MECHANICS
Vol. 24 No. 2 2005

Zbigniew KORUBA*, Jerzy STYCZEŃ*

ACTIVE CONTROL OF VIBRATIONS OF A THREE-AXIS GYROSCOPIC PLATFORM ON BOARD OF A FLYING OBJECT

SUMMARY

The work presents an algorithm for active control of vibrations of a three-axis gyroscopic platform during pre-programmed and stabilised operation on board of a flying object. The considerations focus on disturbances resulting from the flying object manoeuvres and short-duration pulses of external forces under the conditions of friction in the bearings and non-linear platform operation range.

Keywords: *active control, gyroscopic platform, control and pre-programmed moments*

AKTYWNE STEROWANIE DRGANIAMI PLATFORMY TRZYOSIOWEJ NA POKŁADZIE OBIEKTU LATAJĄCEGO

W pracy przedstawiony jest algorytm aktywnego sterowania drganiami trzyosiowej platformy żyroskopowej przy jej ruchu programowym i stabilizującym na pokładzie obiektu latającego. Rozpatrywane są zakłócenia w postaci manewrów obiektu latającego (podstawy platformy) oraz krótkotrwałych impulsów sił zewnętrznych przy istnieniu tarcia w łożyskach i przy nieliniowym zakresie pracy rozważanej platformy.

1. INTRODUCTION

It is required that systems for homing, navigation and field observation used for military or commercial purposes, particularly in aviation, have a reference unit to improve detecting and pinpointing objects, collecting navigational data, and tracking moving targets. The reference unit should maintain constant position and orientation unaffected by any external disturbances and operate in accordance with the predefined program. The most suitable device for such a task is a gyroscopic platform.

The paper discusses a three-axis gyroscopic platform capable of preventing a stabilised device (e.g. a homing head, a tracing head, a head for laser illumination of a ground target, a television camera or an infrared camera) from being affected by angular variations of the flying object. As can be seen from Figure 1, the three-axis gyroscopic platform includes two sensor gyroscopes with three degrees of freedom. The platform has to possess at least two (inner and outer) frames. The platform as well as the frames are equipped with sensors of angular variations and transmitters of control moments. The gyroscopes are arranged inside the platform in such a way that the measurement axes of each gyroscope are parallel to specified axes of the platform frames. Gyroscope 1 has its main axis parallel to the Ox_p axis of the platform, that is why it can measure the platform rotations around the Oy_p and Oz_p axes. The main axis of Gyroscope 2 is parallel to the Oy_p platform axis and can measure the platform rotations around the Ox_p and Oz_p axes. In a three-axis platform there is an interaction of motions around the three axes of suspension. The stabilisation systems affect one another, which means that disturbances on one axis are usually passed on to the other two axes. In addition, assuming that the platform operates under the conditions of vibrations and external disturbances, it becomes clear that the control parameters need to be optimally selected at both design and operation stages.

This work deals with simulations of a controlled gyroscopic platform model with a closed loop system for optimising control parameters by applying the LQR method [1].

2. EQUATIONS OF MOTION FOR A THREE-AXIS GYROSCOPIC PLATFORM ON BOARD OF AN AERIAL VEHICLE

Due to limited space, the paper discusses a linearised platform model. Thus, if we consider only the case of negligible angular deflections of the gyroscope axes and the platform elements axes from their initial positions, negligible velocity products as quantities of lower order, and if we assume that the gyroscopes are astatic and the inertia of their frames is negligible, we have:

