Stanisław FORTUNA[*]

CALCULATION OF ROTOR WORK
ON THE BASIS OF THERMOANEMOMETRIC MEASUREMENT RESULTS.
PART 1

SUMMARY

Article consists of two parts. The first part depicts results of calculations useful for the rotor work determination on the basis of thermoanemometric measurements and measurements of total pressure impact in given sections of the arrangement. Both measurements were made for one point of characteristic in the vicinity of optimal point, but for five different velocities of the rotor. The aim of the article was to verify both measurement methods and adjust results to the same conditions of flow.

Keywords: *rotor work, thermoanemometric and balance methods of measurement*

PORÓWNANIE PRACY WIRNIKOWEJ WYZNACZONEJ
NA PODSTAWIE WYNIKÓW POMIARÓW TERMOANEMOMETRYCZNYCH I BILANSOWYCH.
CZĘŚĆ 1

Artykuł składa się z dwóch części. W pierwszej przedstawiono wyniki obliczeń użytecznych prac wirnikowych wykonanych na podstawie wyników pomiarów termoanemometrycznych i pomiarów spiętrzenia całkowitego w odpowiednich przekrojach układu. Obydwa pomiary wykonano dla jednego punktu charakterystyki w okolicach punktu optymalnego, ale przy pięciu różnych prędkościach wirowania wirnika. Celem artykułu było zweryfikowanie obu metod pomiarowych i skorygowanie ich wyników do tych samych warunków przepływowych.

Słowa kluczowe: *praca jednostkowa, użyteczna praca wirnikowa, metoda termoanemometryczna, metoda bilansowa wyznaczania pracy*

1. INTRODUCTION

Rotor work may be calculated or determined experimentally. Analytical methods consist in determination of angular momentum in Euler's rotor and consideration of absolute velocity deviation because of rotary motion in the real rotor and flow losses. Other analytical methods determining the work apply velocity circulation round the blade, taking into account relative velocities and peripheral components of absolute velocity. At later stages of calculations, approach used in the method of angular momentum increase is applied. Numerical methods consisting in the use of finite elements methods to determine pressure and velocity distribution in blade passages and boundary surfaces are also used. This allows determination of work increase in the rotor.

Though analytical calculation methods are popular, they still do not guarantee sufficiently precise results. Therefor we have to apply balance methods of measurement. They are expensive, arduous, and laborious, but they give the best results of determination the amount of energy transmitted to the medium, with precision to 1%. Which constitutes a very good experimental result.

Thermoanemometric method of determining the peripheral velocity component at the rotor outlet may be used for calculating the useful work transmitted to the fluid in a real rotor. Radial component allows determination of volume of the medium stream leaving the rotor. These two components allow determination of real characteristics of the machine without building special test stands. Future applica-

tion of this method is not apparent, however, because the object of tests consists in a compound turbulent space behind the rotor, which cause measurement errors and difficulties in identification of signals. The article deals with an experiment aiming at determination of gas velocity behind the rotor with three-fibre sensor and calculation of whirl work in one point of characteristics, comparison of this work with the work determined by means of measuring the total pressure impact.

2. STAND FOR THERMOANEMOMETRIC TESTS

The stand includes:
- rotor without casing fixed on a shaft with bushing;
- shaft drive $N = 1.5$ kW, $n = 2880$ r.p.m;
- inverter of Microverter AEG type for changing rotary velocity, with speed control;
- measuring straight pipeline at suction before the rotor;
- cone throttling a flow at the pipeline inlet;
- orifices sized $D = 220$ mm, $d = 155$ mm for flow measurement, and nozzle measuring differential pressure – Δh, negative pressure under the orifice – h_1;
- negative pressure at suction – h_s;
- barometer for measurement of barometric pressure – p_b;
- thermometer for measurement of temperature of air flowing in the pipeline – t;
- sensors for measuring three components and module of vector of outflow stream velocity (three-channel thermoanemometer);

