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Fuzzy Modelling of Thermal Systems with Distributed Parameters

1. Introduction

The research concerns the possibility of using fuzzy set theory and fuzzy logic for modeling of thermal phenomena of chosen electroheat plants – chamber resistance furnaces. Such devices are important in various technologies and widely used in industrial practice. Growing technological requirements along with inherent complexity of this class of dynamical systems imply a need to develop more and more advanced modeling and control methods. Even though several well-known methods are being used successfully for formal description of dynamics of such objects [5, 8], the simplifications introduced in these models become unacceptable at times. Therefore, searching for more sophisticated solutions constitutes a big challenge.

The classical approach to describe heat exchange in the analyzed class of objects is based on well-known physical laws [1, 9]. Many features of the plant, including its spatial nature (distributed parameters), can be taken into account in this way. Unfortunately taking into consideration all constructional details of the device becomes a very difficult task. Moreover, as parameters of materials the thermal plants are made of are temperature-dependent, this approach can lead to non-linear partial differential equations which are not convenient for control system design and implementation.

On the other hand transfer function parametric models are often used for simplified description of dynamic properties of thermal plants. Among others, first-order inertia transfer function with time delay plays an important role in this field [6, 8]. Easily interpretable parameters of such model become a good basis for controller design. Therefore, this type of modeling is sometimes considered as a reasonable compromise between complexity and adequacy of description. However for some advanced thermal technologies this modeling of thermal plants occurs to be not accurate enough.

A promising approach to modeling of electroheat objects arises from a possibility to combine simple and readable transfer function model with additional, qualitative information

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about nature of the object. For example, it is a well-known fact that the typical step input response of a chamber resistance furnace can be hardly approximated with high accuracy by the step input response of a single first-order inertia block, even if delay time is negligible. Instead, descriptive terms like "initial time constant" and "saturation time constant" are intuitively used by experienced engineers to express behavior of such objects in different thermal conditions. These terms are naturally related to a degree of thermal saturation of device elements, especially insulation walls. However identifying thermal states of such a plant and distinguishing the resulting two time constants cannot be straightforward because of imprecise nature of these terms. It makes fuzzy set theory and fuzzy logic appropriate tools for solving this problem.

In this paper the results of some introductory research on the possibility of the use of the two mentioned above time constants in a formal description of chamber furnace behavior have been presented. The main goal is to show the adequacy of the fuzzy model combining two first-order inertia transfer functions with different time constants for modeling of dynamic properties of a thermal plant whose main constructional elements constitute insulation walls. Performed simulations compare the result obtained using proposed fuzzy model with those resulting from one dimensional heat conduction low representing in simplified way the properties of insulation wall of the thermal plant.

2. Modeling of thermal plants

2.1. Using physical laws

As insulation walls, forming the furnace chamber, are main constructional elements of electric resistance furnaces, their properties determine the behavior of the entire plant. In order to describe furnace dynamics the temperature field inside the insulation layers should be taken into account. The heat flow through the furnace wall can be described by partial differential equations of parabolic type with proper boundary conditions [1, 9]. Neglecting many constructional detail, it can be regarded as the one-dimensional spatial model as follows:

$$\frac{d\vartheta(t,x)}{dt} = \frac{\lambda}{\rho c}\frac{\partial^2\vartheta(t,x)}{\partial x^2} \tag{1}$$

where:

- ϑ – temperature,
- λ – thermal conductivity,
- ρ – thermal density,
- c – specific heat of the insulation material.

For simulation purposes, the numerical solution of the above equation [9] can serve as a source of data for an analysis of thermal phenomena inside the thermal plant, taking into account distributed character of its parameters. Having temporary temperature distribution within insulation wall of the furnace the temperature of the inner layer of the wall can represent to some extent the temperature of the furnace chamber which is measured in a real plant by a temperature sensor.

2.2. Transfer function model of chamber furnace

Chamber electric resistance furnaces as industrial devices are often treated as lumped parameter systems especially while designing and tuning the dedicated temperature controller. In this case such a plant is considered as single-input single-output element where the total power $P(t)$ generated by heating elements placed on the furnace walls plays the role of the input signal and the temperature $\vartheta(t)$ measured by the sensor inside the furnace chamber constitutes its output. The relation between such defined signals is then typically expressed in the form of a transfer function, most often as first-order inertial with time delay [6]:

$$G(s) = \frac{\vartheta(s)}{P(s)} = \frac{K}{1 + sN} e^{-sL} \tag{2}$$

where:

K – gain of the thermal plant [K/W],
N – equivalent time constant [s],
L – equivalent time delay [s].