– equations describing the motions of the gyroscopes;

$$J_{gk}\left(\ddot{\vartheta}_{g1}-\ddot{\psi}_p-\dot{r}^*\right)+J_{go}n_{g1}\left(\dot{\psi}_{g1}+\dot{\vartheta}_p+q^*\right)= M_{kg1_2}-M_{r2_{g1}} \tag{1a}$$

$$J_{gk}\left(\ddot{\vartheta}_{g2}+\ddot{\phi}_p+\dot{p}^*\right)+J_{go}n_{g2}\left(\dot{\psi}_{g2}-\dot{\psi}_p-r^*\right)= M_{kg2_2}-M_{r2_{g2}} \tag{1b}$$

$$J_{gk}\left(\ddot{\psi}_{g1}+\ddot{\vartheta}_p+\dot{q}^*\right)+J_{go}n_{g1}\left(\dot{\psi}_p-\dot{\vartheta}_{g1}+r^*\right)= M_{kg1_1}-M_{r1_{g1}} \tag{1c}$$

$$J_{gk}\left(\ddot{\psi}_{g2}-\ddot{\psi}_p-\dot{r}^*\right)+J_{go}n_{g2}\left(\dot{\phi}_p-\dot{\vartheta}_{g2}+p^*\right)= M_{kg2_1}-M_{r1_{g2}} \tag{1d}$$

* Faculty of Mechatronics and Machinery Design, Kielce University of Technology, Kielce, Poland, ksmzko@tu.kielce.pl

– equations describing the motion of the platform elements (the platform itself, the inner and outer frames);

$$\begin{aligned}&\left(J_{x_p}+J_{gk}+m_p l_p^2+l_{g1p}^2(m_{1_{g1}}+m_{2_{g1}}+m_{3_{g1}})+l_{g2p}^2(m_{1_{g2}}+m_{2_{g2}}+m_{3_{g2}})\right)\left(\ddot{\phi}_p+\dot{p}^*\right)+J_{gk}\ddot{\vartheta}_{g2}-\\&-J_{go}n_{g2}\left(\dot{\psi}_p+\dot{\psi}_{g2}+r^*\right)+V_p m_p l_p\left(\dot{\psi}_p+r^*\right)+V_p(m_{1_{g1}}+m_{2_{g1}}+m_{3_{g1}})l_{g1p}\left(\dot{\psi}_{g1}+\dot{\vartheta}_p+q^*\right)-\\&-V_p(m_{1_{g2}}+m_{2_{g2}}+m_{3_{g2}})l_{g2p}\left(\dot{\vartheta}_p+q^*\right)=M_{kp3}-M_{rp}\end{aligned} \tag{1e}$$

$$\begin{aligned}&\left(J_{y_{rw}}+J_{y_p}+J_{gk}+m_p l_p^2\right)\left(\ddot{\vartheta}_p+\dot{q}^*\right)+J_{gk}\ddot{\psi}_{g1}+J_{go}n_{g1}\left(\dot{\psi}_p-\dot{\vartheta}_{g1}+r^*\right)+m_p l_p\dot{V}_p+\\&+V_p\left[-2(m_{1_{g1}}+m_{2_{g1}}+m_{3_{g1}})l_{g1p}\left(\dot{\phi}_p+p^*\right)+(m_{1_{g2}}+m_{2_{g2}}+m_{3_{g2}})l_{g2p}\dot{\phi}_p\right]=M_{kp2}-M_{rrw}\end{aligned} \tag{1f}$$

$$\begin{aligned}&\left(J_{z_{rz}}+J_{z_{rw}}+J_{z_p}+2J_{gk}+l_{g1p}^2(m_{1_{g1}}+m_{2_{g1}}+m_{3_{g1}})+l_{g2p}^2(m_{1_{g2}}+m_{2_{g2}}+m_{3_{g2}})\right)\left(\ddot{\psi}_p+\dot{r}^*\right)-\\&-J_{gk}\ddot{\vartheta}_{g1}-J_{gk}\ddot{\psi}_{g2}-J_{go}n_{g1}\left(\dot{\vartheta}_p+\dot{\psi}_{g1}+q^*\right)+J_{go}n_{g2}\left(\dot{\phi}_p+\dot{\vartheta}_{g2}+p^*\right)+\\&+\left(l_{g1p}(m_{1_{g1}}+m_{2_{g1}}+m_{3_{g1}})-l_{g2p}(m_{1_{g2}}+m_{2_{g2}}+m_{3_{g2}})\right)\dot{V}_p+V_p m_p l_p\left(\dot{\vartheta}_p-\dot{\phi}_p\right)=M_{kp1}-M_{rrz}\end{aligned} \tag{1g}$$