---

[*] AGH University of Science and Technology, Kraków
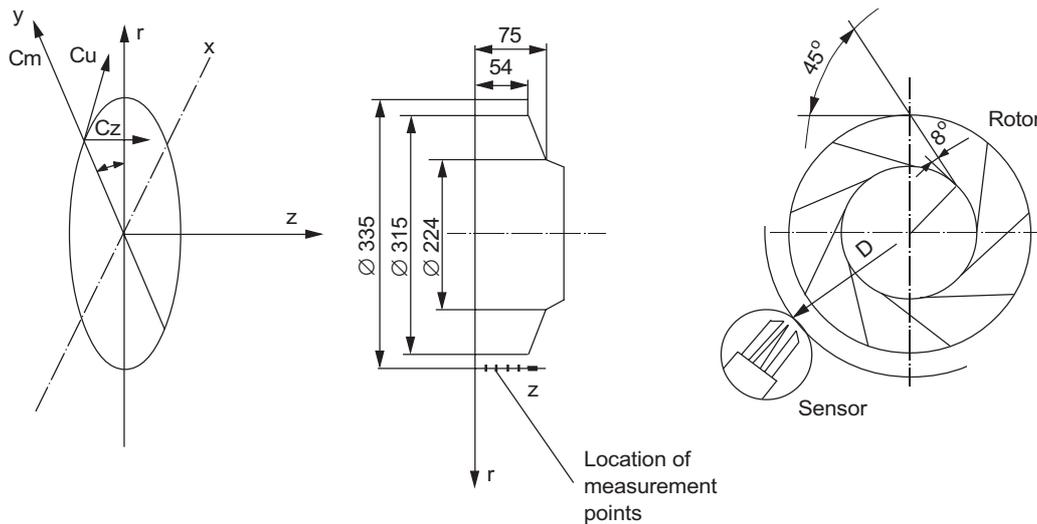
Fig. 1. Characteristic geometrical dimensions of paddle wheel. Assumed system of coordinates and its location against the tested wheel, sensor location: c_m – radial component of velocity vector V_y green, c_z – axial component of velocity vector V_x blue, c_u – peripheral component of velocity vector V_z red

– sensor holder with lathe carriage which allows positioning of the sensor against the rotor disk; distance between the sensor and the rotating paddle wheel during the experiment was 10 mm , the set served to repositioning of the sensor against the back disk of the rotor;
– TURBULENCE METER type ATM 94;
– A/C PC LabCard PLC 814 with software;
– PC with the program ("Program for calculations of velocity components and frequency analysis" prepared by Flow Metrology department of Institute of Orogene Mechanics of Polish Academy of Sciences in Kraków).

Measurement route was composed of an anemometric sensor (three mutually perpendicular fibres) , anemometer – TURBULENCE METER type ATM 94, A/C PC LabCard PLC 814 and PC. Some elements of the measurement stand for testing rotor wheels are shown in Figure 1.

The system is equipped with three anemometric sensors. The program uses procedures which allow calculation of velocity vector components in the co-ordinate system determined with sensor fibres. Sensor fibres are marked with colours (blue, green, red), vector co-ordinates in the program are marked with the same colours. The sensor is connected to the card slot. Number of the connected sensor is given in the program.

The program allows measurement of velocity vector components, and calculates turbulence coefficient. Results of measurements and calculations may be observed at he monitor screen.

3. LIST OF THERMO-ANEMOMETER MEASURED VELOCITIES

Anemometer allows determination of three velocity components and a module in $\pi D'b$ section. These are:

1) c'_m component (radial component),
2) c'_u component (peripheral component)
3) c'_z axial component, which has not been used.

Components were measured along the rotor width for five velocities from 439 to 2260 r.p.m. Results averaging after 30 ms of sampling are shown in Figure 2. Averaging values of velocity components in relation to time and width are shown in Table 1.

Figure 3 depicts peripheral and radial velocities at the diameter of sensor location – D', and at the external rotor diameter – D_2.

