Zdzisław GOSIEWSKI*, Mariusz ŻOKOWSKI*

SLIDING MODE CONTROL FOR ACTIVE MAGNETIC BEARING SYSTEM

SUMMARY

Sliding mode control of a single-axis type bang-bang magnetic bearing actuator is reported in this paper. The two electromagnets are driven by switching between a positive and a negative constant voltage. Sliding mode control using these two switching surfaces in turn is shown to be possible. The sliding mode turns out to take place in a subregion of state space defined by $s_1(x)s_2(x) \le 0$ rather than on a surface defined by $s(x) = 0$ as in most standard cases.

Keywords: *active magnetic bearing, sliding mode control, bang-bang control*

STEROWANIE ŚLIZGOWE DLA SYSTEMU AKTYWNEGO ŁOŻYSKA MAGETYCZNEGO

Praca przedstawia ideę sterowania ślizgowego jednoosiowego siłownika aktywnego łożyska magnetycznego z symulowanym sterowaniem typu przekaźnikowego (bang-bang). Dwa elektromagnesy zasilane są poprzez przełączanie z dużą częstotliwością na przemian ujemnym i dodatnim napięciem stałym. Sterowanie ślizgowe opiera się na płaszczyźnie fazowej, która charakteryzuje fazę ruchu symulowanego (badanego) układu. Ruch ślizgowy odbywa się w obszarze płaszczyzny fazowej zdefiniowanej jako $s_1(x)\cdot s_2(x) \le 0$, a nie jak w większości przypadków, gdzie płaszczyzna fazowa jest zdefiniowana w następujący sposób: $s(x) = 0$.

1. IDEA OF ACTIVE MAGNETIC BEARING

A single-axis active magnetic bearing as shown in Figure 1 is considered. In this section the equations of this object is given, which provides a basis for further discussions.

In Figure 1, the motion of the ferromagnetic moving shaft, which is of mass m, is limited in the x-axis only. The voltages u_1 and u_2 are taken as the input of the system. The external resistors of resistance R_{Cu} denote copper resistance of the coils and may include possible current-sampling resistors. It is assumed that the permeability of the magnetic material is constant, and the flux density over the air gap is uniformly distributed. It is also assumed that the effects of eddy current, flux stray and flux leak are small. We ignore such effects in this paper in order to presentation of idea sliding mode control. Then, the equations of the system are [1]:

$$m\ddot{x} = \frac{1}{\mu_0 A}\left(\phi_2^2 - \phi_1^2\right) + f_d \tag{1a}$$

$$u_1 = N\dot{\phi}_1 + Ri_1, \quad u_2 = N\dot{\phi}_2 + Ri_2 \tag{1b}$$

$$i_1 = \frac{2}{\mu_0 AN}(x_0 + x)\phi_1, \quad i_2 = \frac{2}{\mu_0 AN}(x_0 - x)\phi_2 \tag{1c}$$

where:

$x_0 = g + l$;
l – accounts for the effect of the finite of permeability;
f_d – disturbance force;
φ_1, φ_2 – fluxes due to i_1 and i_2 respectively;
A – the electromagnet pole face area.

For succinctness in following discussions, let (1) be in normalized variables. Choosing a nominal biasing current I_0, the variables are normalized as:

$\xi = x/x_0$, $\varepsilon_1 = u_1/RI_0$,

$\eta_1 = I_1/I_0$, $\varepsilon_2 = u_2/RI_0$,

$\eta_2 = i_2/I_0$, $\psi_1 = \varphi_1/\Phi_0$,

$\theta = f_d/mx_0$, $\psi_2 = \varphi_2/\Phi_0$,

where $\Phi_0 = \mu_0 ANI_0/2x_0$ is the nominal biasing flux.

With the normalized variables (1) becomes

$$\ddot{\xi} = \alpha\left(\psi_2^2 - \psi_1^2\right) + \theta \tag{2a}$$

$$\varepsilon_1 = \frac{1}{\beta}\dot{\psi}_1 + \eta_1, \quad \varepsilon_2 = \frac{1}{\beta}\dot{\psi}_2 + \eta_2 \tag{2b}$$

$$\eta_1 = (1+\xi)\psi_1, \quad \eta_2 = (1-\xi)\psi_2 \tag{2c}$$

where:

$\alpha = \mu_0 AN^2 I_0^2/4mx_0^3$,

$\beta = 2R\xi_0/\mu_0 AN^2$.