In many cases the main source of time delay L in (2) is a temperature sensor [5]. Sometimes, especially for the purpose of simplified analysis, it can be assumed that the temperature inside the furnace is measured by a sensor with negligibly small inertia. It enables to take $L = 0$ so that the formula (2) reduces to the form:

$$G(s) = \frac{K}{1 + sN} \tag{3}$$

As it was mentioned above models (2) and (3) are willingly used in practice because of their straightforward interpretation and understanding as well as because of many tuning rules existing for controller design. However in many technologies the accuracy of such models comparing to experimental results becomes unsatisfactory. Spatial nature of real thermal device can be found as one of the sources of this divergence.

2.3. Identification of temporary dynamic parameters of the furnace

The parameters of (3) can be in obvious way determined from the step input response of the first-order inertia element. When the response of a chamber resistance furnace is taken into account various parameters of (3) can be used to adjust the model, depending on the part of the furnace response being of interest. On the other hand step response of the furnace exhibits a wide variety of thermal states of the plant. This problem can be illustrated more specifically by the simulation analysis in which the step input response resulting from simplified model of thermal phenomena inside an insulation wall (denoted as $\vartheta(t)$[1]) is piecewise approximated by this obtained from equation (3) (denoted as $\vartheta^{(m)}(t)$). For this purpose the output of first-order inertia element is expressed in a discrete manner as follows [2]:

$$\vartheta_k^{(m)} = w_k \theta$$
$$\theta = \left[K\left(1 - e^{-\frac{\Delta}{N}}\right), e^{-\frac{\Delta}{N}} \right]^T \tag{4}$$
$$w_k = \left[P_{k-1}, \vartheta_{k-1}\right]$$

where:

- $\vartheta_k^{(m)}$ – output value of first-order inertia model in time step k,
- θ – vector of model parameters,
- w_k – model input vector,
- Δ – time interval.

For given time step r having set of pairs $\{\vartheta_i, P_i\}_r$ $i = r \ldots r+n$, resulting from model (1) it is possible to determine by the least square estimation the vector of model parameters θ_r. This vector provides the best local representation of the wall temperature by model (4) in the r-th time step and using interval comprising n samples. Static gain K_r and time constant N_r of the object obtained for an appropriately small value of n can be regarded as instantaneous dynamic parameters of the thermal plant in r-th time step. Figures 1 and 2 show changes of these parameters as a function of r and n.

It can be seen that for different ranges of the signal being approximated the calculated parameters N and K differ significantly. Lower values are obtained for the initial part of furnace step response than for its final part. It explains to some extent the existence of mentioned above descriptive terms: "initial time constant" and "saturation time constant". The results show also that these terms are associated with a varying degree of thermal saturation of the components of the object.

Qualitative analysis presented above encourages developing such a model of thermal plant in which this specific information about operation conditions of the plant would be used to enrich simple description based on a first-order inertia model.

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[1] as the signal $\vartheta(t)$ the temperature of inner (heated) surface of the insulation wall has been chosen, being representative of temperature of the furnace chamber.

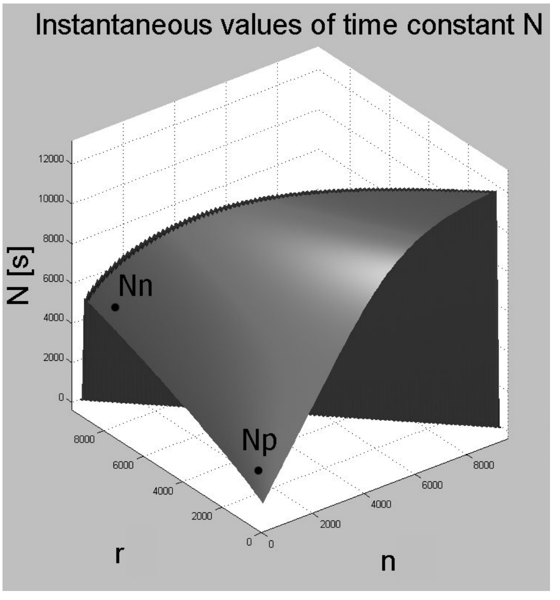

Fig. 1. Instantaneous values of time constant N as a function of r and n where respectively r is the beginning and n is length of the interval in which the value N was calculated. N_n and N_p are initial values used as a start point for the genetic algorithm