Peripheral and radial velocities were recalculated for the external diameter of the rotor according to angular momentum preserving principle

$$\frac{D_2}{2} c_{2u} = \frac{D'}{2} c'_u = \text{const}$$

hence   (1)

$$c_{2u} = \frac{D'}{D_2} c'u$$

and stream continuity equation

$$\dot{m} = 2\pi r' b' c'_m \rho' = 2\pi r_2 b_2 c_{2m} \rho_2 = \text{const}$$

for constant density $\rho_2 = \rho'$

$$c_{2m} = c'_m \frac{r'b'}{r_2 b_2} \tag{2}$$

Assuming that the stream will not have time to expand $(b_2 = b')$

$$c_{2m} = c'_m \frac{r'}{r_2} \tag{3}$$

Recalculation results are shown in Table 2.
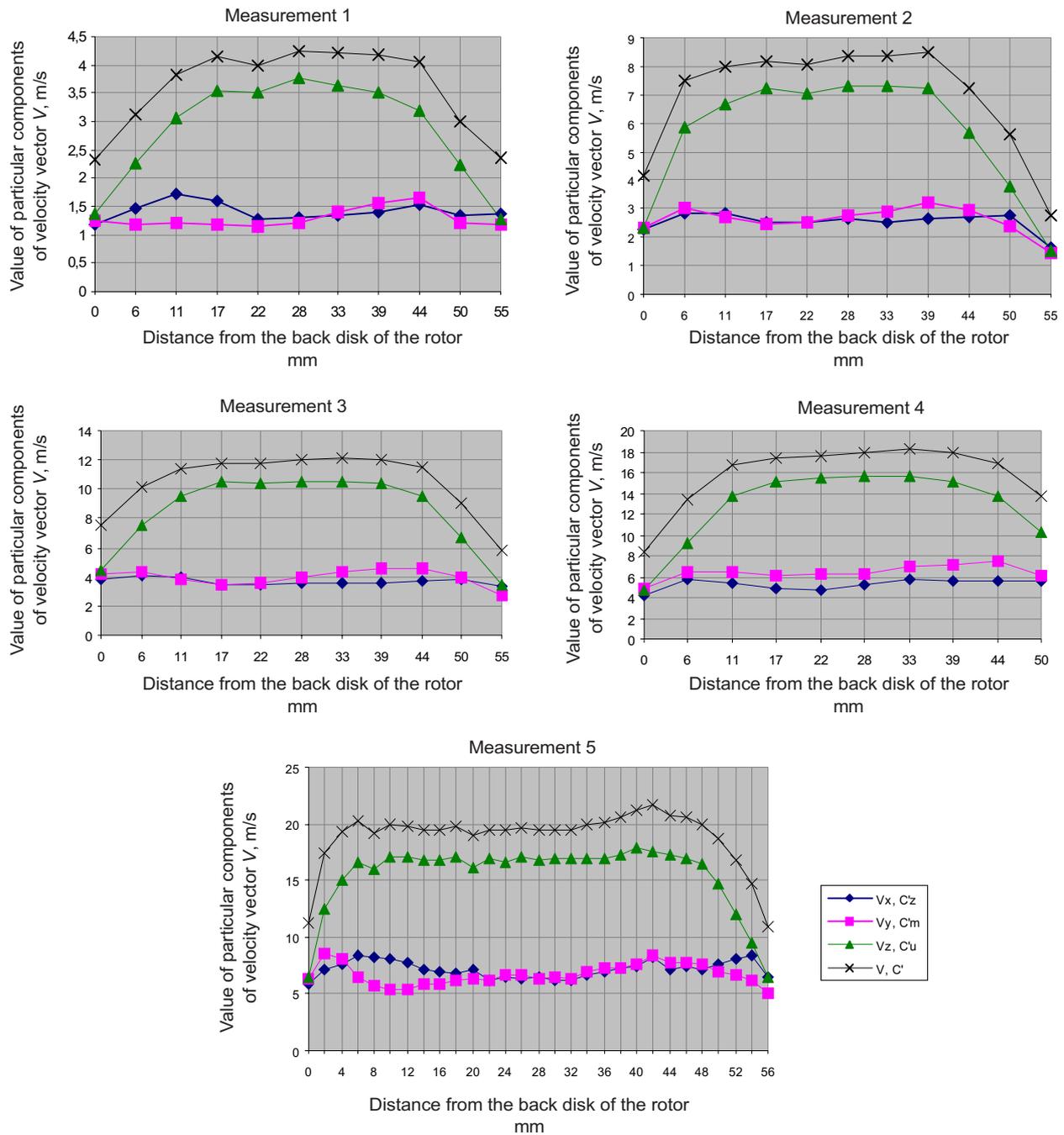
Fig. 2. Velocities measured for revolutions 439(1), 897(2), 1368(3), 2038(4), 2260 r.p.m (5 subseqvent from the top). The diagram shows averaging values after rotation time 0–30 ms along the blade width

