_g}-M_{r_g} \tag{1a}$$

$$\left(J_{gk}+J_{z_p}\right)\left(\ddot{\psi}_p+\dot{r}^*\right)-J_{go}n_g\left(\dot{\vartheta}_g+q^*\right)= =M_{k_p}-M_{r_p} \tag{1b}$$

where:

- J_{go}, J_{gk} – moments of inertia of the gyroscope rotor;
- $J_{z_{rz}}$ – moment of inertia of the platform in relation to the Oz_p axis;
- ϑ_g, ψ_p – angles determining the positions of the axes of the gyroscope frame and the platform;
- n_g – angular velocity of the gyroscope rotor;
- p^*, q^*, r^* – angular velocities of the flying object;
- M_{rg}, M_{rp} – moments of friction forces in the bearings of the spin axes of the gyroscope frame and the platform;
- M_{kg}, M_{kp} – stabilisation moments produced by the correction motors of the gyroscope frame and the platform.

3. EQUATIONS OF MOTION FOR A TWO-AXIS GYROSCOPIC PLATFORM ON BOARD OF A FLYING OBJECT

With the same assumptions of linearization as those presented in Section 2, the equations of motion for a two-axis gyroscopic platform are as follows:

- equations describing the motion of the gyroscope frames:

$$J_{gk}\left(\ddot{\vartheta}_g+\ddot{\vartheta}_p+\dot{q}^*\right)+J_{go}n_g\left(\dot{\Psi}_g+\dot{\Psi}_p+r^*\right)= =M_{kg1}-M_{rg2} \tag{2a}$$

$$J_{gk}\left(\ddot{\Psi}_g+\ddot{\Psi}_p+\dot{r}^*\right)-J_{go}n_g\left(\dot{\vartheta}_g+\dot{\vartheta}_p+q^*\right)= =M_{kg2}-M_{rg1} \tag{2b}$$

– equations describing the motion of the platform and its frame:

$$J_{gk}\left(\ddot{\psi}_p+\ddot{\psi}_g+\dot{r}^*\right)+J_{z_p}\left(\ddot{\psi}_p+\dot{r}^*\right)-$$
$$-J_{go}n_g\left(\dot{\vartheta}_g+\dot{\vartheta}_p+q^*\right)= \qquad (2c)$$
$$=M_{kp2}-M_{rp}$$

$$J_{gk}\left(\ddot{\vartheta}_p+\ddot{\vartheta}_g+\dot{q}^*\right)+$$
$$+\left(J_{y_p}+J_{y_r}+m_r l_r^2+m_p l_p^2\right)\left(\ddot{\vartheta}_p+\dot{q}^*\right)+ \qquad (2d)$$
$$+J_{go}n_g\left(\dot{\psi}_g+\dot{\psi}_p+r^*\right)+\left(m_r l_r+m_p l_p\right)\dot{V}_p=$$
$$=M_{kp1}-M_{rr}$$

where:

$J_{y_p}, J_{z_p}, J_{y_r}$ – moments of inertia of the platform elements;
m_p, m_r – masses of the platform and frame platform;
l_p, l_r – distance of the centre of gravity of the platform and frame platform from the platform geometric rotation centre;
$\vartheta_g, \psi_g, \vartheta_p, \psi_p$ – angles determining the positions of the particular spin axes of the gyroscope frames, the platform and its frame;
n_g – angular velocities of the rotors of gyroscope;
M_{ri} – moments of friction forces in the bearings of the spin axes of the particular gyroscope and platform elements;
M_{kgi} – stabilisation moments produced by the correction motors of the gyroscope frames;
M_{kpi} – stabilisation moments produced by the correction motors of the platform and its frame.