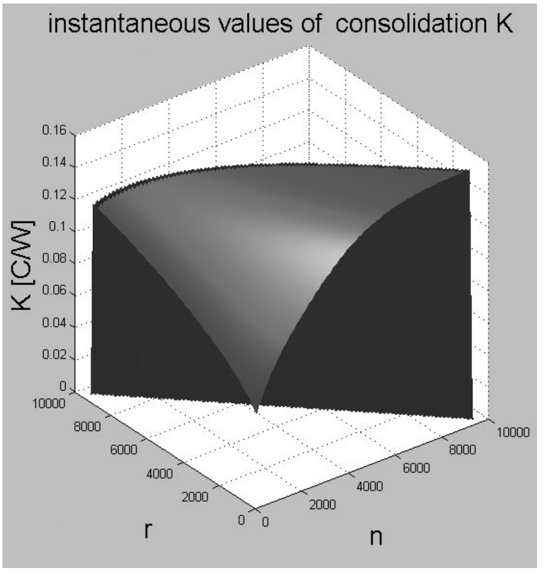

Fig. 2. Instantaneous values of gain K as a function of r and n where respectively r is the beginning and n is the length of the interval in which the value K was calculated

3. Fuzzy modelling of dynamic properties of the furnace

3.1. Model description

Changes of the parameter *N*, resulting from various thermal states of the plant, can be qualitatively described by linguistic terms mentioned above. This is a good basis for the use of fuzzy set theory and fuzzy logic to incorporate into a model this kind of additional, imprecise information. Fuzzy system enables a combination of two inertial first-order transfer functions of "initial time constant" and "saturation time constant" respectively. The first one is responsible for the initial thermal state of the plant while the latter for the saturation state. Applied fuzzy inference machine assures smooth transition from one block to another when the thermal state of the plant gradually changes. Such a model has been proposed in [3, 4]. The thermal state of the plant can be identified basing on the observation of temperature changes $d\vartheta$ in the consecutive time steps of step input response. Near the thermal saturation state the changes in temperature are smaller then in the unsaturated state. In the space of temperature changes two fuzzy sets *small* and *big* can be defined representing imprecise terms describing thermal state of the plant. These sets are defined by the formulas (5) and are shown in Figure 3.

$$\mu_{small}(d\vartheta)=\begin{cases} 1 & \text{for} \quad d\vartheta \le a \\ \dfrac{b-d\vartheta}{b-a} & \text{for} \quad a < d\vartheta \le b \\ 0 & \text{for} \quad d\vartheta > b \end{cases}$$
$$\mu_{big}(d\vartheta)=\begin{cases} 0 & \text{for} \quad d\vartheta \le a \\ \dfrac{d\vartheta-a}{b-a} & \text{for} \quad a < d\vartheta \le b \\ 1 & \text{for} \quad d\vartheta > b \end{cases} \tag{5}$$

where:

$\mu_{small}(d\vartheta)$ – membership function of the fuzzy set "small change in temperature",

$\mu_{big}(d\vartheta)$ – membership function of the fuzzy set "big change in temperature",

$d\vartheta$ – Temperature change in consecutive time steps.

It is then possible to formalize the description of thermal plant dynamics, including imprecise information concerning its behavior in various thermal states, using a fuzzy model in the form of Takagi-Sugeno-Kanga system (e.g. [7]).