Table.1. Average values of axial, radial, and peripheral components and module in [m/s] , diameter D'

It.	Value	Revolutions n, r.p.m	$V_x = c_z'$	$V_y = c_m'$	$V_z = c_u'$	$V = c$
1.	average	439	1.4140	1.2940	2.8564	3.5898
2.	average	897	2.5352	2.6149	5.6297	6.9655
3.	average	1368	3.6820	3.9535	8.5190	10.4724
4.	average	2038	5.2724	6.4202	12.8854	15.8014
5.	average	2260	7.1530	6.6900	15.4800	18.8800

Table 2. List of values of peripheral and radial velocities recalculated for $D_2 = 315$ mm

It.	Velocities		Recalculation of c'_u velocity		Recalculation of c'_m velocity	
	Rotational n	Periphetal u_2	c'_u, m/s	c_{2u}, m/s	c'_m, m/s	c_{2m}, m/s
	r.p.m	m/s	(335 mm)	(315 mm)	(335 mm)	(315 mm)
1	439	7.2	2.85	3.03	1.29	1.37
2	897	14.8	5.62	5.98	2.61	2.78
3	1368	22.5	8.51	9.05	3.95	4.20
4	2038	33.6	12.88	13.70	6.42	6.82
5	2260	37.2	15.48	16.46	6.69	7.11

Fig. 3. Velocities at D_2, D' diameters
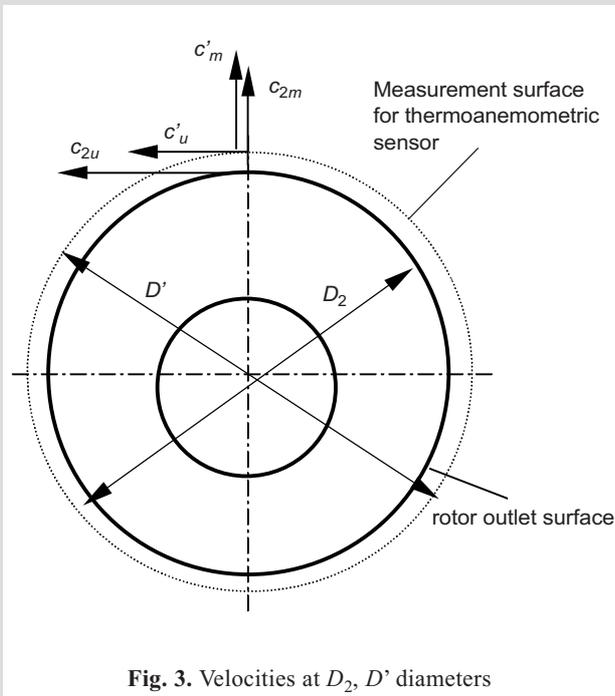
4. CALCULATION OF CAPACITY AND WHIRL WORK ON THE BASIS OF VELOCITY COMPONENTS c_{2m} AND c_{2u}

Data for calculations:

$D_2 = 315$ mm – sensor location diameter,
$b_2 = 54$ mm – rotor width,
$\chi_2 = 0.98$ – degree of outlet covering,
$\rho = 1.2$ kg/m^3 – air density.