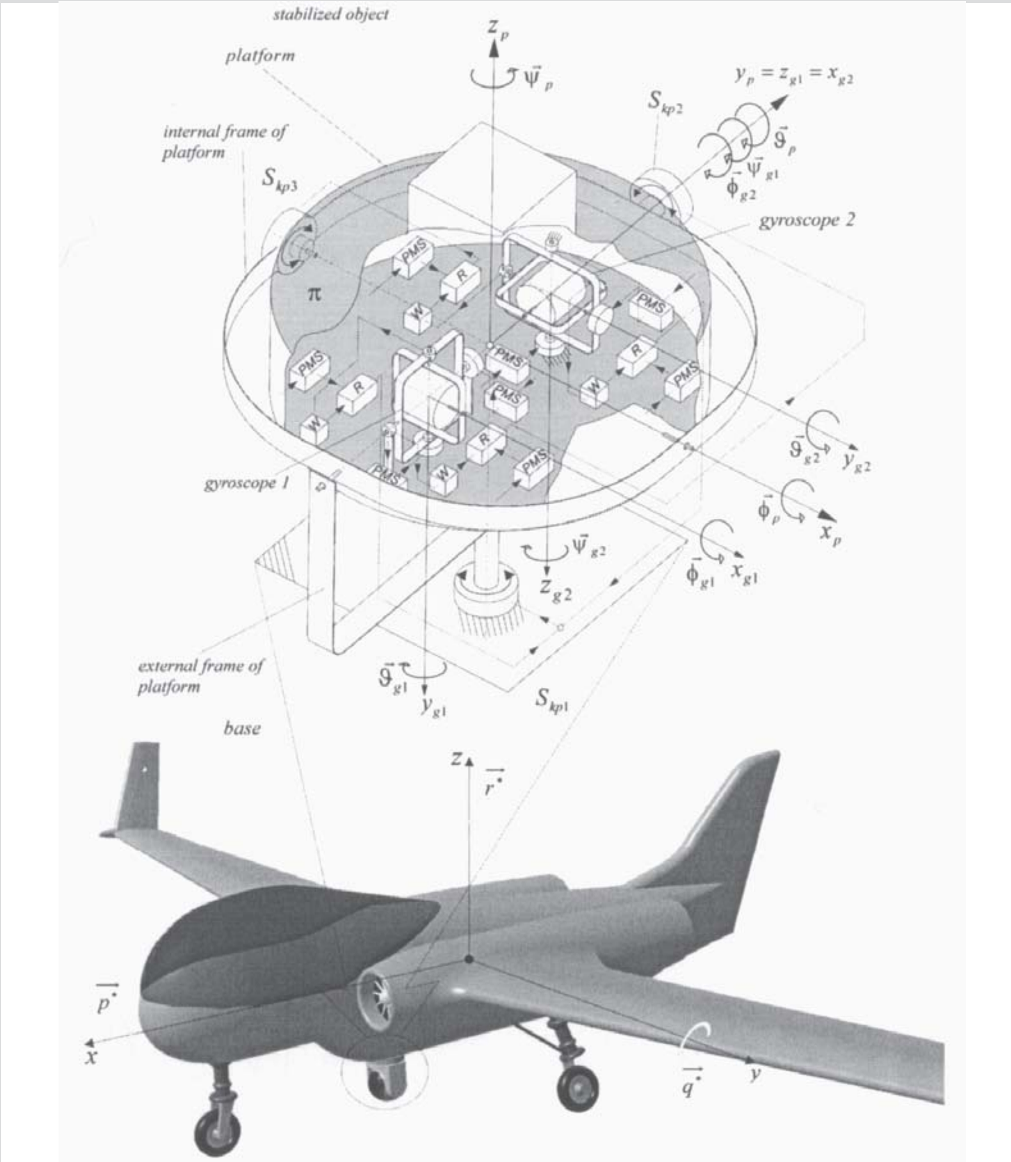

Fig. 1. Schematic diagram of a gyroscopic platform on board of a flying object

where:

J_{go}, J_{gk} – moments of inertia of the gyroscope rotors;

$J_{x_p}, J_{y_p}, J_{z_p}, J_{y_{rw}}, J_{z_{rz}}$ – moments of inertia of the platform elements;

$m_{1_{g1}}, m_{2_{g1}}, m_{3_{g1}}$ – rotor masses of the inner and outer frames of Gyroscope 1;

$m_{1_{g2}}, m_{2_{g2}}, m_{3_{g2}}$ – rotor masses of the inner and outer frames of Gyroscope 2;

l_p, l_{g2p}, l_{g2p} – distance of the centres of gravity of the platform, Gyroscope 1, and Gyroscope 2 from the geometric platform rotation centre;

$\vartheta_{g1}, \psi_{g1}, \vartheta_{g2}, \psi_{g2}, \phi_p, \vartheta_p, \psi_p$ – angles determining the position of the particular spin axes of the gyroscope elements and the platform elements;

n_{g1}, n_{g2} – angular velocities of the rotors of Gyroscopes 1 and 2;

V_p – linear velocity of the flying object;

p^*, q^*, r^* – angular velocities of the flying object;

M_{ri} – moments for friction forces in the bearings of the spin axes of the particular gyroscope and platform elements;

M_{kgi} – stabilisation moments produced by the correction motors of the particular gyroscope elements;

M_{kpi} – stabilisation moments produced by the correction motors of the particular platform elements.

3. CONTROLLING A THREE-AXIS GYROSCOPIC PLATFORM AND OPTIMISING THE CONTROL PARAMETERS OF A THREE-AXIS PLATFORM ON A MOVABLE BASE

The equations of motion for a controlled platform are written in the vector-matrix form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{2}$$

The vector **u** illustrates a pre-programmed open-loop control system. Pre-programmed controls **u** will be determined from an inverse problem of mechanics. According to the general definition, inverse problems of dynamics involve *estimating the external forces acting upon a mechanical system, the system parameters and the constraints imposed on the system,* with the predetermined motion being *the only possible motion of the system.*

Thus, it is essential to formulate the desired controls, the vector of which is denoted by $\mathbf{u}_p$. A control problem involves determining time-dependent vector components of $\mathbf{u}_p$, being the control moments that will cause motion of the gyroscope axis defined with the desired angles (determining the required position of the platform in space).

Then, the equation of control is transformed as follows

$$\mathbf{B}\mathbf{u}_p = \dot{\mathbf{x}}_p - \mathbf{A}\mathbf{x}_p.$$

The case under consideration is an open-loop control system of the gyroscopic platform. Figure 2 shows a schematic diagram of such a system.

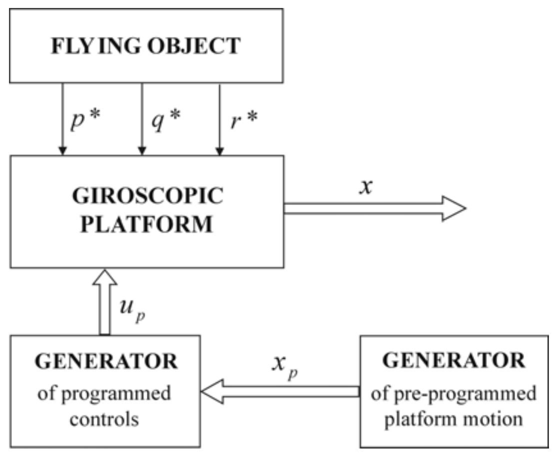

Fig. 2. Schematic diagram of the gyroscopic platform open-loop control system

The system operates correctly if the initial conditions of the axis angular positions coincide with those of the desired positions: $x(0) = x_p(0)$. Except for the above mentioned inconsistencies of program control, it is significant to analyse the effect of external disturbances, particularly the kinematic impact of the base and non-linear platform motions (considerable angles of deflections of the gyroscope axes), friction in the suspension bearings, technological inaccuracies, errors of the measuring devices, etc.