4. OPTIMISING THE CONTROL PARAMETERS AND THE CONTROLLER COEFFICIENTS

Let us present the equations of motion concerning the open-loop platform control system in the following vector-matrix form

$$\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu} \qquad (3)$$

The same equation for the closed-loop control system will be

$$\dot{\mathbf{x}}^*=\mathbf{Ax}^*+\mathbf{Bu}_k \qquad (4)$$

The control law for stabilisation, $\mathbf{u}_k$, will be determined using the method of linear and square optimisation with the functional in the form

$$J=\int_0^\infty\left[\left(\mathbf{x}^*\right)^T\mathbf{Qx}^*+\mathbf{u}_k^T\mathbf{Ru}_k\right]dt \qquad (5)$$

Let us present the law in the following form

$$\mathbf{u}_k=-\mathbf{K}\cdot\mathbf{x}^* \qquad (6)$$

The matrix of coupling **K** given by Equation (6) is determined from the following relationship

$$\mathbf{K}=\mathbf{R}^{-1}\cdot\mathbf{B}^T\cdot\mathbf{P} \qquad (7)$$

Matrix **P** is a solution of the algebraic Riccati equation

$$\mathbf{A}^T\mathbf{P}+\mathbf{PA}-2\mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P}+\mathbf{Q}=\mathbf{0} \qquad (8)$$

Substituting (6) into (4), we get the state equations in a new form

$$\dot{\mathbf{x}}^*=\left(\mathbf{A}-\mathbf{BK}\right)\mathbf{x}^*=\mathbf{A}^*\cdot\mathbf{x}^* \qquad (9)$$

where

$$\mathbf{A}^*=\mathbf{A}-\mathbf{BK} \qquad (10)$$

Recall that the platform motion should be as stable as possible. It is crucial, then, to reduce the transitional processes to a minimum, whenever a disturbance occurs and control is applied. Select optimal parameters of the system described by Eq. (9). Then, apply a modified Golubencev optimisation method [2], the algorithm of which is shown in Figure 4.

In accordance with the modified Golubencev method, introduce a new exchange determined by the formula