Recalculation formulas:

$$\dot{V}_2 = \pi D_2 b_2 \chi_2 c_{2m} = 0.0523 c_{2m} \qquad (4)$$

$$l_u = u_2 c_{2u} \qquad (5)$$

where:

$$u_2 = \frac{\pi D_2 n}{60} = 0.01648 n,$$

n – rotor revolutions r.p.m.,
c_{2u} – velocity vector peripheral component,
c_{2m} – velocity vector radial component.

Capacity of $\dot{V}_2$ rim was recalculated to capacity of $\dot{V}_s$ fan, asking into consideration volumetric loss as shown in Figure 4.

According to the balance principle, capacity behind $\dot{V}_2$ rotor is equal to the capacity determined in $\dot{V}_s$ suction pipe plus $\dot{V}_v$ slot leakage, so we obtain

$$\dot{V}_s = \dot{V}_2 - \dot{V}_v \qquad (6)$$

In order to determine the leakage, value of volumetric efficiency $\eta_v = 0.93$ was adopted, and value of $\dot{V}_v$ leakage was calculated ex definitions

$$\eta_v = \frac{\dot{V}_s}{\dot{V}_s + \dot{V}_v} = \frac{\dot{V}_s}{\dot{V}_2} \qquad (7)$$

where:

$\dot{V}_s$ – gas stream at the fan inlet,
$\dot{V}_v$ – volumetric loss stream,
$\dot{V}_2$ – stream calculated from anemometer.

Results of the calculations are shown in Table 3.

Fig. 4. Consideration of volumetric loss
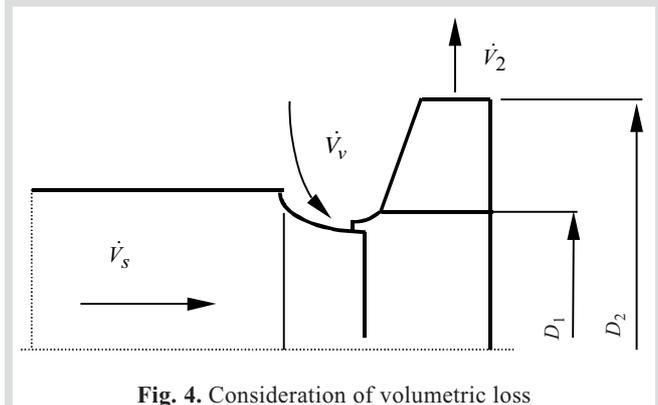
Table 3. List of capacities and whirl work defined on the basis of anemometric tests

It.	Revolutions n r.p.m	Capacity $\dot{V}_2$ m³/s	Capacity $\dot{V}_s$ m³/s	Value of volumetric loss $\dot{V}_v$ m³/s	Whirl work l_u m²/s²
1	439	0.0721	0.0671	0.0050	23.8
2	897	0.1456	0.1354	0.0102	88.5
3	1368	0.2202	0.2047	0.0155	203.6
4	2038	0.3566	0.3316	0.0250	460.3
5	2260	0.3720	0.3460	0.0254	612.3

5. VALUES OF DIFFERENTIAL PRESSURE, NEGATIVE PRESSURE BEFORE ORIFICE AND NEGATIVE PRESSURE MEASURED PARALLEL TO ANEMOMETRIC MEASUREMENT

Values of suction negative pressure h_s, differential pressure Δh, negative pressure before orifice h_1 measured parallel to thermo-anemometric measure are shown in Table 4. The data allowed calculations of capacity, total pressure impact and useful work of the tested rotor.

Algorithm for calculation of the characteristics is presented, and results are shown in Table 4.