Concept about reasonable orders of the normalized variables is useful in subsequent sections. It is seen that $|\xi| < 1$, $\psi's$ and $\eta's$ are both on the order of 1, and maximum possible values of $\varepsilon's$ are on the order of 10 (probably greater than ten due to the requirement of a sufficient force slew rate). For most practical systems α and β are on the orders of 10^4 and 10^2 respectively. Figure 2 provides the proposed design of a system for controlling the active magnetic bearing.

* Department of Mechanical, Technical University of Koszalin, gosiewski@post.pl

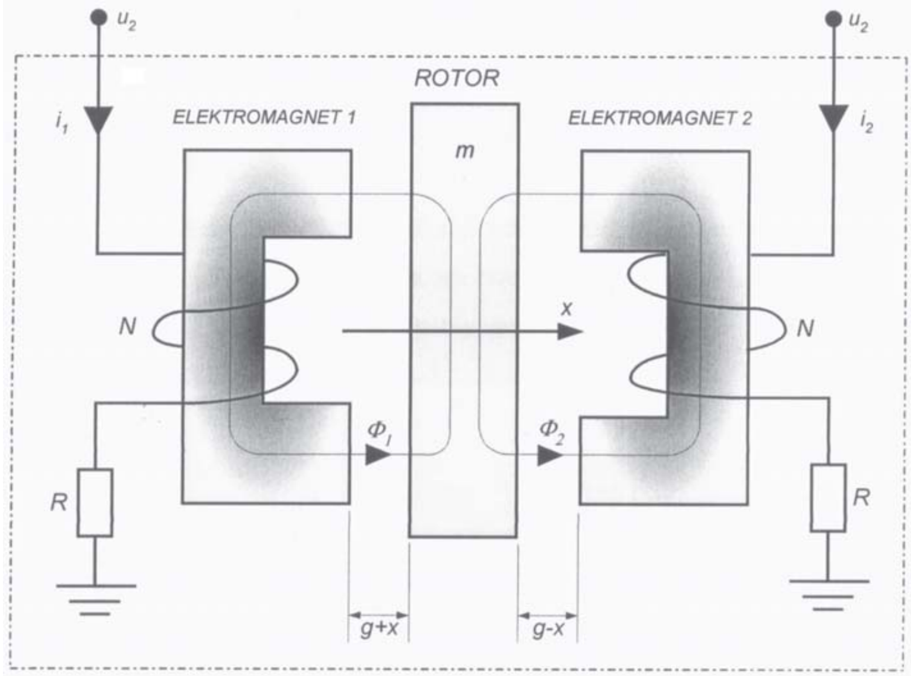

Fig. 1. Schematic of active magnetic bearing

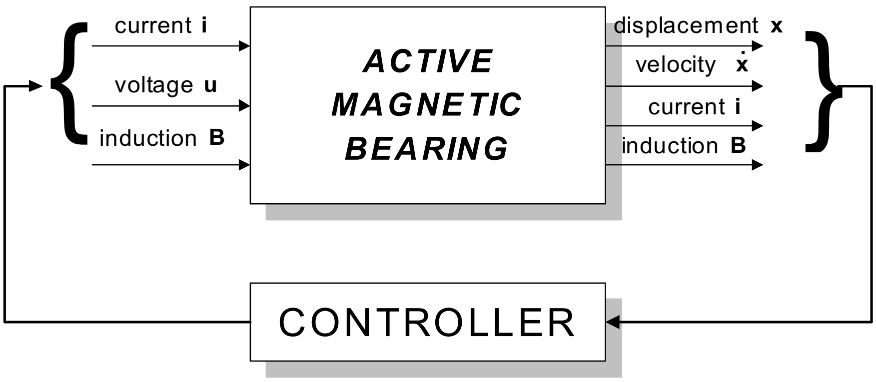

Fig. 2. Active magnetic bearing as closed structure of automatic control

2. SLIDING MODE CONTROL

It is assumed that the internal of the turned dashed block in Figure 1 is not available. We have only accessible the coil tips for control. This may be an adequate situation we face for sliding control. We search, starting from (2), for equalities that at one side the quantity is useful for control while at the other side the quantity is readily or easily measured outside the dashed block. Rearranging (2c) with the term $\xi\psi's$ alone on the left-hand side, we have:

$$\xi\psi_1 = \eta_1 - \psi_1 \quad \xi\psi_2 = -\eta_2 + \psi_2 \tag{3}$$

Then, differentiating (3) with respect of time yields, we have next expressions:

$$\psi_1\dot{\xi} + \dot{\psi}_1\xi = \dot{\eta}_1 - \dot{\psi}_1 = \dot{\eta}_1 - \beta(\varepsilon_1 - \eta_1) \tag{4a}$$

$$\psi_2\dot{\xi} + \dot{\psi}_2\xi = -\dot{\eta}_2 + \dot{\psi}_2 = -\dot{\eta}_2 + \beta(\varepsilon_2 - \eta_2) \tag{4b}$$

It is seen that the left-hand sides of (4) are linear in displacement and velocity, and may serve as part of switching functions for sliding mode control (the other part is accele-ration, as under voltage control the system is of third-order).

Besides, the variables on the right-hand sides are easily measured: current and its change rate are measured by connecting sampling resistors and inductors in series with the coils, both places outside the turned dashed block. How-ever, as coefficients the fluxes and their change rates are not of constant values. This may cause problem. Even worse is that flux change rate cannot have a single sign all the time.

If driving current is limited to be unidirectional, then ψ_1 and ψ_2 can be nonnegative. If further a substantial biasing current is maintained, then the fluxes can be positive and relatively stable. Besides, if switching driving is used, which is in consistent with sliding mode control, then problems associated with the flux change may be relieved. Let the input voltages be switched between constant positive and negative voltages, that is:

$$\varepsilon_1 = 1 - \left(\frac{U}{RI_0}\right)\mu, \quad \varepsilon_2 = 1 + \left(\frac{U}{RI_0}\right)\mu \tag{5}$$

where:
U – positive physical voltage;
$\mu \in \{-1, 1\}$ – new control input.

This is the constant voltage sum (CVS) configuration for maintaining the nominal biasing current I_0 [1]. Note that (5) implies $\varepsilon_1 + \varepsilon_2 = 2$. The physical driving voltages are switched between $RI_0 + U$ and $RI_0 - U$. If U/RI_0 is much greater than 1, then between following switching instants, $d\psi_1/dt$ and $d\psi_2/dt$ are close to some constants, since in (2b) compared with $\varepsilon's$ the values of $\eta's$ are much smaller. It is also noticed that the orders of $\{\psi_1, \psi_2\}$ and $\{d\psi_1/dt, d\psi_2/dt\}$ are 1 and 10^3 respectively, being desirable as coefficients of linear switching surfaces. Another fact to notice is that at any time either $d\psi_1/dt$ or $d\psi_2/dt$ has positive sign. This motivates an attempt of using equations (4a) and (4b) in turn, depending on the sign of μ, for sliding mode control.

Some effect of acceleration should be included in switching surfaces. To this end, (2c) is manipulated to yield

$$\psi_2 - \psi_1 = \eta_2 - \eta_1 + \xi(\psi_1 + \psi_2) \tag{6}$$

Substitute equation (6) into (2a) and assume $\theta = 0$, we get

$$\ddot{\xi} = \alpha(\psi_1 + \psi_2)(\eta_1 - \eta_2) + \alpha\xi(\psi_1 + \psi_2)^2 \tag{7}$$

The flux sum $\psi_1 + \psi_2$ is very close to 2 under CVS. The second term on the right-hand side, which is linear in ξ, can be included in the displacement. Thus, $C(\eta_2 - \eta_1)$, where C is a positive constant, may be used in switching surfaces for acceleration.