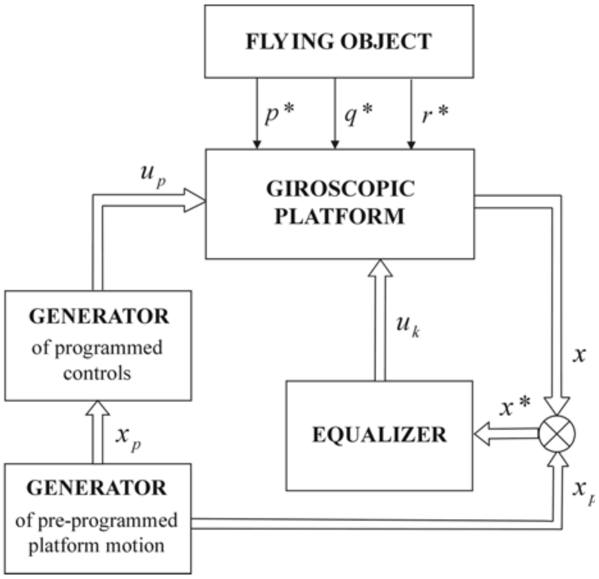

Fig. 3. Schematic diagram of the gyroscopic platform closed-loop control system

To maintain the platform stability, however, it is necessary to apply additional correction control $\mathbf{u}_k$ in a closed-loop system (Fig. 3). Then, the equations describing the motion of a controlled platform will be in the form

$$\dot{\mathbf{x}}^* = \mathbf{A}\mathbf{x}^* + \mathbf{B}\mathbf{u}_k \tag{3}$$

The system of platform control employs PD controllers for which the equations of control moments produced by the correction motors can be written as

$$M_{ki} = h_i\dot{\alpha}_i + k_i\alpha_i \tag{4}$$

where:

- h_i, k_i – parameters of a PD controller,
- $\alpha_i = \vartheta_{g1}, \psi_{g1}, \vartheta_{g2}, \psi_{g2}, \phi_p, \vartheta_p, \psi_p$ – angular coordinate of the particular gy-gyroscope and platform ele-elements.

The stabilisation control law $\mathbf{u}_k$ will be defined by means of the linear and square optimisation method with the functional in the form

$$J = \int_0^\infty \left[\left(\mathbf{x}^*\right)^T \mathbf{Q}\mathbf{x}^* + \mathbf{u}_k^T \mathbf{R}\mathbf{u}_k \right] dt \tag{5}$$

The law can be represented by

$$\mathbf{u}_k = -\mathbf{K}\cdot\mathbf{x}^* \tag{6}$$

where:

$$\mathbf{u} = \left[M_{k1}\ M_{ki+1}\ M_{kk}\right]^T$$

$$\mathbf{x} = \left[\psi_{g1}\ \dot{\psi}_{g1}\ \vartheta_{g1}\ \dot{\vartheta}_{g1}\ \psi_{g2}\ \dot{\psi}_{g2}\ \vartheta_{g2}\ \dot{\vartheta}_{g2}\ \dot{\Phi}_p\ \dot{\vartheta}_p\ \dot{\psi}_p\right]^T$$

The matrix of coupling **K** in Equation (6) is derived from the following relationship

$$\mathbf{K} = \mathbf{R}^{-1}\cdot\mathbf{B}^T\cdot\mathbf{P} \tag{7}$$

The matrix **P** is a solution of the algebraic Riccati equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - 2\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{8}$$