This model enables to combine two first-order transfer functions in one structure assuring smooth transition between them. Taking into account two time constants: the initial N_p and saturation N_n, and using (4) the following rules can be created:

$$IF \quad d\vartheta \quad is \quad big \quad THEN \quad \vartheta_k = w_k \theta_p$$

$$IF \quad d\vartheta \quad is \quad small \quad THEN \quad \vartheta_k = w_k \theta_n$$

$$\theta_p = \left[K \left(1 - e^{-\frac{\Delta}{N_p}} \right), e^{-\frac{\Delta}{N_p}} \right]^T \tag{6}$$

$$\theta_n = \left[K \left(1 - e^{-\frac{\Delta}{N_n}} \right), e^{-\frac{\Delta}{N_n}} \right]^T$$

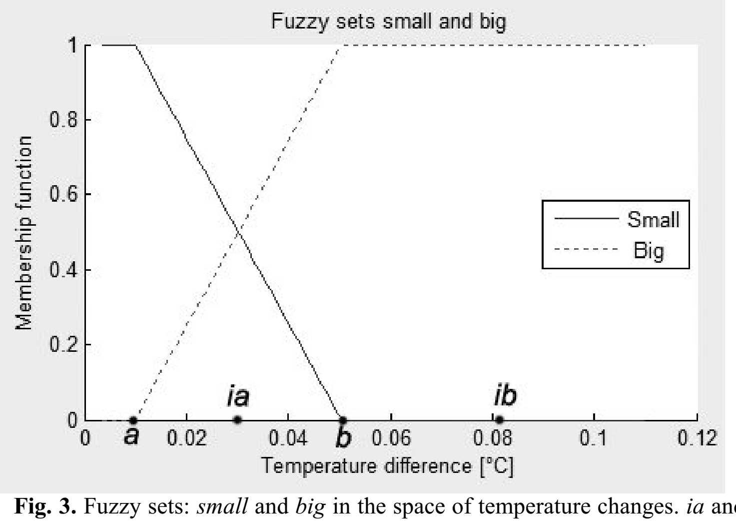

Fig. 3. Fuzzy sets: *small* and *big* in the space of temperature changes. *ia* and *ib* are initial parameters *a* and *b* used in genetic optimization

Implementing typical forms of logical operations in (6) [7] the final version of the model can be expressed as:

$$\vartheta_k = w_k [\mu_{big}(d\vartheta_k) \cdot \theta_p + \mu_{small}(d\vartheta_k) \cdot \theta_n] \tag{7}$$

3.2. Optimization of model parameters

Least squares estimation of the parameters of the model (7) requires solution of the following optimization problem:

$$\{a, b, N_p, N_n\} = \arg \min_{a,b,N_p,N_n} \sum_{k=1}^{n} (\vartheta_k - w_k [\mu_{big}(d\vartheta_k) \cdot \theta_p + \mu_{small}(d\vartheta_k) \cdot \theta_n])^2 \tag{8}$$

Since the solution of (8) using Matlab *lsqnonlin* function were not satisfactory genetic optimization methods were adopted. Evolutionary algorithm with real coding was applied with roulette wheel selection method, single point crossing with probability 0.9 and Gaussian mutation with probability of 0.02. Number of individuals in the population was 60 and the number of generations 300. Initial values of N_p, N_n, a, b given in Table 1 were chosen basing on the analysis presented in sec. 2.3. The approximation quality was assessed using various numbers of data from the entire step input response. The initial parameters of defined fuzzy sets and its best values achieved by genetic algorithm are marked on Figure 3.

Table 2 shows the obtained values of model parameters for three chosen cases. It can be noticed that the evolutionary optimization is quite insensitive to amount of data used for the error calculation.

Table 1

Initial values of fuzzy model parameters

a [°C]	b [°C]	N_p [s]	N_n [s]
0.03	0.08	2800	5300

Table 2

The set of best values obtained for the parameters and the average error of a single data point for different number of points taken into account error counting.

Number of points taken into account in the counting error	a [°C]	b [°C]	N_p [s]	N_n [s]	Average error of a single point
1000	0.01	0.05	2051	7138	0.39
333	0.011	0.053	1893	6958	0.61
200	0.01	0.061	1682	7170	0.54

4. Application created for modeling of electroheat objects

The application designed and implemented for the research calculates and stores the results of various types of electroheat objects models. For future work the application is also ready to import and process data resulting from experiments performed on the real thermal plants: industrial and semi-industrial chamber electric resistance furnaces available in laboratory of Computer Engineering Department, TUL. Temperature signals originating from various above mentioned sources can be portrayed in the two or three charts of the application window shown in Figure 5. The main application functionality, illustrated by the activity diagram shown in Figure 4, covers numerical calculation of the insulation wall temperature basing on its physical parameters, calculation of first-order inertia model and proposed fuzzy model (7), as well as optimization procedure using evolutionary algorithm.