Data:

A_s = 0.038 m² – inlet section area,
A_t = 0.0534 m² – outlet section area,
ρ_1 = 1.1941 kg/m³ – air density before orifice,
p_b = 99 400 Pa – barometric pressure,
ϕ = 0.65 – air relative humidity,
t_1 = 14.4°C – ambient temperature,
p_p'' =1 650 Pa – saturated steam pressure for t_1,
Re = 3 · 10⁵ – initial Reynold's value,
β – reducer throat,
κ = 1.4; ρ_n = 1.277 kg/m³

T_n = 273 K; p_n =10⁵ Pa – conventional thermal parameters,
T_1 = 287 K – temperature before reducer,
ρ_{alk} = 825 kg/m³ – manometer fluid density,
d = 0.155 m – orifice diameter,
g = 9.81 m/s² – acceleration of gravity,
Δh_1 – differential pressure height,
h_1 – negative pressure height before orifice,
h_s – suction negative pressure height,
η_v = 0.93 – volumetric efficiency.

Exemplary calculation for:

Δh = 53 mm alc,
h_1 = 0.054 m,
h_s = 0.0815 m.

– calculation of rotor capacity:
• capacity at orifice V_1:

$$\dot{V}_1 = K\sqrt{\Delta h_{mmalc}},$$

$$K = \left(\frac{C}{\sqrt{1-\beta}}\right)\varepsilon_1\left(\frac{\pi d^2}{4}\right)\left(\frac{\sqrt{\rho_{alk}\cdot g\cdot 10^{-3}}}{\sqrt{\rho_1}}\right)\sqrt{2}\ \ \text{m}^{2.5}\cdot\text{s}^{-1},$$

Table 4. List of h_s, Δh, h_1 values $\dot{V}_s$ and l_u

It.	Wave frequency Hz	Revolutions n, r.p.m	h_s mm, alc.	Δh mm, alc.	h_1 mm, alc.	V_s m³/s	Δp_{st} Pa	Δp_d Pa	Δp_c Pa	l_u m²/s²
1	10	439	3	2	3	0.083	24.3	–1.4	23.0	19.2
2	20	897	12	8	10	0.135	97.1	–3.7	93.4	77.8
3	30	1368	28	20	20	0.214	226.5	–9.3	217.2	181.0
4	45	2038	65	45	42	0.322	525.8	–21.2	504.6	420.5
5	50	2260	80	48	56	0.350	659.3	–15.0	644.2	537.0

$$C = 0.5959 + 0.31(\beta)^{2.1} - 0.184(\beta)^8 +$$

$$+ 0.029(\beta)^{2.5}\left(\frac{10^6}{\text{Re}}\right)^{0.75},$$

$$C = 0.6029,$$

$$\Delta h_{\acute{s}r} = \frac{\Delta h_1 + \dots + \Delta h_5}{5} =$$

$$= \frac{2 + 8 + 20 + 45 + 48}{5} = 0.0297 \text{ m},$$

$$\Delta p_{\acute{s}r} = \rho_{alc}\, g\, \Delta h_{\acute{s}r} = 240.50 \text{ Pa},$$

$$\varepsilon_1 = 1 - \left[0.41 + 0.35\beta^4\right]\frac{\Delta p_{\acute{s}r}}{\kappa \cdot p_1} = 0.9991,$$

$$K = 0.48 \text{ m}^{2.5} \cdot \text{s}^{-1},$$

$$\dot{V}_1 = K\sqrt{\Delta h_{mmalc}} = 0.3495 \text{ m}^3 \cdot \text{s}^{-1},$$

$$p_1 = p_b - h_1\, \rho_{alc}\, g = 98\,963 \text{ Pa},$$

$$p_s = p_b - h_s\, \rho_{alc}\, g = 98\,741 \text{ Pa},$$

$$\dot{V}_s = \dot{V}_1\frac{p_1}{p_s} = 0.3502 \text{ m}^3 \cdot \text{s}^{-1};$$