The weight matrices **R** and **Q** in Equations (7) and (8) reduced to the diagonal form are selected experimentally, with the search starting from even values:

$$q_{ii} = \frac{1}{2x_{i_{\max}}},\quad r_{ii} = \frac{1}{2u_{i_{\max}}},\qquad (i = 1, 2, ..n) \tag{9}$$

where:

- $x_{i_{\max}}$ – maximum variation range of the i-th value of the state variable,
- $u_{i_{\max}}$ – maximum variation range of the i-th value of the control variable.

After solving the Riccati matrix equation numerically and determining the matrix of coupling **K**, one can write the equations of optimised control moments as follows

$$\bar{M}_{ki} = \bar{h}_i\dot{\alpha}_i^* + \bar{k}_i\alpha_i^* \tag{10}$$

4. RESULTS AND CONCLUSIONS

Figures 4 through 7 illustrate the performance of the stabilisation platform. It is clear that if the controller parameters are not optimised, the transitional period following a disturbance is considerably longer.

If the controller parameters are optimised using the LQR method, the platform immediately regains the initial position (Fig. 5). The optimised correction moments in a function of time are more or less steady (Fig. 7).

Figures 8 and 9 present the performance of a platform in a pre-programmed motion around a circular cone. In Figure 8 we can see a predetermined and actual trajectory in the ψ_p and ϑ_p coordinates. In the case of optimised controller parameters, the platform, after a short transitional process, performs a predetermined motion with great accuracy. Figure 9 shows time-dependent correction moments produced by the stabilisation motors of the platform and Gyroscope 1.

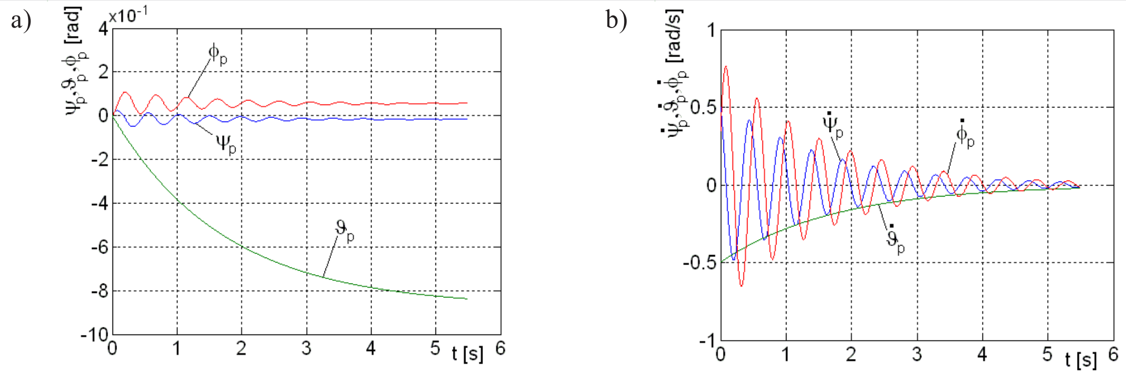

Fig. 4. Angular variations of the platform for the initially selected parameters of the controllers: a) angular variations; b) angular velocity variations