Now, the above mentioned facts are pieced together to formulate our possible switching functions as:

$$s_1 = C(\eta_2 - \eta_1) + \psi_1\dot{\xi} + \dot{\psi}_1\xi \tag{8a}$$

$$s_2 = C(\eta_2 - \eta_1) + \psi_2\dot{\xi} + \dot{\psi}_2\xi \tag{8b}$$

Equations $s_1 = 0$ and $s_2 = 0$ are surfaces in the state space when μ is constant. It is obvious from (4) that the switching functions s_1 and s_2 can be evaluated by the measurement of η_1, η_2, $d\eta_1/dt$ and $d\eta_2/dt$. The coefficients in (8a) are all positive when $\mu = -1$, and so are those in (8b) when $\mu = 1$. The attempted sliding mode control algorithm is thus constructed in discrete time as below.

At each sampling instant →

if $\mu = -1$ and $s_1 < 0$, **then** let $\mu = 1$ (9)

else if $s_2 > 0$, **then** let $\mu = -1$

It is noted that a continuous time version of (9) is also possible, but in discrete time it is easier to describe. It is assumed that the sampling rate is very high that there is no significant difference between discrete and continuous time implementations.

3. REACHING CONDITION

The switching functions s_1 and s_2 are directly related to the flux change rates, which are not state variables. In order to facilitate the analysis we give equivalent switching functions that are functions of state variables alone. Because s_1 is checked only when $\mu = -1$ and s_2 is checked only when $\mu = 1$, these switching functions can be redefined as:

$$s_1 = C(\eta_2 - \eta_1) + \psi_1\dot{\xi} + \beta(\rho - \eta_1)\xi \tag{10a}$$

$$s_2 = C(\eta_2 - \eta_1) + \psi_2\dot{\xi} + \beta(\rho - \eta_2)\xi \tag{10b}$$

without any influence on the algorithm (9), where $\rho = 1 + U/RI_0$ is the positive amplitude of the normalized voltage inputs. From here s_1 and s_2 mean those defined in (10). It is not hard to see that under first-order approximation about the equilibrium point s_1 and s_2 become identical and are linear functions of $d^2\xi/dt^2$, $d\xi/dt$ and ξ. With the redefined switching functions, the previous problem that flux change rate cannot have a single sign all the time is now transformed as only one of the switching functions in (10) is available at any time through the measurement of driving current and its change rate, except for instants μ being switched between –1 and 1.

For the algorithm (9) to work a basic requirement is that the conditions for the if is can actually become satisfied. It is therefore required that $ds_1/dt < 0$ when $\mu = -1$ and $ds_2/dt > 0$ when $\mu = 1$. But this is not enough. For the reasons to be clear below, it is also required that $ds_2/dt < 0$ when $\mu = -1$ and $ds_1/dt > 0$ when $\mu = 1$.

It is therefore assumed that $|d\xi/dt|$ is bounded and U is sufficiently high such that the following conditions hold:

$$\dot{s}_1 < 0 \qquad \dot{s}_2 < 0 \qquad \text{when} \qquad \mu = -1 \tag{11a}$$

$$\dot{s}_1 > 0 \qquad \dot{s}_2 > 0 \qquad \text{when} \qquad \mu = 1 \tag{11b}$$

It will be seen bellow that this is the reaching condition for the sliding mode control.

In sliding mode control, the number of switching surfaces is equal to that of control variables. But here there are two switching surfaces while there is only one control input. It is generally impossible to keep both $s_1 = 0$ and $s_2 = 0$. The algorithm (9) can at most maintain s_1 and s_2 close to zero. It actually tries to get out of the situation $s_1 \cdot s_2 > 0$ in the reaching phase. Then, if the switching is infinitely fast, it maintains $s_1 \cdot s_2 \leq 0$ in the sliding phase.

Under infinitely fast switching, the running of the algorithm is more clearly seen:

– if $s_1 < 0$ and $s_2 < 0$, then $\mu = 1$ until $s_2 = 0$;
– if $s_1 > 0$ and $s_2 > 0$, then $\mu = -1$ until $s_1 = 0$;
– if $s_1 \le 0$ and $s_2 \ge 0$, then μ is switched between –1 and 1 with 50% duty cycle;
– if $s_1 \ge 0$ and $s_2 \le 0$, then μ is switched between –1 and 1 such that s_1 and s_2 just reach zero in turn.