– calculation of total pressure impact Δp_c of the rotor and useful work:

$$\Delta p_c = \Delta p_{st} + \Delta p_d,$$

$$\Delta p_{st} = h_s\, \rho_{alc}\, g = 659.0 \text{ Pa},$$

$$\Delta p_d = \left(\frac{1}{2}\right)\rho_{\acute{s}r}\dot{V}_s'^2\left(\frac{1}{A_t^2} - \frac{1}{A_s^2}\right),$$

$$\rho_s = \rho_1\frac{p_s}{p_1} = 1.199 \text{ kg} \cdot \text{m}^{-3},$$

$$p_t = \rho_1\frac{p_b}{p_1} = 1.202 \text{ kg} \cdot \text{m}^{-3},$$

$$\rho_{\acute{s}r} = \frac{(\rho_s + \rho_t)}{2} 1.201 \text{ kg} \cdot \text{m}^{-3},$$

$$\Delta p_d = -24.5 \text{ Pa},$$

$$\Delta p_c = \Delta p_{st} + \Delta p_d = 634.5 \text{ Pa},$$

$$l_u = \frac{\Delta p_c}{\rho_{\acute{s}r}} = 529 \text{ m}^2 \cdot \text{s}^{-2}.$$

Figure 5 shows comparison of characteristics of rotor works obtained from thermoanemometric measurements "1" and balance measurements "2".
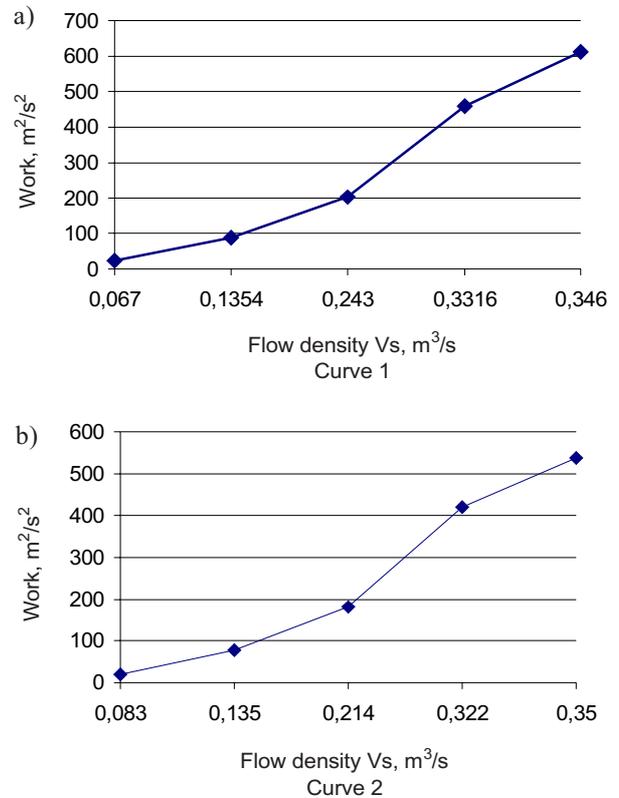
Fig. 5. Comparison of whirl work based on thermoanemometric measurements (a) and balance measurements (b)

The presented results prove that thermoanemometric measurements of flows through a radial machine determine reliable characteristics with precision to 10% when compared with the balance method. In this connection, these results with be adjusted in Part 2, taking into account losses occuring between sections "s-s" and "1-1" in order to obtain conformity of results derived from both methods.

Literature

[1] Fortuna S. i in.: *Badania prędkości i poziomu turbulencji w strugach powietrza wypływających z koła promieniowego do swobodnej oraz ograniczonej przestrzeni.* Kraków, 2003 (praca niepublikowana)
[2] Poleszczyk E.: *Termoanemometryczna metoda wyznaczania prędkości przepływu gazu.* Kraków, Prace Instytutu Mechaniki Górotworu PAN 2002
[3] Ligęza P.: *Pomiary termoanemometryczne w przepływach nieizotermicznych.* Pomiary, Automatyka, Kontrola, 1995
[4] Gumuła S., Prync-Skotniczny K., Fortuna S.: *Prędkość i turbulencja przepływu powietrza w otoczeniu kół łopatkowych wentylatorów promieniowych.* Materiały VIII Międzynarodowej Konferencji „Przepływowe Maszyny Wirnikowe", Politechnika Rzeszowska, Rzeszów, 1998
[5] PN-76/M-34034: *Zasady obliczania strat ciśnienia*