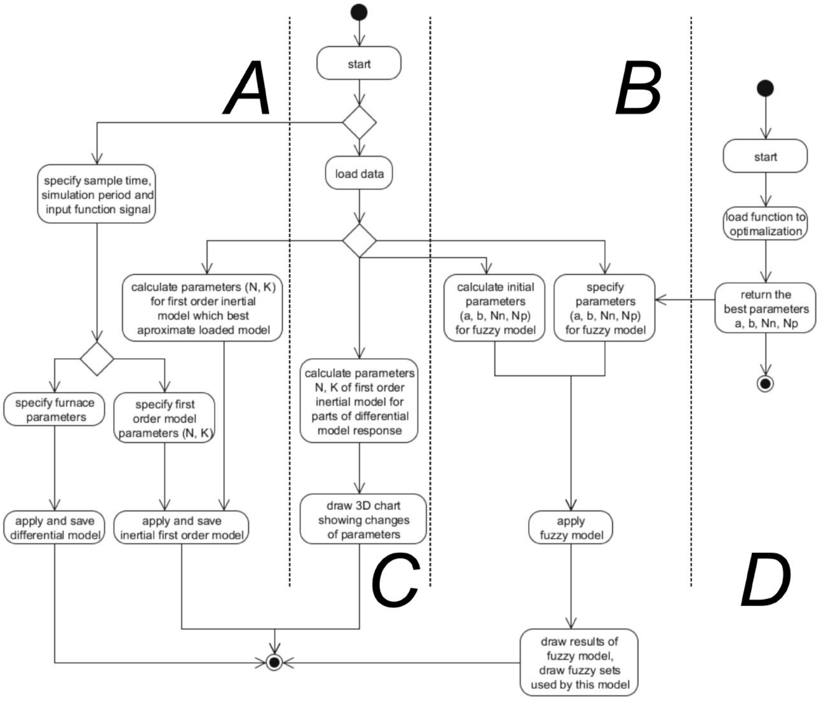

Fig. 4. An activity diagram of the application created for modeling of electroheat object. Part *A* represents data generation using different models. Part *B* applies fuzzy model. Part *C* calculates or draw 3D charts. Part *D* illustrates an outside genetic algorithm

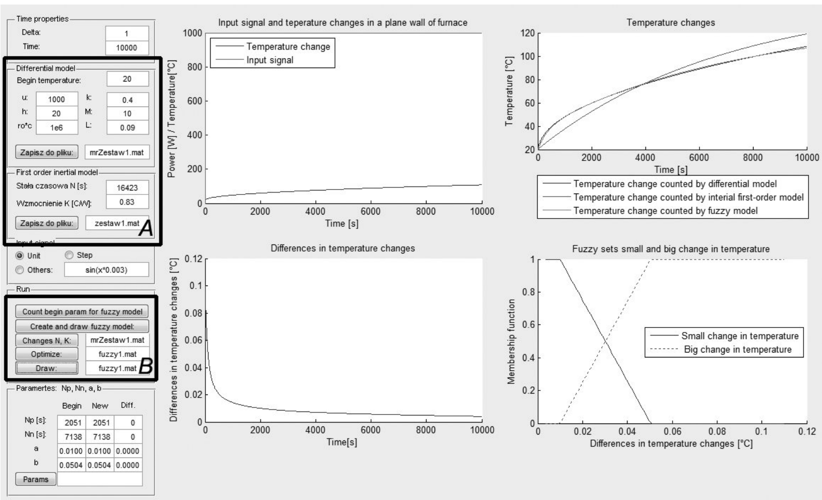

Fig. 5. View of the main application window. The frame with letter *A* shows the part of data generation later used as an input data, frame with letter *B* shows part of counting results by the implemented fuzzy model

5. Results

Several simulation experiments have been performed using the prepared application. In order to assess the adequacy of the proposed fuzzy model of thermal objects its results have been compared to those of distributed parameter model of insulation wall (1) as well as to a single first-order inertia transfer function.