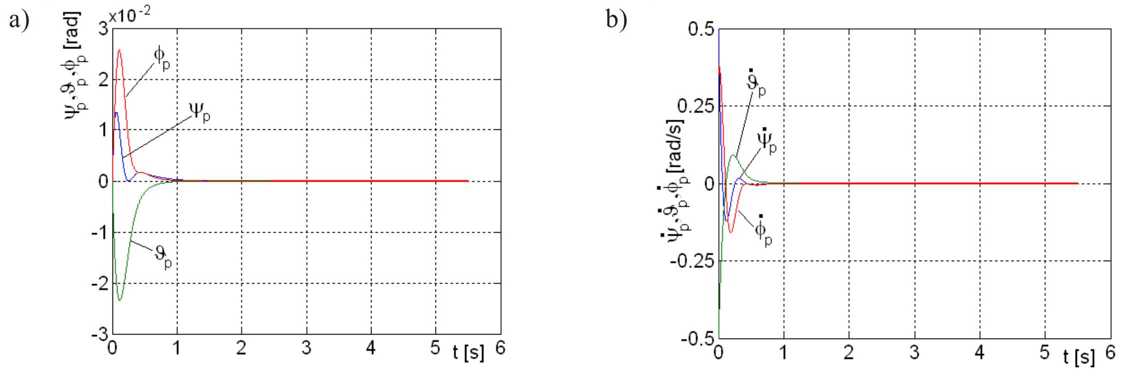

Fig. 5. Angular variations of the platform in a function of time for optimised parameters of the controllers: a) angular variations; b) angular velocity variations

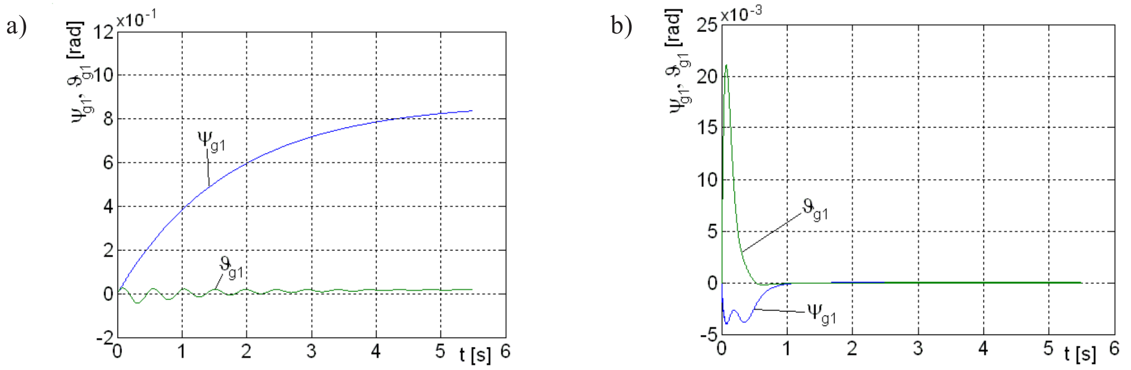

Fig. 6. Angular variations of Gyroscope 1: a) for non-optimised controller parameters; b) for optimised controller parameters