Thus, once $s_1 \cdot s_2 \le 0$ becomes true it will remain true in future time and this indicates the entering of sliding mode. These switching laws are summarized in Figure 3. It is not hard to notice that the reaching condition (11) must be true to guarantee the algorithm to run properly. Unlike standard sliding mode control where sliding occurs on surfaces or in subspaces of the state space, here the sliding occurs in a subregion of the state space. One may also notice, as shown in Figure 3, that the reaching behaviors in the two reaching regions are symmetrical while the sliding behaviors in the two sliding regions are not.

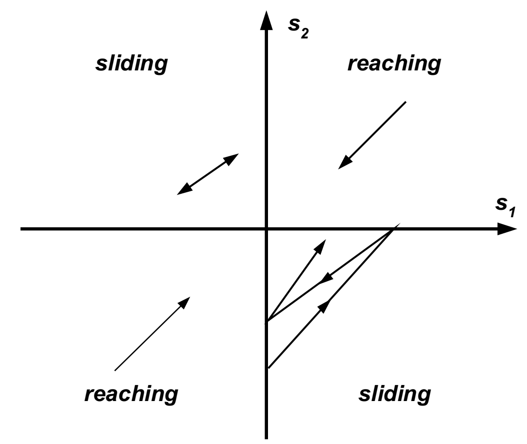

Fig. 3. Reaching and sliding areas

4. STABILITY OF SLIDING DYNAMICS

In order for succinctness of equations, let $\psi_0 = (\psi_1 + \psi_2)/2$ and $\psi = (\psi_2 - \psi_1)/2$, referred to as biasing and actuating flux respectively. Similarly, let $\eta_0 = (\eta_1 + \eta_2)/2$ and $\eta = (\eta_2 - \eta_1)/2$, referred to as biasing and actuating current respectively. Note that at equilibrium we have $\psi_0 = \eta_0 = 1$. From (2c) we also have:

$$\eta_0 = \psi_0 - \xi\psi \tag{12}$$

$$\eta = \psi - \xi\psi_0 \tag{13}$$

Let $s_0 = (s_1 + s_2)/2$. Then it follows from (10) and (12)–(13) that

$$s_0 = \frac{2C}{4\alpha\psi_0}\ddot{\xi} + \psi_0\dot{\xi} + [\beta\rho - (2C+\beta)\psi_0]\xi \tag{14}$$

Adding the two equations in (2b) and substituting (5) and (12)–(13), we arrive at

$$\frac{1}{\beta}\dot{\psi}_0 + \psi_0 = 1 + \xi\psi \tag{15}$$

Equations (14)–(15) are looked upon as state equations of a 3rd-order system with inputs on the right-hand sides. It is seen that if $s_0 = 0$ then the autonomous system is locally stable about the equilibrium point $\xi = d\xi/dt = 0$ and $\psi_0 = 1$. Since in sliding mode $s_1 \cdot s_2 \le 0$ is true, which implies $|s_1 + s_2| \le |s_2 - s_1|$, that is

$$|s_0| \le \left|\xi\dot{\psi} - \beta\psi\xi + \beta\psi_0\xi^2\right| \tag{16}$$

Thus s_0 is bounded by a quadratic function of the states and the sliding dynamics is locally stable. It is noted that though the system (14)–(15) is of 3rd-order, the sliding dynamics is of 4th-order. As (16) is an inequality, s_0 is not a function of the state variables ξ, $d\xi/dt$ and ψ_0. Thus $d^2\xi/dt^2$ in (14) is not a function of the state variables but a fourth state variable. The sliding dynamics is not completely determined by the switching surfaces. Even if the reaching condition (11) is true, the robustness associated with standard sliding mode control is lost.