$$\mathbf{x}^*(t)=\mathbf{y}^*(t)\cdot e^{\delta(t)} \qquad (11)$$

while

$$\delta=\frac{1}{n}Tr\mathbf{A}^* \qquad (12)$$

After the transformation, we get

$$\frac{d\mathbf{y}}{dt}=\mathbf{B}^*\cdot\mathbf{y} \qquad (13)$$

where

$$\mathbf{B}^*=\mathbf{A}^*-\mathbf{I}\cdot\delta \qquad (14)$$

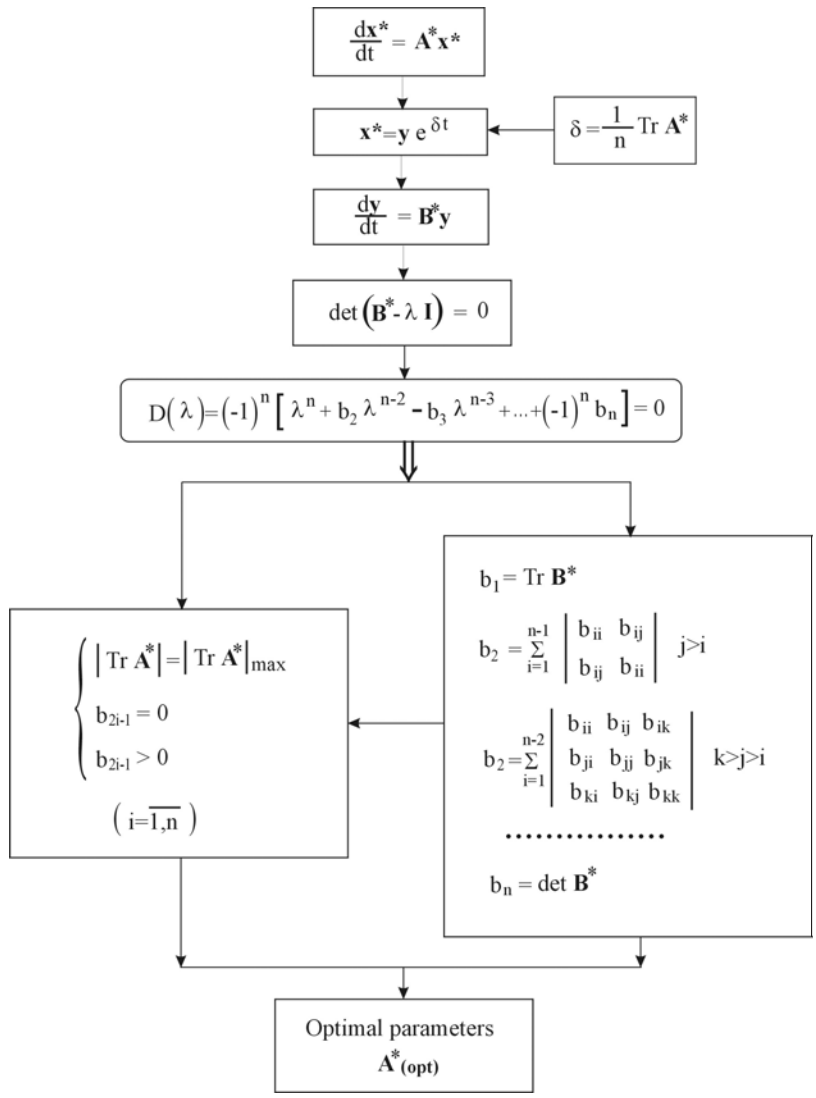

Fig. 4. Algorithm for the optimal selection of control parameters using the Golubencev method

The characteristic equation of matrix $\mathbf{B}^*$, when $Tr\mathbf{B}^* = 0$, is transformed into a characteristic polynomial in the form

$$(-1)^n \left[\lambda^n + b_2\lambda^{n-2} - b_3\lambda^{n-3} + ... + (-1)^n b_n\right] = 0 \qquad (15)$$

For matrix $\mathbf{B}^*$ we need to find selected values of elements (appropriate coefficients of attenuation and amplification) so that characteristic equation (14) will have imaginary roots or ones equal to zero [2]. Therefore, the coefficients of characteristic equation (8) b_2, b_3, ..., b_n (coefficient $b_1 = Tr\mathbf{B}^* = 0$) have to be defined as sums of all possible combinations of principal minor determinants of the successive 2nd, 3rd, … and *nth* degree of matrix $\mathbf{B}^*$ described by (14).

Furthermore, we need to take account of a very important condition of maximisation of the absolute value of the trace of matrix $\mathbf{A}^*$ determined by expression (10):

$$\left|Tr\mathbf{A}^*\right| = \left|a_{11} + a_{22} + ... + a_{nn}\right| = \max \qquad (16)$$

The parameters platform optimised in the way described above assure minimisation of dynamic effects and prompt attenuation of transitional processes. The method is particularly useful when maintaining the desired position of a platform on board of an aerial vehicle performing manoeuvres and affected by external disturbances.

5. RESULTS AND CONCLUSIONS

Figures 5 to 7 show selected results concerning a one-axis gyroscopic platform. In Figures 5, we can see that at the initial stage the platform rotates around its axis with an angular velocity of 0.5 rad/s.

It is clear that before a transitional process is damped, both the gyroscope axis and the stabilisation axis slightly deflect from their set (zero) positions (see Fig. 5a), which is due to friction in the suspension bearings. Only by applying optimal correction control can an operational error of a one-axis gyroscopic platform be minimised (Fig. 5b).

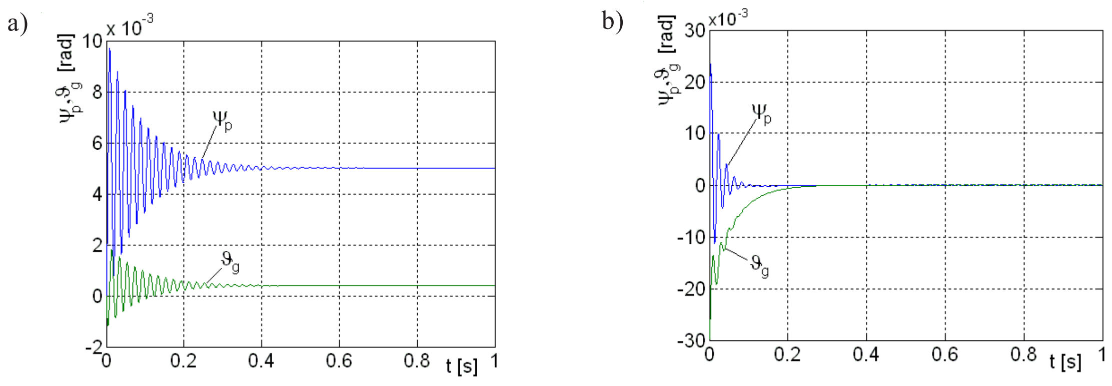

Fig. 5. Motion of a one-axis gyroscopic platform caused by the initial conditions: a) without correction control; b) with correction control