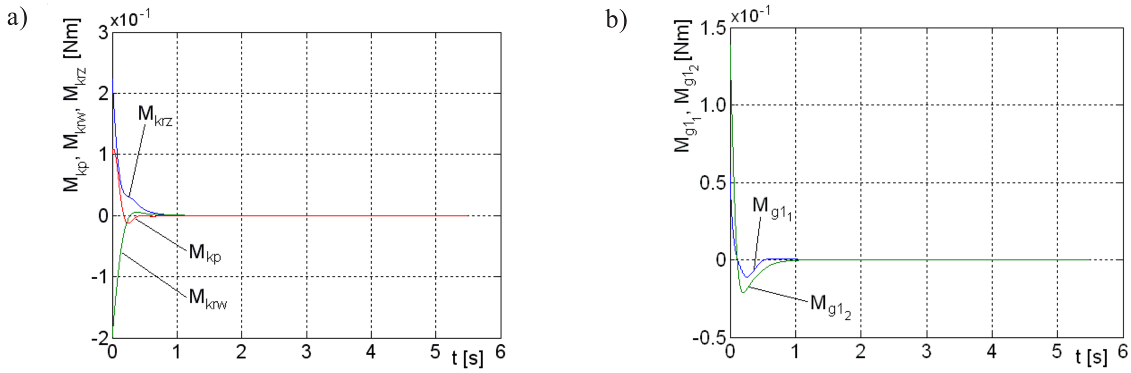

Fig. 7. Optimised correction moments of: a) the platform; b) Gyroscope 1

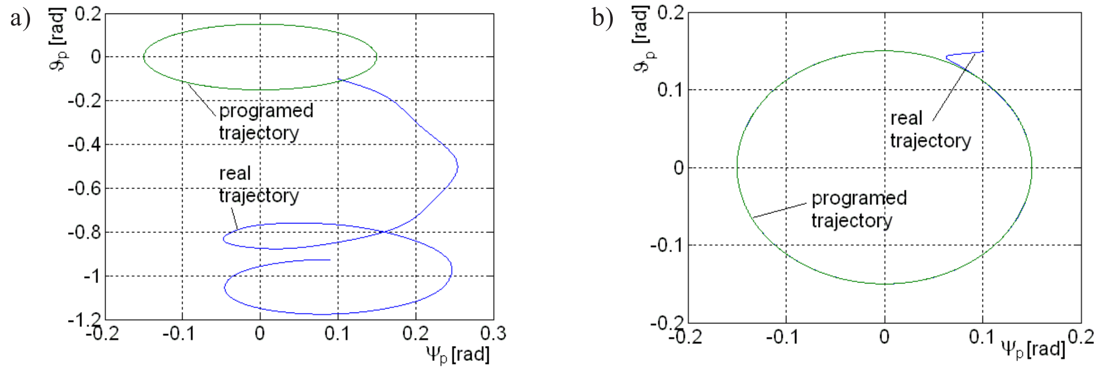

Fig. 8. Pre-programmed motion of the platform around a circular cone: a) for non-optimised controller parameters; b) for optimised controller parameters

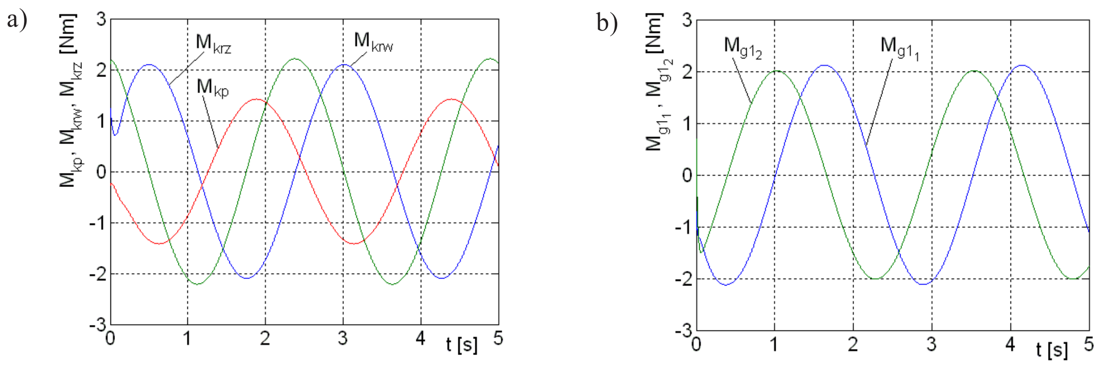

Fig. 9. Optimised correction moments in the pre-programmed motion of: a) the platform; b) Gyroscope 1

References

[1] Koruba Z.: *Dynamics and Control of a Gyroscope on Board of an Aerial Vehicle.* Monographs, Studies, Dissertations, No. 25, Kielce University of Technology 2001, 285 (in Polish)

[2] Koruba Z., Osiecki J.: *Construction, Dynamics and Navigation of Close-Range Missiles – part I.* University course book, No. 348, Kielce University of Technology Publishing House, 1999, PL ISSN 0239-6386, 168 (in Polish)

[3] Павлов В.А.: *Авиационные гироскопические приборы*. Москва, Гос. Издат. Оборонной Промышленности 1954

[4] Павловский М.А.: *Теория гироскопов.* Киев, Выща школа 1986