5. COMPUTER SIMULATION

Formulate an assumption about parameters of the active magnetic bearing which $A = 10^{-3}$ m^2; $N = 300$; $m = 0.3$ kg; $x_0 = 0.3 \times 10^{-3}$ m and $R = 0.27\ \Omega$. The effect of the finiteness of permeability is not considered, that is, $l = 0$. For other parameters, $I_0 = 0.8$ A, $U = 25$ V and $C = 300$ are chosen. Transformers with an inductance of 0.1 mH in both sides are used for the measurement of current change rate. The sampling rate is 100 kHz. In simulation is done on the base equation (10) and then with a term z as determined by

$$\tau\ \dot{z} + z + -C(\eta_2 - \eta_1) \tag{17}$$

being added to the right-hand sides of (10a) and (10b), thus canceling the term $C(\eta_2 - \eta_1)$ under steady-state. The value of τ is 0.005 s. This is the similar concept of disturbance observation for achieving zero steady-state error under external disturbance force θ. Simulation results of startup from $\xi = -0.9$, with other initial states being zero, are shown in Figure 4. It is assumed that the motion axis is vertical, so that the mass of the moving part causes a static displacement in Figure 4a.

It is observed in simulation that performance is sensitive to some of the parameters and the symmetry of the two sides. The switching function values are close to zero under steady-state. But they are based on the canceling of two large values: the two terms on the right-hand sides of (4). A small percentage error on β used for the construction of the bright-hand sides of (4) will cause large errors of s_1 and s_2 when the state is close to the equilibrium point.

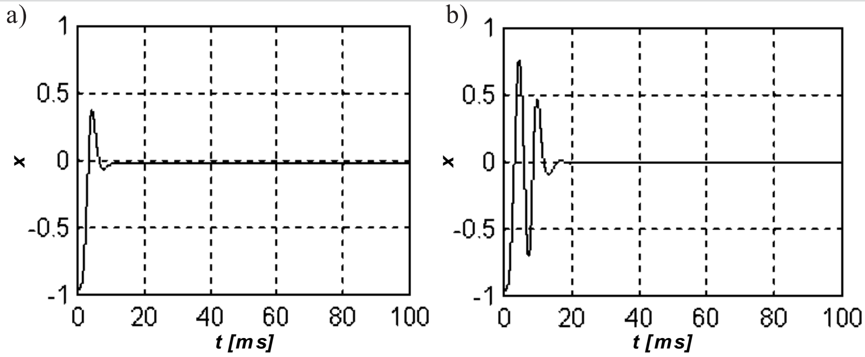

Fig. 4. Simulation results: a) startup without disturbance observer; b) startup with disturbance observer

6. CONCLUSIONS

Under switching driving, as the input voltages are piecewise constant, some functions of state variables are available through the measurement of driving current and its change rate. Using these functions as switching functions, the system state is conducted towards and then kept in a subregion of the state space, in which the system dynamics is stable.

Sliding mode control is applied for magnetic bearings for the associated robustness. In this paper, however, sliding mode control is used due to the very format of the available information under switching driving. As a result, two switching surfaces are used in turn and the state is only kept within a subregion other than a subspace. Thus the good properties of sliding mode control are lost. The sliding dynamics is dependent on disturbance and plant parameters such as biasing current and amplitude of driving voltage.

References

[1] Li L.: *Linearizing Magnetic Bearing Actuators by Constant Current Sum, Constant Voltage Sum, and Constant Flux Sum*. IEEE Trans. Magnetics, vol. 35, No. 1, January 1999, 528–535

[2] Li L., Allaire P.E.: *Sensorless Sliding Mode Control of Magnetic Bearing Actuators using Implicit Switching Surfaces*. Seventh International Symp. on Magnetic Bearings, August 2000, 311–316

[3] Gosiewski Z., Hanc D.: *Sliding Mode Control of an Aircraft Electric Drive.* Recent Research and Design Progress in Aeronautical Engineering and its Influence on Education, Warsaw, Poland, 2000

[4] Gosiewski Z., Falkowski K.: *Wielofunkcyjne łożyska magnetyczne*. Warszawa, Biblioteka Naukowa Instytutu Lotnictwa 2003

[5] Kurman K.: *Teoria regulacji – podstawy, analiza, projektowanie.* Warszawa, WNT 1975

[6] Nonlinear Control Design Blockset User's Guide Version 5. MathWorks 1999