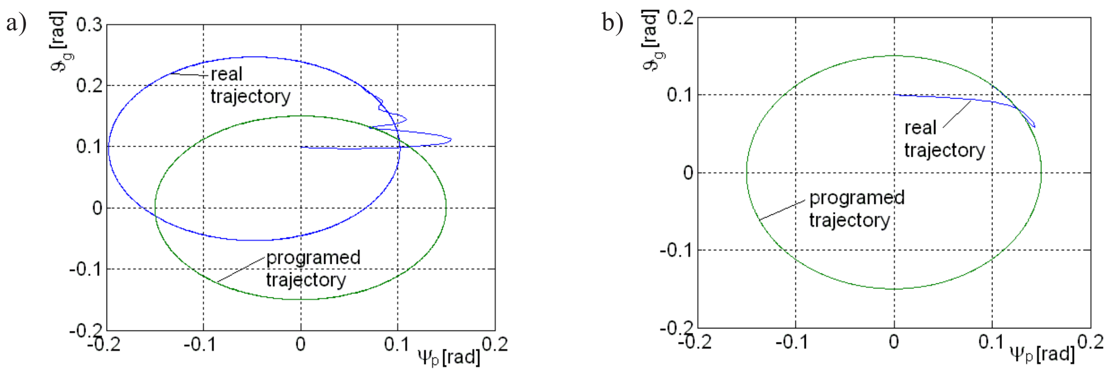

Fig. 6. Unaffected pre-programmed motion of a one-axis gyroscopic platform: a) in the open-loop system; b) with feedback

Figure 6 presents set and actual motions of the stabilisation axis of a one-axis gyroscopic platform under initial conditions inconsistent with the set conditions. The stabilisation axis of the platform performs a pre-programmed motion deflected by a certain angular value in relation to the preset motion (Fig. 6a). When the optimal closed-loop control system is applied, the stabilisation axis of a one-axis gyroscopic platform performs the preset motion exactly (Fig. 6b). Figure 7 illustrates the influence of a short-duration kinematic impact of the base (i.e. vibrations of the vehicle board) on the pre-programmed motion of the platform axis.

Also in this case, optimal control reduces an unfavourable effect of a disturbance to a necessary minimum (Fig. 7b).

Figures 8 through 10 present results of an investigation into two-axis gyroscopic platforms. Again, the effectiveness of an optimal closed-loop control system is confirmed. Optimal control minimises each error to an admissible value somewhere between that of the actual motion and that of the preset motion. It allows reducing the influence of the kinematic excitation of the base (i.e. the vehicle board).

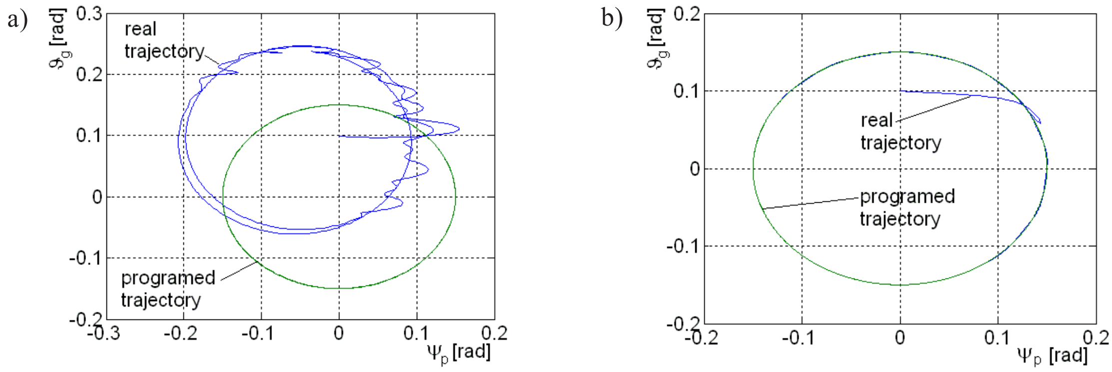

Fig. 7. Pre-programmed motion of a one-axis gyroscopic platform affected by disturbances: a) in the open-lop system; b) with feedback