Various sets of material parameters as well as transfer function and fuzzy model parameters were examined. Chosen examples of the step input responses of analyzed models are shown in Figure 6 and corresponding temperature errors are given in Figure 7.

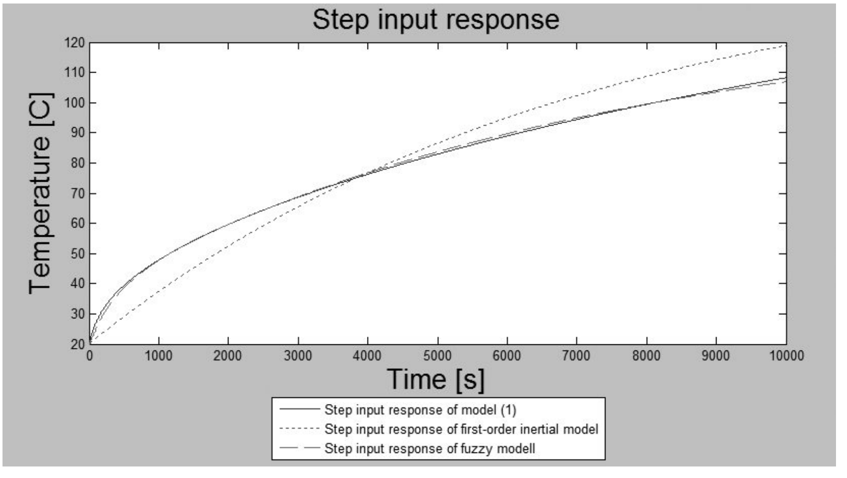

Fig. 6. Temperature signals yielded by various models of thermal plant

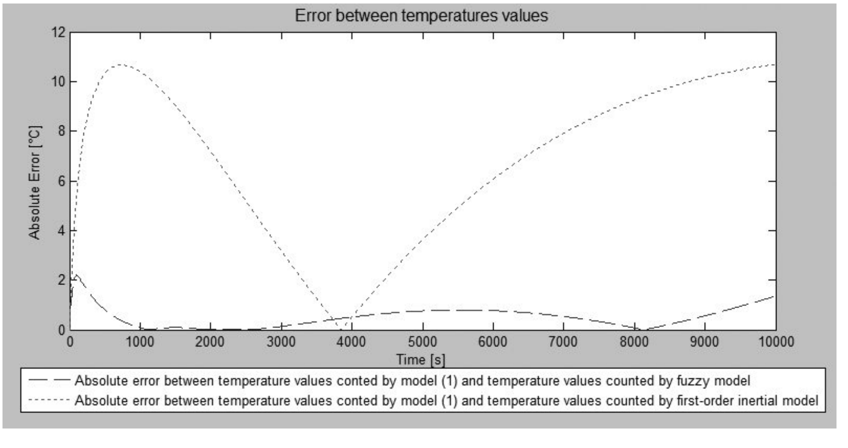

Fig. 7 . Error between the values obtained using the model (1) compering to the values obtained using first-order inertia model and the proposed fuzzy model

It is noticeable that the graphs of the temperature signal calculated basing on model (1) and obtained from the fuzzy model (7) almost overlap whereas first-order transfer function model gives unsatisfactory results. It proves that proposed fuzzy modeling can be a very promising approach to describe the properties of thermal devices with distributed parameters.

6. Conclusions

Fuzzy approach to modeling of dynamic properties of thermal systems has been proposed. This approach enables to take into account the influence of distributed parameters of a real thermal system on its dynamic behavior keeping relatively simple structure of the model. The qualitative description of thermal plant in various working condition (expressed in practice by linguistic terms: "initial time constant" and "saturation time constant") can be formalized using Takagi-Sugeno-Kang fuzzy structure. The antecedents of IF-THEN rules refer to different thermal states of insulation walls of the plant while their consequences realize I-st order inertia dynamics including both "initial" and "saturation" time constants. It has been proved that such an approach enables to describe with high accuracy the behavior of a distributed parameters thermal system in various thermal conditions. The fuzzy model can also be used for quantitative identification of two characteristic time constants of the thermal plant basing on its step input response.

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