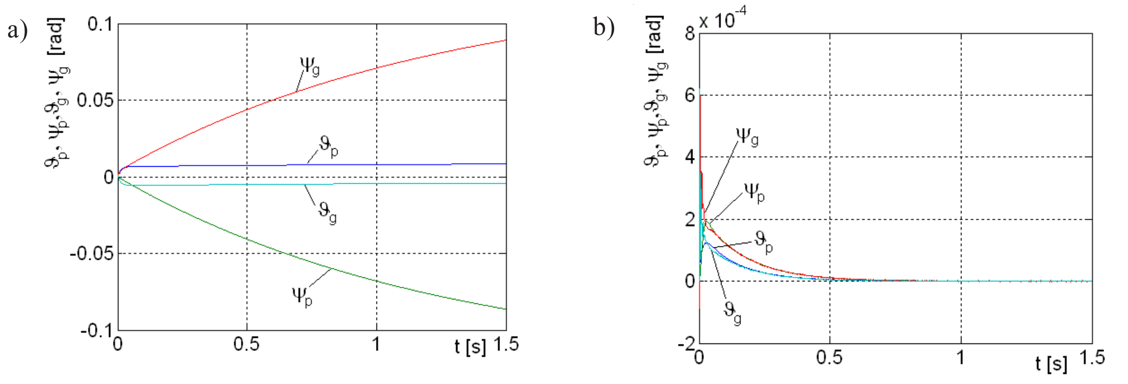

Fig. 8. Angular position variations of a two-axis gyroscopic platform in the time function under initial conditions: a) without correction control; b) with correction control

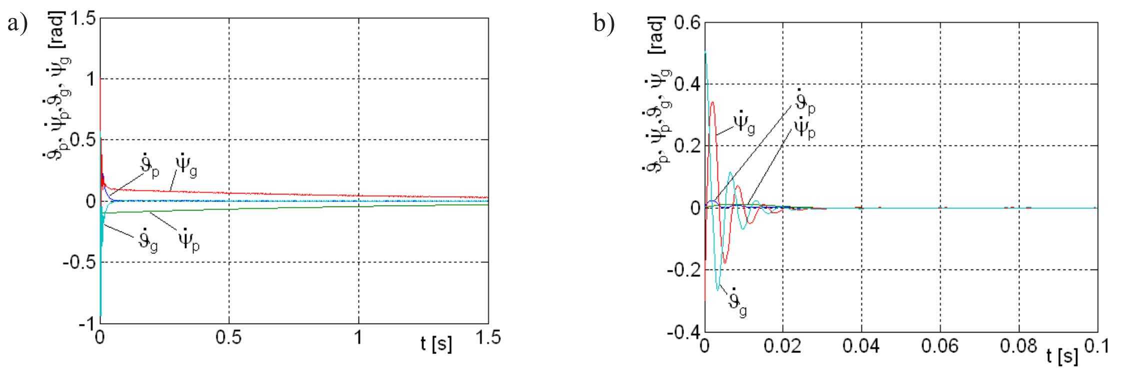

Fig. 9. Time-dependent angular velocities of a two-axis gyroscopic platform under initial conditions: a) without correction control; b) with correction control

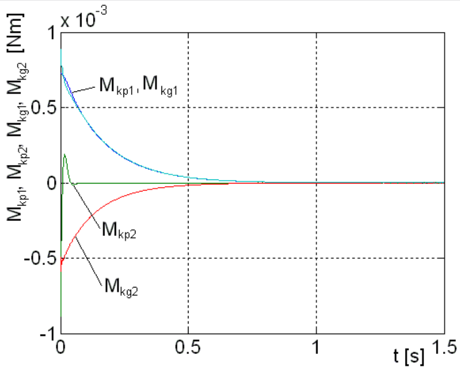

Fig. 10. Time-dependent correction moments of a two-axis gyroscopic platform

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